## Cosmological Hydrogen Recombination: The effect of extremely high-n states

arXiv:0911.1359, submitted to Phys. Rev. D.
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## OUTLINE

* Motivation: CMB anisotropies and recombination spectra
* Recombination in a nutshell
* Breaking the Peebles/RecFAST mold
* RecSparse: a new tool for high-n states
* Forbidden transitions
* Results
* Ongoing/future work


## WALK THE PLANCK



* Planck (launched May 2009) will make cosmic-variance limited CMB anisotropy measurements up to $1 \sim 2500$ (T), and l~1500 (E)
* Wong 2007 and Lewis 2006 show that $x_{e}(z)$ needs to be predicted to several parts in $10^{4}$ accuracy for Planck data analysis


## RECOMBINATION, INFLATION, AND REIONIZATION

$$
P(k)=A_{s}\left(k \eta_{0}\right)^{n_{s}}
$$

* Planck uncertainty forecasts using MCMC




* Cosmological parameter inferences will be off if recombination is improperly modeled (Wong/Moss/Scott 2007)
* Leverage on new physics comes from high 1. Here the details of recombination matter!
* Inferences about inflation will be wrong if recombination is improperly modeled

$$
\mathrm{n}_{s}=1-4 \epsilon+2 \eta \quad \epsilon=\frac{m_{\mathrm{pl}}^{2}}{16 \pi}\left[\frac{V^{\prime}(\phi)}{V(\phi)}\right]^{2} \quad \mathrm{~A}_{s}^{2}=\left.\frac{32}{75} \frac{V}{m_{\mathrm{pl}}^{4} \epsilon}\right|_{k_{\mathrm{pivot}}=a H}
$$

$$
\text { CAVEAT EMPTOR: } 3 \lesssim ? \lesssim 16
$$

Need to do eV physics right to infer anything about $10^{?} \mathrm{GeV}$ physics! 4

## RECOMBINATION, INFLATION, AND REIONIZATION

* Planck uncertainty forecasts using MCMC



Bad recombination history yields biased inferences about reionization

## PHYSICAL RELEVANCE FOR CMB:

## SmEARING AND MOVING THE SURFACE OF LAST SCATTERING

 (SLS)* Photons kin. decouple when Thompson scattering freezes out

$$
\gamma+e^{-} \Leftrightarrow \gamma+e^{-}
$$

* Acoustic mode evolution influenced by visibility function

$$
g=\dot{\tau} e^{-\tau} \quad \tau(z)=\int_{0}^{\eta(z)} n_{e} \sigma_{T} a\left(\eta^{\prime}\right) d \eta^{\prime}
$$

* $z_{\mathrm{dec}} \simeq 1100$ : Decoupling occurs during recombination

$$
C_{l} \rightarrow C_{l} e^{-2 \tau(z)} \text { if } l>\eta_{\text {dec }} / \eta(z)
$$

## PHYSICAL RELEVANCE FOR CMB:

## SMEARING AND MOVING THE SURFACE OF LAST SCATTERING

 (SLS)* Photons kin. decouple when Thompson scattering freezes out

$$
\gamma+e^{-} \Leftrightarrow \gamma+e
$$



Relic electrons from rec


## PHYSICAL RELEVANCE FOR CMB: The Silk Damping Tail



* Inhomogeneities are damped for $\lambda<\lambda_{D}$


## PHYSICAL RELEVANCE FOR CMB: POLARIZATION

Isotropic radiation

Quadrupole moment

No polarization
Polarization
From Wayne Hus's website

* Need time to develop a quadrapole
$\Theta_{l}(k \eta) \sim \frac{k \eta}{2 \tau} \Theta_{l}(k \eta) \ll \Theta_{l}(\eta)$ if $l \geq 2$, in tight coupling regime
* Need to scatter quadrapole to polarize CMB


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## PHYSICAL RELEVANCE FOR CMB: SPECTRAL DISTORTIONS FROM RECOMBINATION



