COSMOLOGICAL HYDROGEN RECOMBINATION:
The effect of extremely high-n states and forbidden transitions

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in collaboration with Christopher M. Hirata

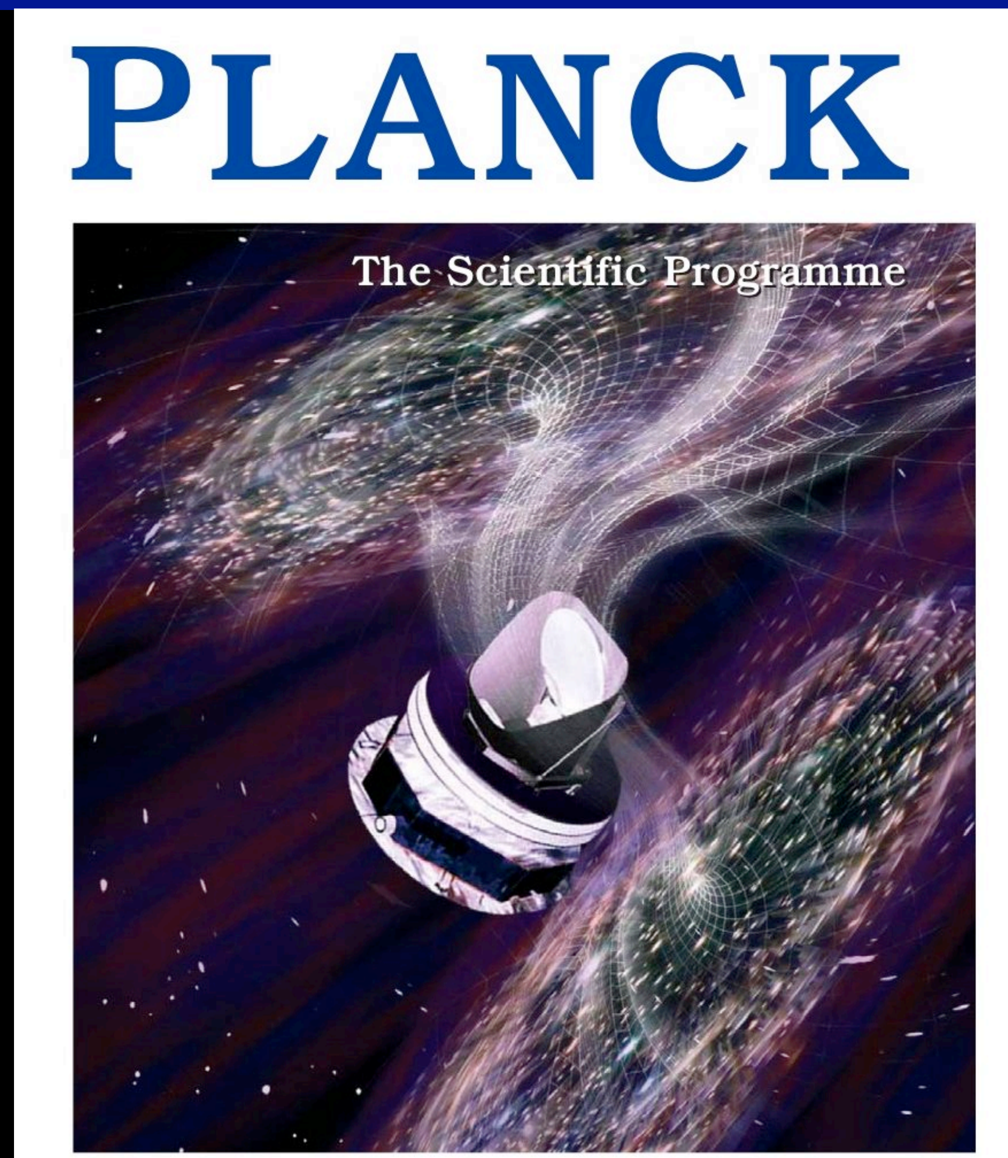
Princeton Gravity Group Seminar

12/4/09

OUTLINE

- Motivation: CMB anisotropies and recombination spectra
- Recombination in a nutshell
- Breaking the Peebles/RecFAST mold
- RecSparse: a new tool for high-n states
- Forbidden transitions
- Results
- Ongoing/future work

WALK THE PLANCK

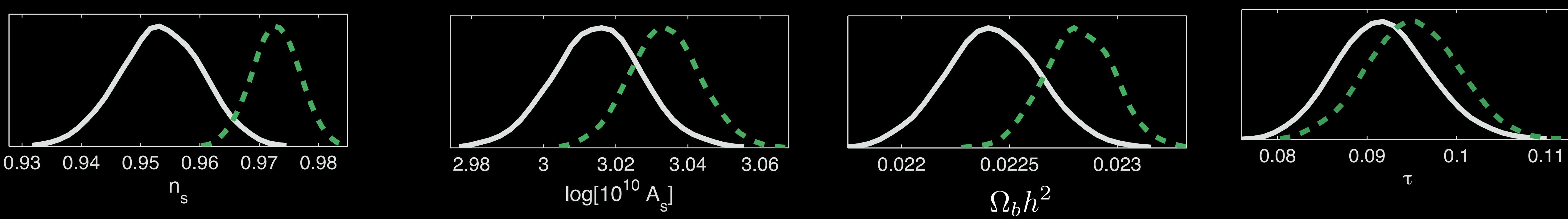


- Planck (launched May 2009) will make cosmic-variance limited CMB anisotropy measurements up to l~2500 (T), and l~1500 (E)
- Wong 2007 and Lewis 2006 show that $x_e(z)$ needs to be predicted to several parts in 10^4 accuracy for Planck data analysis

RECOMBINATION, INFLATION, AND REIONIZATION

$$P(k) = A_s\,(k\eta_0)^{n_s}$$

- Planck uncertainty forecasts using MCMC



- Cosmological parameter inferences will be off if recombination is improperly modeled (Wong/Moss/Scott 2007)

- Leverage on new physics comes from high l. Here the details of recombination matter!

- Inferences about inflation will be wrong if recombination is improperly modeled

$$\mathrm{n}_s = 1 - 4\epsilon + 2\eta \qquad \epsilon = \frac{m_{\rm pl}^2}{16\pi}\left[\frac{V'(\phi)}{V(\phi)}\right]^2 \qquad \mathrm{A}_s^2 = \frac{32}{75}\frac{V}{m_{\rm pl}^4\epsilon}\bigg|_{k_{\rm pivot}=aH}$$

CAVEAT EMPTOR: $3 \lesssim \;?\; \lesssim 16$

Need to do eV physics right to infer anything about $10^?$ GeV physics! 4

RECOMBINATION, INFLATION, AND REIONIZATION

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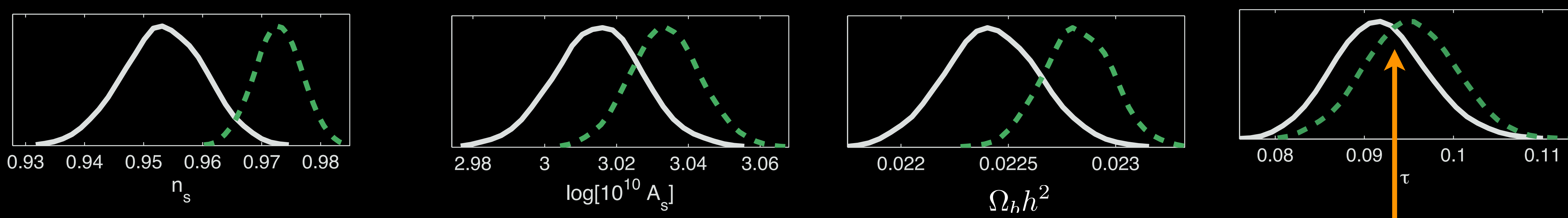

Bad recombination history yields biased inferences about reionization

Physical relevance for CMB: Smearing and moving the surface of last scattering (SLS)

- Photons kin. decouple when Thompson scattering freezes out

$$\gamma + e^- \Leftrightarrow \gamma + e^-$$

- Acoustic mode evolution influenced by visibility function

$$g = \dot{\tau} e^{-\tau} \qquad \tau(z) = \int_0^{\eta(z)} n_e \sigma_T a(\eta') d\eta'$$