## SAHA EQUILIBRIUM IS INADEQUATE

$$
p+e^{-} \leftrightarrow H^{(n)}+\gamma^{(n c)}
$$

* Chemical equilibrium does reasonably well predicting "moment of recombination"

$$
\frac{x_{e}^{2}}{1-x_{e}}=\left(\frac{13.6}{T_{\mathrm{eV}}}\right)^{3 / 2} e^{35.9-13.6 / T_{\mathrm{eV}}}
$$

$$
x_{e}=0.5 \text { when } T=T_{\text {rec }} \simeq 0.3 \mathrm{eV} \quad z_{\mathrm{rec}} \simeq 1300
$$

*Further evolution falls prey to reaction freeze-out

$$
\Gamma<H \text { when } T<T_{\mathrm{F}} \simeq 0.25 \mathrm{eV}
$$

## BOTTLENECKS/ESCAPE ROUTES

BOTTLENECKS

* Ground state recombinations are ineffective

$$
\Gamma_{c \rightarrow 1 s}=10^{-1} \mathrm{~s}^{-1} \gg H \simeq 10^{-12} \mathrm{~s}^{-1}
$$

*Resonance photons are re-captured, e.g. Lyman $\alpha$

$$
\Gamma_{2 p \rightarrow 1 s}=10^{-2} \mathrm{~s}^{-1} \gg H \simeq 10^{-12} \mathrm{~s}^{-1}
$$

ESCAPE ROUTES (e.g. n=2)

* Two-photon processes

$$
H^{2 s} \rightarrow H^{1 s}+\gamma+\gamma \quad \Lambda_{2 \mathrm{~s} \rightarrow 1 \mathrm{~s}}=8.22 \mathrm{~s}^{-1}
$$

* Redshifting off resonance

$$
R \sim\left(n_{\mathrm{H}} \lambda_{\alpha}^{3}\right)^{-1} H
$$

## THE PEEBLES PUNCHLINE

* Only n=2 bottlenecks are treated
*Net Rate is suppressed by bottleneck vs. escape factor

$$
-\frac{d x_{e}}{d t}=\mathcal{S} \sum_{n, l>1 s} \alpha_{n l}(T)\left\{n x_{e}^{2}-x_{1 s} f(T)\right\}
$$

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\text { Recombination rate }
\end{gathered}
$$

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lonization rate

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$$

## THE PEEBLES MODEL

*Net Rate is suppressed by bottleneck vs. escape factor
$\mathcal{S}=\frac{\frac{8 \pi}{\lambda_{\alpha}^{3} n_{1 s}} H+\Lambda_{2 s \rightarrow 1 s}}{\frac{8 \pi}{\lambda_{\alpha}^{3} n_{1 s}} H+\left(\Lambda_{2 s \rightarrow 1 s}+\beta_{c}\right)}$

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Redshifting term

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$$

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$$

## THE PEEBLES MODEL

*Net Rate is suppressed by bottleneck vs. escape factor
$\mathcal{S}=\frac{\frac{8 \pi}{\lambda_{\alpha}^{3} n_{1 s}} H+\Lambda_{2 s \rightarrow 1 s}}{\frac{8 \pi}{\lambda_{\alpha}^{3} n_{1 s}} H+\left(\Lambda_{2 s \rightarrow 1 s}+\beta_{c}\right)}$ Ionization Term

$$
\frac{\text { redshift term }}{2 \gamma \text { term }} \simeq 0.02 \frac{\Omega_{m}^{1 / 2}}{\left(1-x_{e}[z]\right)\left(\frac{1+z}{1100}\right)^{3 / 2}}
$$

$2 \gamma$ process dominates until late times $(z \lesssim 850)$

## THE PEEBLES MODEL

* Peebles 1967: State of the Art for 30 years!