- $z_{\rm dec} \simeq 1100$: Decoupling occurs during recombination

$$C_l \to C_l e^{-2\tau(z)} \text{ if } l > \eta_{\rm dec}/\eta(z)$$

PHYSICAL RELEVANCE FOR CMB:
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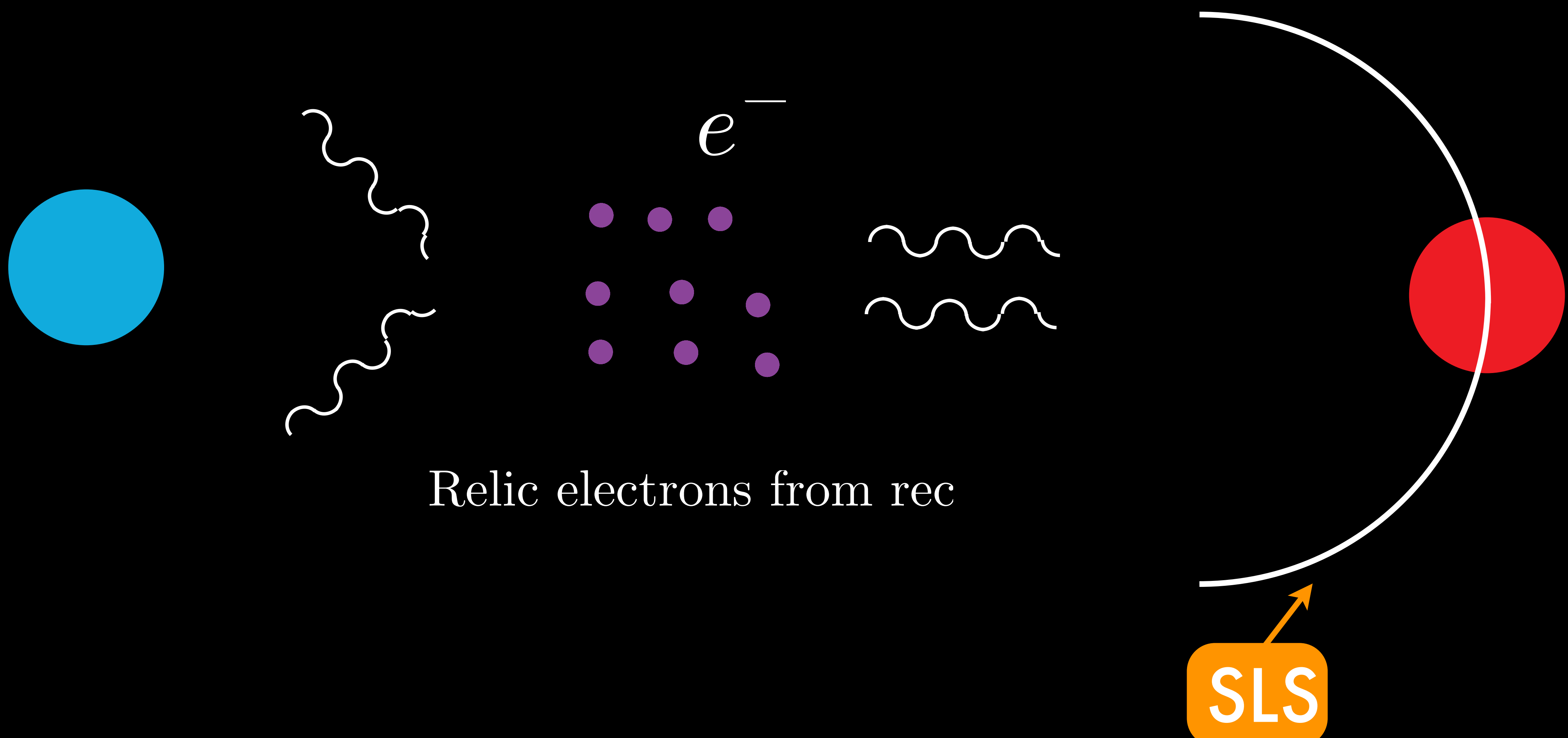

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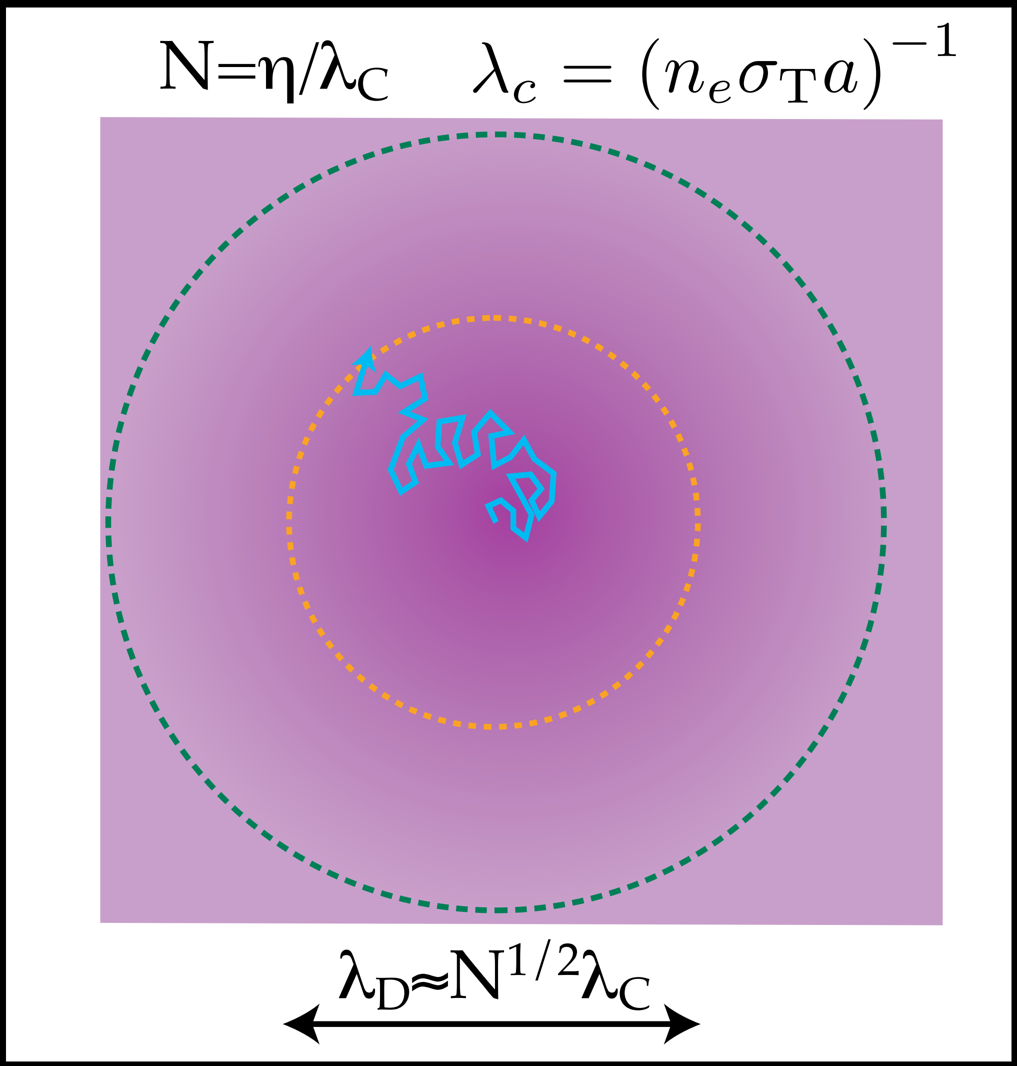
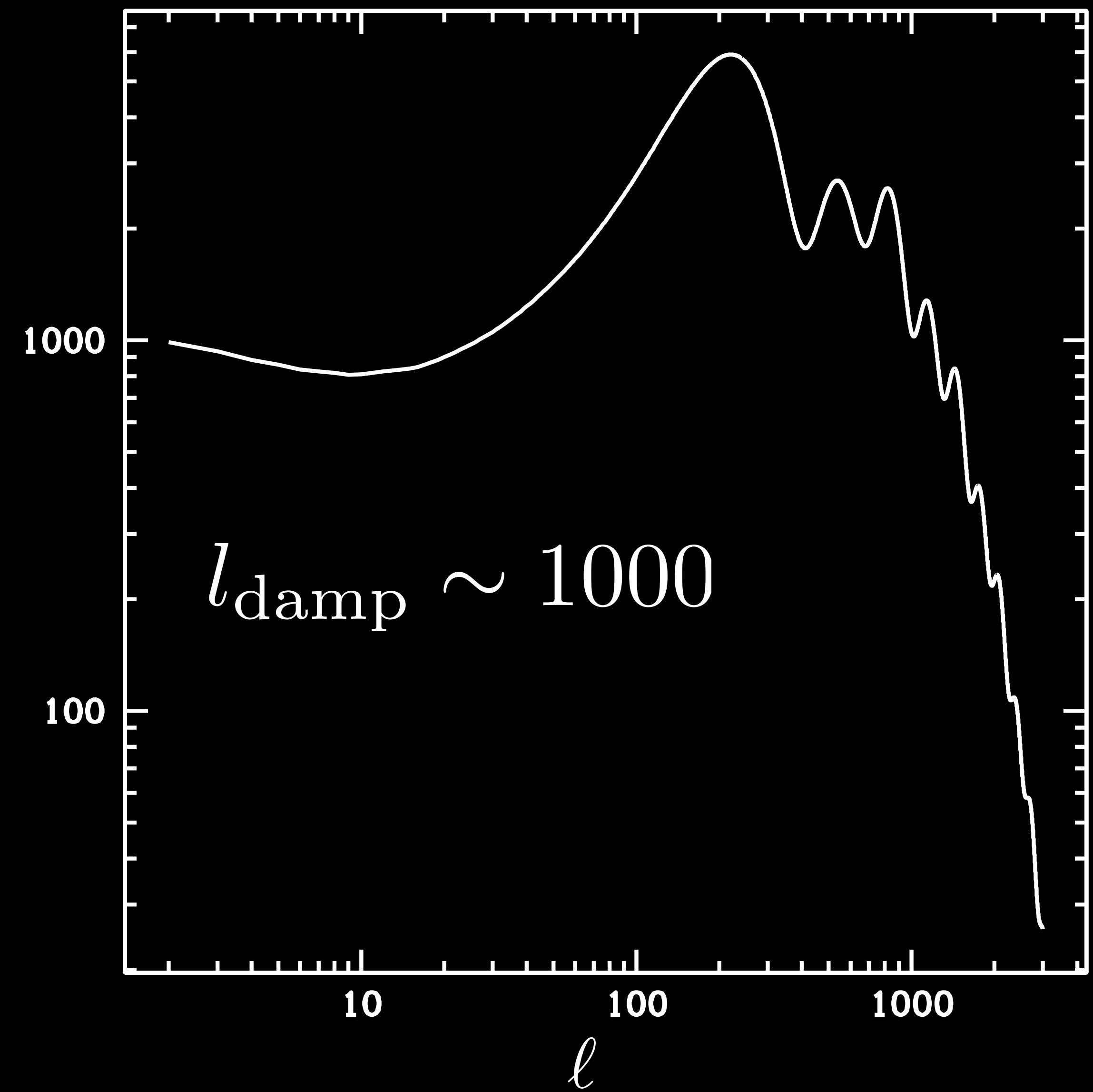
Physical relevance for CMB: The Silk Damping Tail





- Inhomogeneities are damped for $\lambda < \lambda_D$

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Physical relevance for CMB: Polarization