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## EQUILIBRIUM ASSUMPTIONS

*Radiative/collisional eq. between different 1

$$
\mathcal{N}_{n l}=\mathcal{N}_{n} \frac{(2 l+1)}{n^{2}}
$$

* Radiative eq. between different n-states

$$
\mathcal{N}_{n}=\sum_{l} \mathcal{N}_{n l}=\mathcal{N}_{2} e^{-\left(E_{n}-E_{2}\right) / T}
$$

*Matter in eq. with radiation due to Thompson scattering

$$
T_{m}=T_{\gamma} \text { since } \frac{\sigma_{\mathrm{T}} a T_{\gamma}^{4} c}{m_{e} c^{2}}<H(T)
$$

## EQUILIBRIUM ASSUMPTIONS

*Radiative/collisional eq. between different 1

$$
\mathcal{N}_{n l}=\mathcal{N}_{n} \frac{(2 l+1)}{n^{2}}
$$

## Seager/Scott/Sasselov 2000/RECFAST!

* Radiative eq. between different n-states

$$
\mathcal{N}_{n}=\sum_{l} \mathcal{N}_{n l}=\mathcal{N}_{2} e^{-\left(E_{n}-E_{2}\right) / T}
$$

Non-eq rate equations
*Matter in eq. with radiation due to Thompson scattering

$$
T_{m}=T_{\gamma} \text { since } \frac{\sigma_{\mathrm{T}} a T_{\gamma}^{4} c}{m_{e} c^{2}}<H(T)
$$

## BREAKING EQUILIBRIUM

* Chluba et al. $(2005,6)$ follow $1, \mathrm{n}$ separately, get to $n_{\max }=100$
* 0.1 \%-level corrections to CMB anisotropies at $n_{\max }=100$
* Equilibrium between $l$ states: $\Delta l= \pm 1$ bottleneck
* Beyond this, testing convergence with $n_{\max }$ is hard!

```
t _ { \text { compute } } ^ { \sim } \sim \mathcal { O } \text { (years) for } \mathrm { n } _ { \operatorname { m a x } } = 3 0 0
```

How to proceed if we want $\mathcal{O}(1) \times 10^{-4}$ accuracy in $C_{\ell}$ ?

## THESE ARE REAL STATES

* Still inside plasma shielding length for $\mathrm{n}<100000$
* $r \sim a_{0} n^{2}$ is as large as $2 \mu \mathrm{~m}$ for $n_{\max }=200$
* $\frac{\left.\Delta E\right|_{\text {thermal }}}{E}<\frac{2}{n^{3}}$
* Similarly high n are seen in emission line nebulae


## THE EFFECT OF RESOLVING $l$ - SUBSTATES

## Resolved I vs unresolved I



* 'Bottlenecked' l-substates decay slowly to 1s: Recombination is slower; Chluba al. 2006


Continuum


* We implement a multi-level atom computation in a new code, RecSparse!
* Boltzmann eq. solved for $\mathrm{T}_{m}\left(T_{\gamma}\right)$
* Spontaneous/stimulated emission/absorption included

RECSPARSE AND THE MULTI-LEVEL ATOM
l=0


Continuum


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## RECSPARSE AND THE MULTI-LEVEL ATOM



Continuum
Other bound states
$n \uparrow$

$2 s \rightarrow 1 s, 2 \gamma$ transition

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## RECSPARSE AND THE MULTI-LEVEL ATOM

*Free electron fraction evolved according to

$$
\begin{aligned}
\dot{x}_{e} & =-\dot{x}_{1 s} \\
& =-\Lambda_{2 s \rightarrow 1 s}\left(x_{2 s}-x_{1 s} e^{-E_{2 s \rightarrow 1 s} / T_{\gamma}}\right)+\sum_{n, l>1 s} A_{n 1}^{l 0} P_{n 1}^{l 0}\{g(T, n, l)\}
\end{aligned}
$$

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& 2 \text { 2s-1s decay rate }
\end{aligned}
$$

## RECSPARSE AND THE MULTI-LEVEL ATOM

* Free electron fraction evolved according to

$$
\begin{aligned}
& \dot{x}_{e}=-\dot{x}_{1 s} \\
&=-\Lambda_{2 s \rightarrow 1 s}\left(x_{2 s}-x_{1 s} e^{-E_{2 s \rightarrow 1 s} / T_{\gamma}}\right)+\sum_{n, l>1 s} A_{n 1}^{l 0} P_{n 1}^{l 0}\{g(T, n, l)\} \\
& \text { Einstein coeff. }
\end{aligned}
$$

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& \text { Escape probability }
\end{aligned}
$$

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& \text { Lyman series current to ground state }
\end{aligned}
$$

## RADIATION FIELD: BLACK BODY +

* Escape probability treated in Sobolev approx: depends on steady-state and incoherent scattering approximations

$$
P_{n, n^{\prime}}^{l, l^{\prime}}=\frac{1-e^{-\tau_{s}}}{\tau_{s}}
$$

$$
\tau_{s} \propto \frac{n_{\mathrm{H}} x_{n}^{l} A_{n n^{\prime}}^{l l^{\prime}}}{H(z)} \quad n^{\prime}>n
$$

* RecSparse includes radiative feedback
* Ongoing work in field focuses on corrections to simple radiative transfer picture
* Ultimate goal is to combine all new atomic physics effect in one fast recombination code


## RADIATION FIELD: BLACK BODY +

* Escape probability treated in Sobolev approx: depends on steady-state and incoherent scattering approximations


## Resonant absorber density

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## RADIATION FIELD: BLACK BODY +

* Escape probability treated in Sobolev approx: depends on steady-state and incoherent scattering approximations


## Cosmological expansion

$$
\tau_{s} \propto \frac{n_{\mathrm{H}} x_{n}^{l} A_{n n^{\prime}}^{l l^{\prime}}}{H(z)^{\prime}} \quad n^{\prime}>n
$$

* RecSparse includes radiative feedback
* Ongoing work in field focuses on corrections to simple radiative transfer picture
* Ultimate goal is to combine all new atomic physics effect in one fast recombination code


## STEADY-STATE FOR EXCITED LEVELS

* Evolution equations may be re-written in matrix form

$$
\frac{d \vec{x}}{d t}=\mathbf{R} \vec{x}+\vec{s}
$$

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$$
\begin{gathered}
\frac{d \vec{x}}{d t}=\mathbf{R} \vec{x}+\vec{s} \\
\\
\vec{x}_{l}=\left(\begin{array}{c}
x_{l, l+1} \\
\cdots \\
x_{l, n_{\max }}
\end{array}\right) \\
\stackrel{\rightharpoonup}{x}=\left(\begin{array}{c}
\vec{x}_{0} \\
\vec{x}_{1} \\
\cdots \\
\vec{x}_{l} \\
\cdots \\
\vec{x}_{l_{\max }}
\end{array}\right)
\end{gathered}
$$

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For state 1, includes BB transitions out of 1 to all other 1", photo-ionization, $2 \gamma$ transitions to ground state

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## STEADY-STATE FOR EXCITED LEVELS

* Evolution equations may be re-written in matrix form

$$
\frac{d \vec{x}}{d t}=\mathbf{R} \vec{x}+\hat{s}
$$

Includes recombination to 1 , 1 and $2 \gamma$ transitions from ground state

## STEADY-STATE FOR EXCITED LEVELS

* Evolution equations may be re-written in matrix form

$$
\frac{d \vec{x}}{d t}=\underbrace{\mathbf{R} \vec{x}+\vec{s}}_{\text {For } \mathrm{n}>1, \mathbf{R}, \vec{s} \geq 1 \mathrm{~s}^{-1} \text { e.g. Lyman- } \alpha}
$$

## STEADY-STATE FOR EXCITED LEVELS

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## RAPID MATRIX INVERSION: SPARSITY TO THE RESCUE

* Matrix is $\sim n_{\max }^{2} \times n_{\max }^{2}$
* Brute force would require $\mathrm{A} n_{\max }^{6} \sim 10^{5} \mathrm{~s}$ for $n_{\max }=200$ for a single time step
* Dipole selection rules: $\Delta l= \pm 1$ $\mathrm{M}_{l, l-1} \vec{x}_{l-1}+\mathbf{M}_{l, l} \vec{x}_{l}+\mathbf{M}_{l, l+1} \vec{x}_{l+1}=\vec{s}_{l}$

* Physics imposes sparseness on the problem. Solved in closed form to yield algebraic $\vec{x}_{l_{\max }}$, then $\vec{x}_{l}$ in terms of $\vec{x}_{l+1}$


## RAPID MATRIX INVERSION: SPARSITY TO THE RESCUE

* Einstein coefficients to states with $n>n_{\max }$ are set $A=0$ : more later!
* RecSparse generates rec. history with computation time $\sim \mathrm{n}_{\max }{ }^{2.5}$ : Huge improvement!
* Case of $n_{\max }=100$ runs in less than a day, $n_{\max }=200$ takes $\sim 4$ days.