Isotropic radiation

Quadrupole moment

No polarization

Polarization

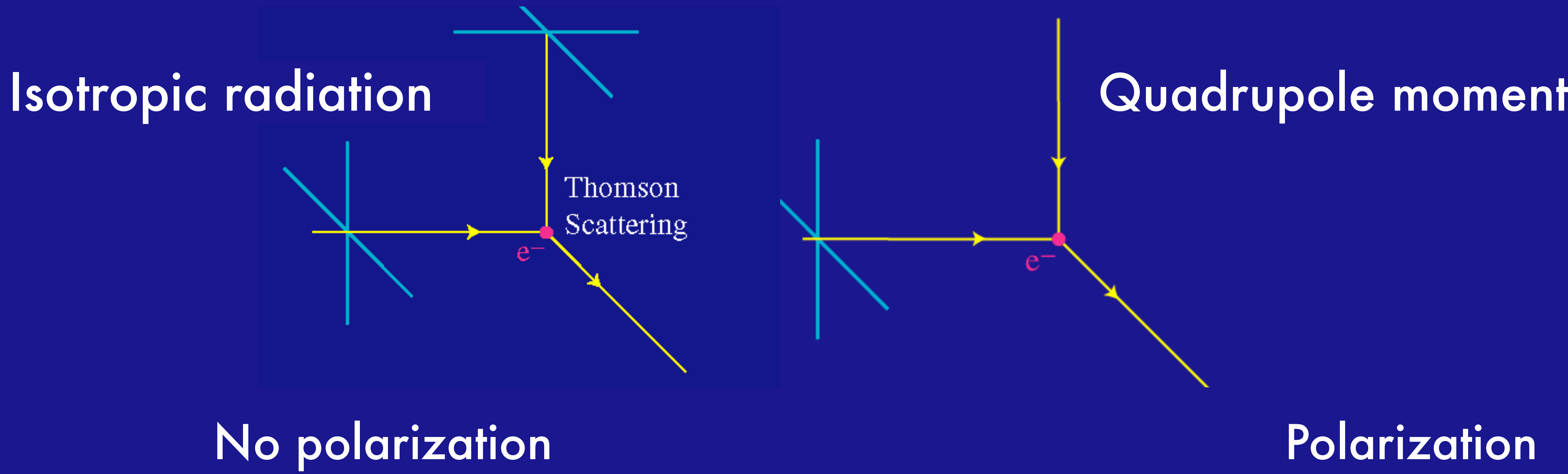
From Wayne Hu's website

* Need time to develop a quadrapole

$$\Theta_l\,(k\eta) \sim \frac{k\eta}{2\tau}\Theta_{l+1}\,(k\eta) \ll \Theta_{l+1}\,(k\eta) \;\text{ if } l \geq 2, \text{ in tight coupling regime}$$

* Need to scatter quadrapole to polarize CMB

PHYSICAL RELEVANCE FOR CMB: POLARIZATION



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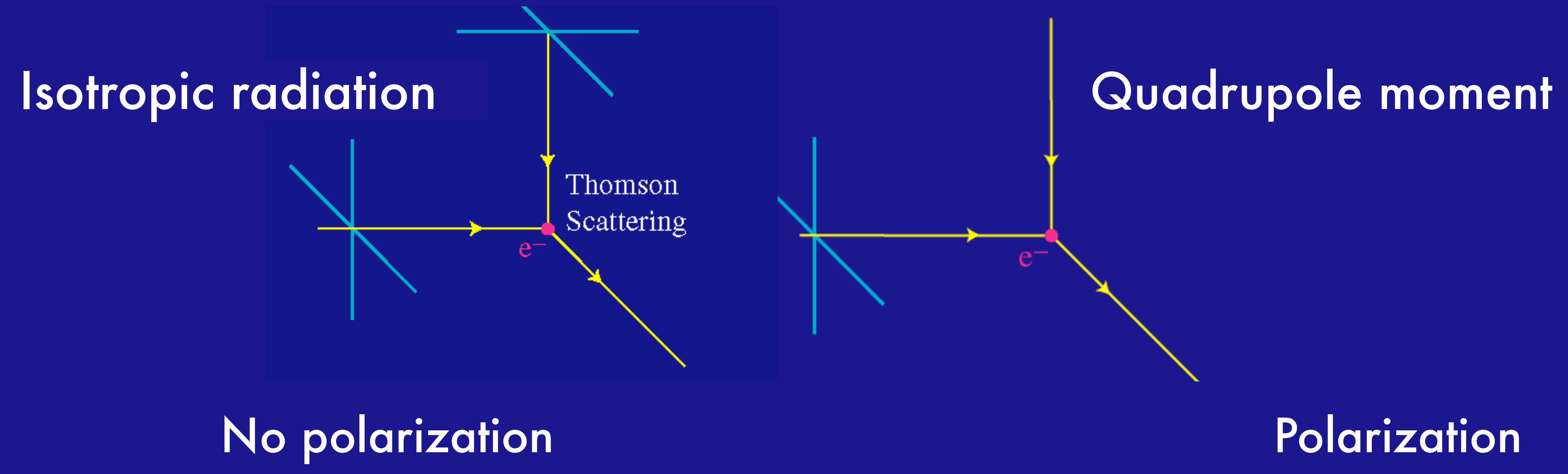