## FORBIDDEN TRANSITIONS AND RECOMBINATION

* Higher-n $2 \gamma$ transitions in H important at 7- $\sigma$ for Planck (TT/EE) data analysis (Hirata 2008, Kholupenko 2006)
* Some forbidden transitions are important in Helium recombination (Dubrovich 2005, Lewis 2006) and would bias cosmological parameter estimation.
* Are other forbidden transitions in hydrogen important, particularly for Planck data analysis? How about electric quadrupole (E2) transitions?


## QUADRUPOLE TRANSITIONS AND RECOMBINATION

* Ground-state electric quadrupole (E2) lines are optically thick!

$$
\begin{gathered}
P_{n, n^{\prime}}^{l, l^{\prime}}=\frac{1-e^{-\tau_{s}}}{\tau_{s}} \\
R \propto A P \propto A / \tau \text { if } \tau \gg 1 \\
\tau \propto A \rightarrow R \rightarrow A / A \rightarrow \text { const }
\end{gathered}
$$

* Coupling to ground state will overwhelmingly dominate:

$$
\frac{A_{n, 2 \rightarrow 1,0}^{\text {quad }}}{A_{n, 2 \rightarrow m, 0}^{\text {quad }}} \propto \frac{\omega_{n 1}^{5}}{\omega_{n m}^{5}} \geq 10^{3} \text { if } m \geq 2
$$

## QUADRUPOLE TRANSITIONS AND RECOMBINATION

* Lyman lines are optically thick, so $n d \rightarrow 1 s$ immediately followed by $1 s \rightarrow n p$, so this can be treated as an effective $d \rightarrow p$ process with rate $A_{n d \rightarrow 1 s} x_{n d}$.
* Same sparsity pattern of rate matrix, similar to l-changing collisions
* Detailed balance yields net rate

$$
R_{n d \rightarrow n p}^{\mathrm{quad}}=A_{n d \rightarrow 1 s}\left(x_{n d}-\frac{5}{3} x_{n p}\right)
$$

## RESULTS: STATE OF THE GAS

## DEVIATIONS FROM BOLTZMANN EQ: L-SUBSTATES

RecSparse outpuł

$$
\begin{aligned}
& \frac{\Delta x_{f i}^{e q}}{x_{f l}^{\text {ef }}} \\
& (\text { in \%) }
\end{aligned}
$$



## DEVIATIONS FROM BOLTZMANN EQ: L-SUBSTATES

RecSparse outpuł

$$
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DEVIATIONS FROM BOLTZMANN EQ: L-SUBSTATES
RecSparse
outpuł

$$
\begin{aligned}
& \frac{\Delta x_{\text {eq }}^{e q}}{x_{n l}^{e q}} \\
& (\text { in \%) }
\end{aligned}
$$

## I=0 can't cascade down, so s states are not as under-populated

## DEVIATIONS FROM BOLTZMANN EQ: L-SUBSTATES

RecSparse outpuł

$$
\begin{aligned}
& \frac{\Delta x_{f i}^{e q}}{x_{f l}^{e q}} \\
& (\text { in \%) }
\end{aligned}
$$



Higher I are bottlenecked by $\Delta l= \pm 1$ (over-pop)

## DEVIATIONS FROM BOLTZMANN EQ: L-SUBSTATES

RecSparse outpuł


Highest I states recombine inefficiently, and are under-populated

## DEVIATIONS FROM BOLTZMANN EQ: L-SUBSTATES

RecSparse outpuł

$$
\begin{aligned}
& \frac{\Delta x_{f i}^{g_{i}}}{x_{f i}^{\text {ef }}} \\
& (\text { in \%) }
\end{aligned}
$$


l-substates are highly out of Bolizmann eqb'm at late times

## DEVIATIONS FROM BOLTZMANN EQ: L-SUBSTATES

RecSparse outpuł

$$
\begin{aligned}
& \frac{\Delta x_{f q}^{e q}}{x_{f l}^{\text {eid }}} \\
& (\text { in \%) }
\end{aligned}
$$