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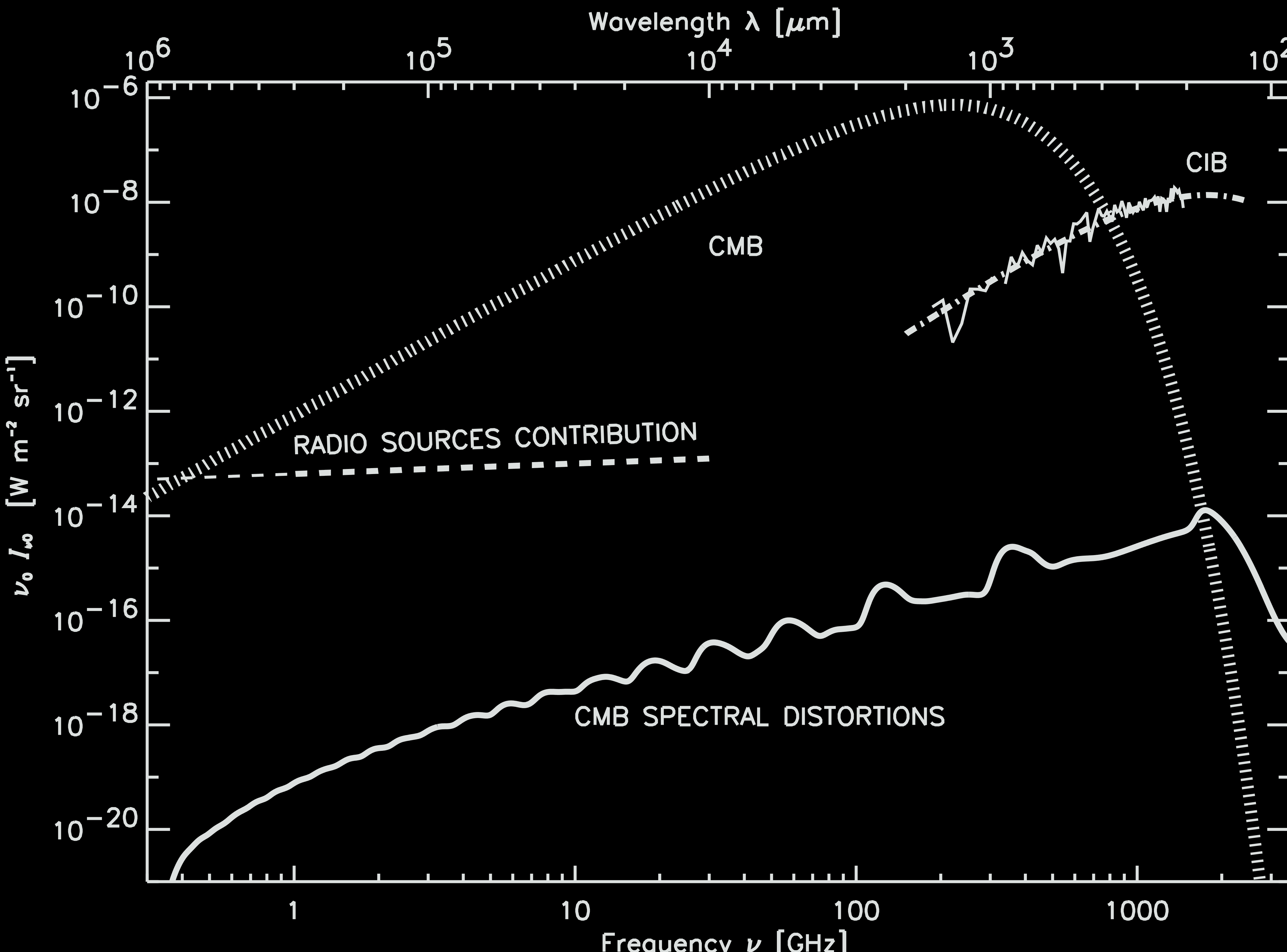
PHYSICAL RELEVANCE FOR CMB: SPECTRAL DISTORTIONS FROM RECOMBINATION



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SAHA EQUILIBRIUM IS INADEQUATE

$$p + e^- \leftrightarrow H^{(n)} + \gamma^{(nc)}$$

- Chemical equilibrium does reasonably well predicting "moment of recombination"

$$\frac{x_e^2}{1 - x_e} = \left(\frac{13.6}{T_{\rm eV}}\right)^{3/2} e^{35.9 - 13.6/T_{\rm eV}}$$

$$x_e = 0.5 \text{ when } T = T_{\rm rec} \simeq 0.3 \text{ eV}$$

$$z_{\rm rec} \simeq 1300$$

- Further evolution falls prey to reaction freeze-out

$$\Gamma < H \text{ when } T < T_{\rm F} \simeq 0.25 \text{ eV}$$

BOTTLENECKS/ESCAPE ROUTES

BOTTLENECKS

✳ Ground state recombinations are ineffective

$$\Gamma_{c\to 1s} = 10^{-1}\ \mathrm{s}^{-1} \gg H \simeq 10^{-12}\ \mathrm{s}^{-1}$$