## WHAT IS THE ORIGIN OF THE $l=2$ DIP?

$$
A_{\mathrm{nd} \rightarrow 2 \mathrm{p}}>A_{\mathrm{np} \rightarrow 2 \mathrm{~s}}>A_{\mathrm{ns} \rightarrow 2 \mathrm{p}}
$$

* $\mathrm{l}=2$ depopulates more rapidly than $\mathrm{l}=1$ for higher ( $\mathrm{n}>2$ ) excited states
* We can test if this explains the dip at $1=2$ by running the code with these Balmer transitions the blip should move to $=1$


## L-SUBSTATE POPULATIONS, BALMER LINES OFF



Dip moves as expected when Balmer lines are off!

## ATOMIC COLLISIONS



* 1-changing collisions bring 1 -substates closer to statistical equilibrium (SE) (Chluba, Rubino Martin, Sunyaev 2006)
* Theoretical collision rates unknown to factors of 2 !


## DEVIATIONS FROM BOLTZMANN EQUILIBRIUM: DIFFERENT $n$-SHELLS




* No inversion relative to $\mathrm{n}=2$ (just over-population)
* Population inversion seen between some excited states: Does radiation stay coherent? Does recombination mase?


## DEVIATIONS FROM BOLTZMANN EQUILIBRIUM: DIFFERENT $n$-SHELLS



Masing could make spectral
distortions detectable!

## DEVIATIONS FROM SAHA EQUILIBRIUM



## HUGE DEVIATIONS FROM SAHA EQ!

* Effect of states with $\mathrm{n}>\mathrm{n}_{\max }$ could be approximated using asymptotic Einstein coeffs. and Saha eq, but Saha is elusive at high n /late times.
* At $\mathrm{z}=200, \mathrm{n}_{\max } \sim 1000$ needed, unless collisions included


## RESULTS: RECOMBINATION HISTORIES

## RESULTS: RECOMBINATION HISTORIES INCLUDING HIGH- $n$



* $x_{e}(z)$ falls with increasing $n_{\max }=10 \rightarrow 250$, as expected.
* Rec Rate>downward BB Rate> Ionization, upward BB rate
* For $n_{\max }=100$, code computes in only 2 hours


## RESULTS: RECOMBINATION HISTORIES INCLUDING HIGH- $n$



* Relative convergence is not the same thing as absolute convergence: Want to see Saha asymptote and impose well-motivated cutoff! Collisions could help
* These are lower limits to the actual error
* $\mathrm{n}_{\max }=300$ under way to further test convergence (more time consuming)


## RESULTS: RECOMBINATION WITH HYDROGEN



## BUILDING INTUITION FOR THE EFFECT OF E2 TRANSITIONS

$$
R_{n d \rightarrow n p}^{\text {quad }}=A_{n d \rightarrow 1 s}\left(x_{n d}-\frac{5}{3} x_{n p}\right)
$$

$n<5$, early times

## BUILDING INTUITION FOR THE EFFECT OF E2 TRANSITIONS



Sub-Dominant decay channel to gs, slows rec down rel. to $n<5$

$$
R_{n d \rightarrow n p}^{\mathrm{quad}}=A_{n d \rightarrow 1 s}\left(x_{n d}-\frac{5}{3} x_{n p}\right)
$$

$n \geq 5$, early times

## BUILDING INTUITION FOR THE EFFECT OF E2 TRANSITIONS



Dominant decay channel to gs, speeds up rec

$$
R_{n d \rightarrow n p}^{\text {quad }}=A_{n d \rightarrow 1 s}\left(x_{n d}-\frac{5}{3} x_{n p}\right)
$$

All $n$, late times

## RESULTS: CMB ANISOTROPIES

## RESULTS: TT $C_{l} s$ WITH HIGH-N STATES



RESULTS: EE $C_{l} s$ WITH HIGH-N STATES


## RESULTS: TEMPERATURE (TT) $C_{l} s$ WITH HYDROGEN QUADRUPOLES,

> Bulk of integral from late times, higher $\mathrm{n}_{\max } \rightarrow$ lower $x_{e}$ $\rightarrow$ lower $\tau \rightarrow$ higher $e^{-2 \tau} \rightarrow$ higher $C_{l}$


## RESULTS: TEMPERATURE (TT) $C_{l} s$ WITH HYDROGEN QUADRUPOLES,

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## Overall effect is negligible for CMB experiments!