✳Resonance photons are re-captured, e.g. Lyman α

$$\Gamma_{2p\to 1s} = 10^{-2}\ \mathrm{s}^{-1} \gg H \simeq 10^{-12}\ \mathrm{s}^{-1}$$

ESCAPE ROUTES (e.g. n=2)

✳ Two-photon processes

$$H^{2s} \to H^{1s} + \gamma + \gamma \qquad \boldsymbol{\Lambda}_{2\mathrm{s}\to 1\mathrm{s}} = \mathbf{8.22}\ \mathrm{s}^{-1}$$

✳ Redshifting off resonance

$$R \sim \left(n_{\mathrm{H}}\lambda_\alpha^3\right)^{-1} H$$

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THE PEEBLES PUNCHLINE

- Only n=2 bottlenecks are treated

- Net Rate is suppressed by bottleneck vs. escape factor

$$-\frac{dx_e}{dt} = \mathcal{S} \sum_{n,l>1s} \alpha_{nl}(T) \left\{ n x_e^2 - x_{1s} f(T) \right\}$$

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Recombination rate

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Ionization rate

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$$\mathcal{S} = \frac{\frac{8\pi}{\lambda_\alpha^3 n_{1s}} H + \Lambda_{2s\to 1s}}{\frac{8\pi}{\lambda_\alpha^3 n_{1s}} H + (\Lambda_{2s\to 1s} + \beta_c)}$$

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Redshifting term

12

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2γ term

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Ionization Term

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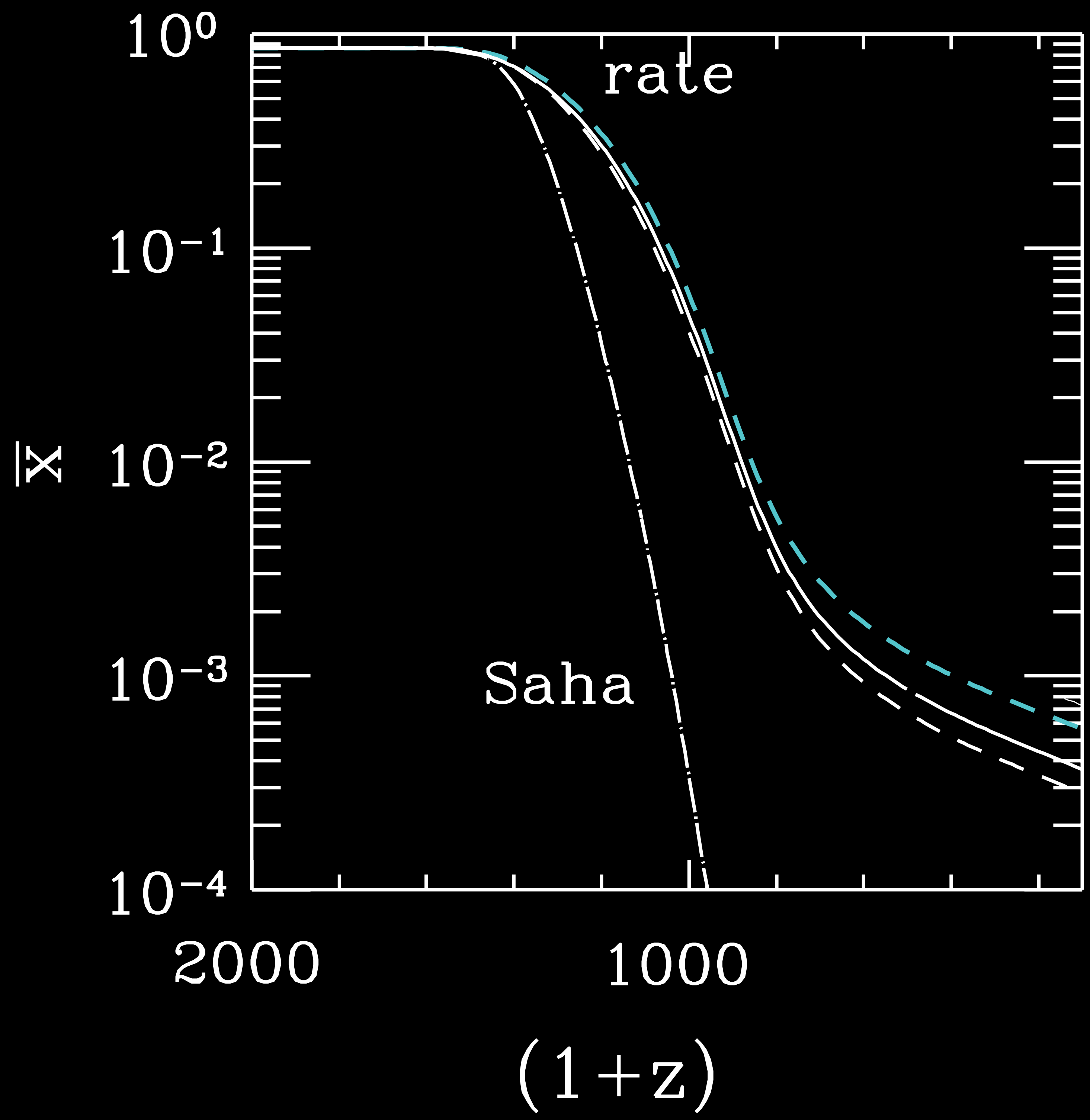
Ionization Term