## RESULTS: POLARIZATION (EE) $C_{l} s$ WITH HYDROGEN QUADRUPOLES



Bulk of integral from late times, higher $\mathrm{n}_{\text {max }} \rightarrow$ lower $x_{e}$ $\rightarrow$ lower $\tau \rightarrow$ higher $e^{-2 \tau} \rightarrow$ higher $C_{l}$

## RESULTS: POLARIZATION (EE) $C_{l} s$ WITH HYDROGEN QUADRUPOLES



Bulk of integral from late times, higher $\mathrm{n}_{\max } \rightarrow$ lower $x_{e}$ $\rightarrow$ lower $\tau \rightarrow$ higher $e^{-2 \tau} \rightarrow$ higher $C_{l}$

## CONVERGENCE



* Relative error well described by power law at high $n_{\max }$

$$
\Delta x_{e} / x_{e} \propto n_{\max }^{-1.9}
$$

* Can extrapolate to absolute error


## THE UPSHOT FOR COSMOLOGY

* Can explore effect on overall Planck likelihood analysis

$$
\begin{gathered}
Z^{2}=\sum_{l l^{\prime}, X, Y} F_{l l^{\prime}} \Delta C_{l}^{\mathrm{X}} \Delta C_{l}^{\mathrm{Y}} \\
Z=1.8 \text { if } n_{\max }=64 \\
Z=0.50 \text { if } n_{\max }=128 \\
Z=0.14 \text { if } n_{\max }=250
\end{gathered}
$$

## CONCLUSIONS

* RecSparse: a new tool for MLA recombination calculations (arXiv:0911.1359)
* Highly excited levels ( $\mathrm{n} \sim 64$ and higher) are relevant for CMB data analysis
* E2 transitions in H are not relevant for CMB data analysis


## FUTURE WORK

* Include line-overlap
* Develop cutoff method for excluded levels
* Generalize RecSparse to calc. rec. line. spectra
* Compute and include collisional rates
* Monte-Carlo analyses
* Cosmological masers


## Bound-free rates

* Using continuum wave functions, bound-free rates are obtained (Burgess 1957)
* Bound-free matrix elements satisfy a convenient recursion relation:
- Matrix elements compared with Burgess 1965 (5 digits) and with WKB approximation (5\%):
- At each temperature, thermal recombination/ionization rates obtained using 11point Newton-Cotes formula, agreement with Burgess to 4 published digits


## BB Rate coefficients: verification

- WKB estimate of matrix elements $\quad \rho\left(n^{\prime} l^{\prime}, n l\right)=a_{0} n^{2} \int_{-\pi}^{\pi} d \tau e^{i \Omega \tau}(1+\cos \eta)$

$$
\Omega=\omega_{n}-\omega_{n^{\prime}}
$$

Fourier transform of classical orbit! Application of correspondence

$$
\begin{array}{r}
r=r_{\max }(1+\cos \eta) / 2 \\
\tau=\eta+\sin \eta
\end{array}
$$ principle!

$$
\begin{array}{r}
\rho^{\text {dipole }}\left(n, l, n^{\prime}, l^{\prime}\right)=\frac{n_{c}^{2}}{s}\left\{J_{s-1}(s \epsilon)-\frac{1 \mp \sqrt{1-\epsilon^{2}}}{\epsilon} J_{s}(s \epsilon)\right\} \\
\epsilon=\left(1-\frac{l(l+1)}{n^{2}}\right)^{1 / 2} \\
s=n-n^{\prime}
\end{array}
$$

- Radial matrix elements checked against WKB (10\%), published rates of Brocklehurst (1971), Green, Rush, and Chandler (1967) (agreement to their published 4 digits)


## DEVIATIONS FROM BOLTZMANN EQ: L-SUBSTATES

## Chluba/Rubino-Martin/Sunyaev 2006



## Quadrupole rates: basic formalism

$$
*_{A_{n_{a}, l_{a} \rightarrow n_{b}, l_{b}}^{\text {quad }}}=\frac{\alpha}{15} \frac{1}{2 l_{a}+1} \frac{\omega_{a b}^{5}}{c^{4}}\left\langle l_{a}\left\|C^{(2)}\right\| l_{b}\right\rangle^{2}\left({ }^{2} R_{n_{b} l_{b}}^{n_{a} l_{a}}\right)^{2}
$$

- Reduced matrix element evaluated using Wigner 3J symbols:

$$
\left\langle l_{a}\left\|C^{(2)}\right\| l_{b}\right\rangle=(-1)^{l_{a}} \sqrt{\left(2 l_{a}+1\right)\left(2 l_{b}+1\right)}\left(\begin{array}{rrr}
l_{a} & 2 & l_{b} \\
0 & 0 & 0
\end{array}\right)
$$

- Radial matrix element evaluated using operator methods

$$
{ }^{2} R_{n_{b} l_{b}}^{n_{a} l_{a}} \equiv \int_{0}^{\infty} r^{4} R_{n_{a} l_{a}}(r) R_{n_{b} l_{b}}(r) d r
$$

## Quadrapole rates: Operator algebra

* Radial Schrödinger equation can be factored to yield:

$$
\begin{aligned}
&-\Omega_{n l}=\frac{1}{l A_{n l}}[ \left.1-l\left(\frac{d}{d r}+\frac{l+1}{r}\right)\right] \quad{ }^{+} \Omega_{n l}=\frac{1}{l A_{n l}}\left[1+l\left(\frac{d}{d r}-\frac{l-1}{r}\right)\right] \\
&-\Omega_{n l} R_{n l}(r)=R_{n l-1}(r) \quad A_{n l}=\frac{\sqrt{n^{2}-l^{2}}}{n l} \\
&+\Omega_{n l-1} R_{n l}(r)=R_{n l}(r) \quad A_{n}
\end{aligned}
$$

- This algebra can be applied to radial matrix elements:


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\end{aligned}
$$

- This algebra can be applied to radial matrix elements:

$$
\begin{aligned}
& { }^{2} R_{n^{\prime} l-1}^{n l-1}=\frac{1}{A_{n l}}\left\{A_{n^{\prime} l}{ }^{2} R_{n^{\prime} l}^{n l}+2^{(1)} R_{n^{\prime} l-1}^{n l}\right\}
\end{aligned}
$$

## Diagonal!

## Quadrapole rates: Operator algebra

* Radial Schrödinger equation can be factored to yield:

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&+\Omega_{n l-1} R_{n l}(r)=R_{n l}(r) \quad A_{n l}=\frac{\sqrt{n^{2}-l^{2}}}{n l}
\end{aligned}
$$

- This algebra can be applied to radial matrix elements:
$l(2 l+3) A_{n^{\prime} l}{ }^{(2)} R_{n^{\prime}}^{n} l-1-1=(2 l+1)(l+2) A_{n} l+2^{(2)} R_{n^{\prime} l}^{n} l+2+2(l+1) A_{n^{\prime}} l+1{ }^{(2)} R_{n^{\prime}}^{n} l+1+1$
$2(2 l+1)(3 l+5)^{(1)} R_{n^{\prime} l}^{n l+1} \quad\left(1 \leq l \leq n^{\prime}-1\right)$
(2) $R_{n^{\prime}}^{n} n_{n^{\prime}+1}^{\prime-1}=0$
(2) $R_{n^{\prime}}^{n} n^{n+1} n^{\prime}-1=(-1)^{n-n^{\prime}} 2^{2 n^{\prime}+4}\left[\frac{\left(n+n^{\prime}+1\right)!}{\left(n-n^{\prime}-2\right)!\left(2 n^{\prime}-1\right)!}\right]^{1 / 2} n^{\prime}\left(n n^{\prime}\right)^{n^{\prime}+3} \frac{\left(n-n^{\prime}\right)^{n-n^{\prime}-3}}{\left(n+n^{\prime}\right)^{n+n^{\prime}+3}}$