$$\frac{\text{redshift term}}{2\gamma \text{ term}} \simeq 0.02 \frac{\Omega_m^{1/2}}{\left(1 - x_e\left[z\right]\right)\left(\frac{1+z}{1100}\right)^{3/2}}$$

2γ process dominates until late times ($z \lesssim 850$)

THE PEEBLES MODEL

- Peebles 1967: State of the Art for 30 years!



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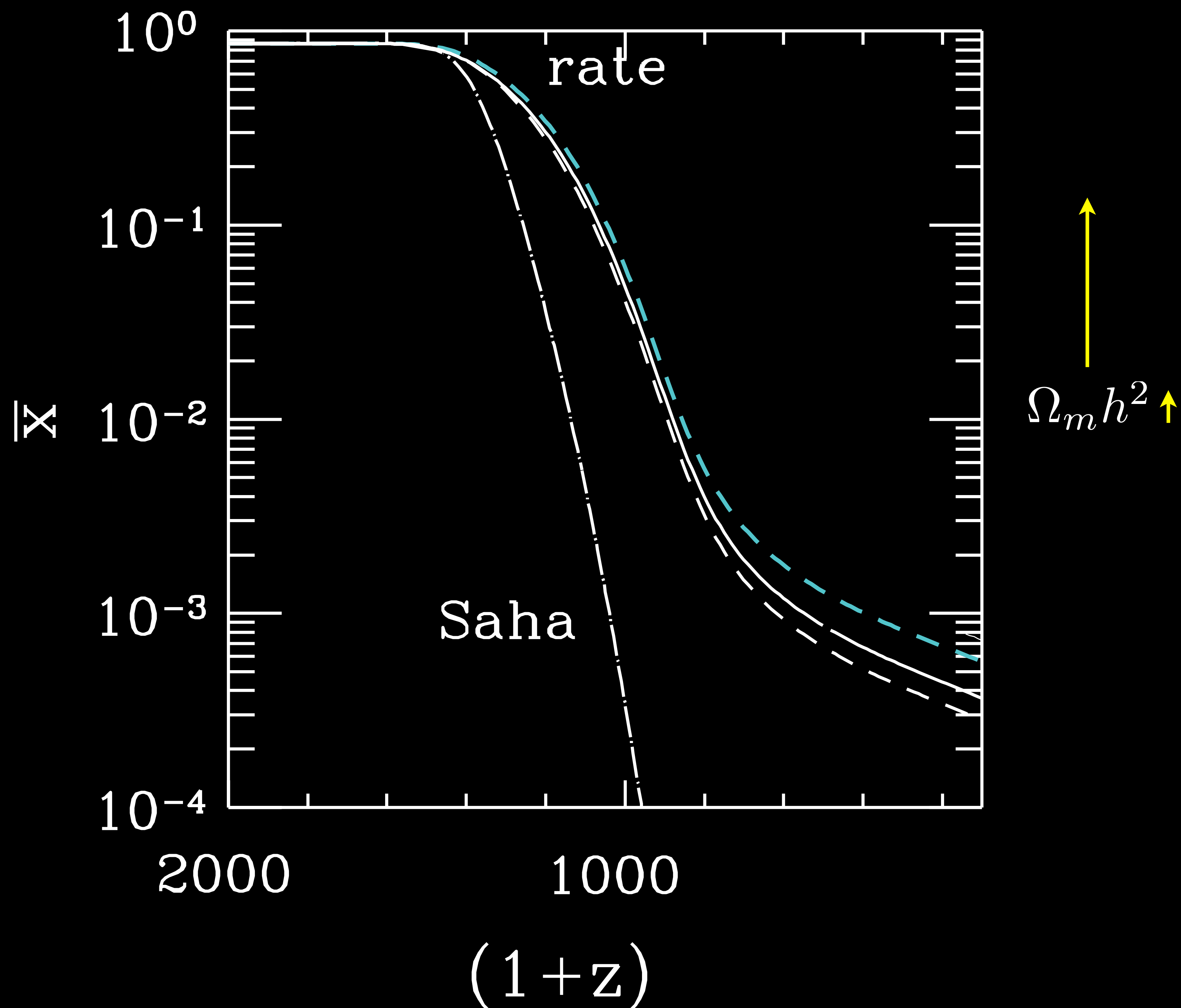

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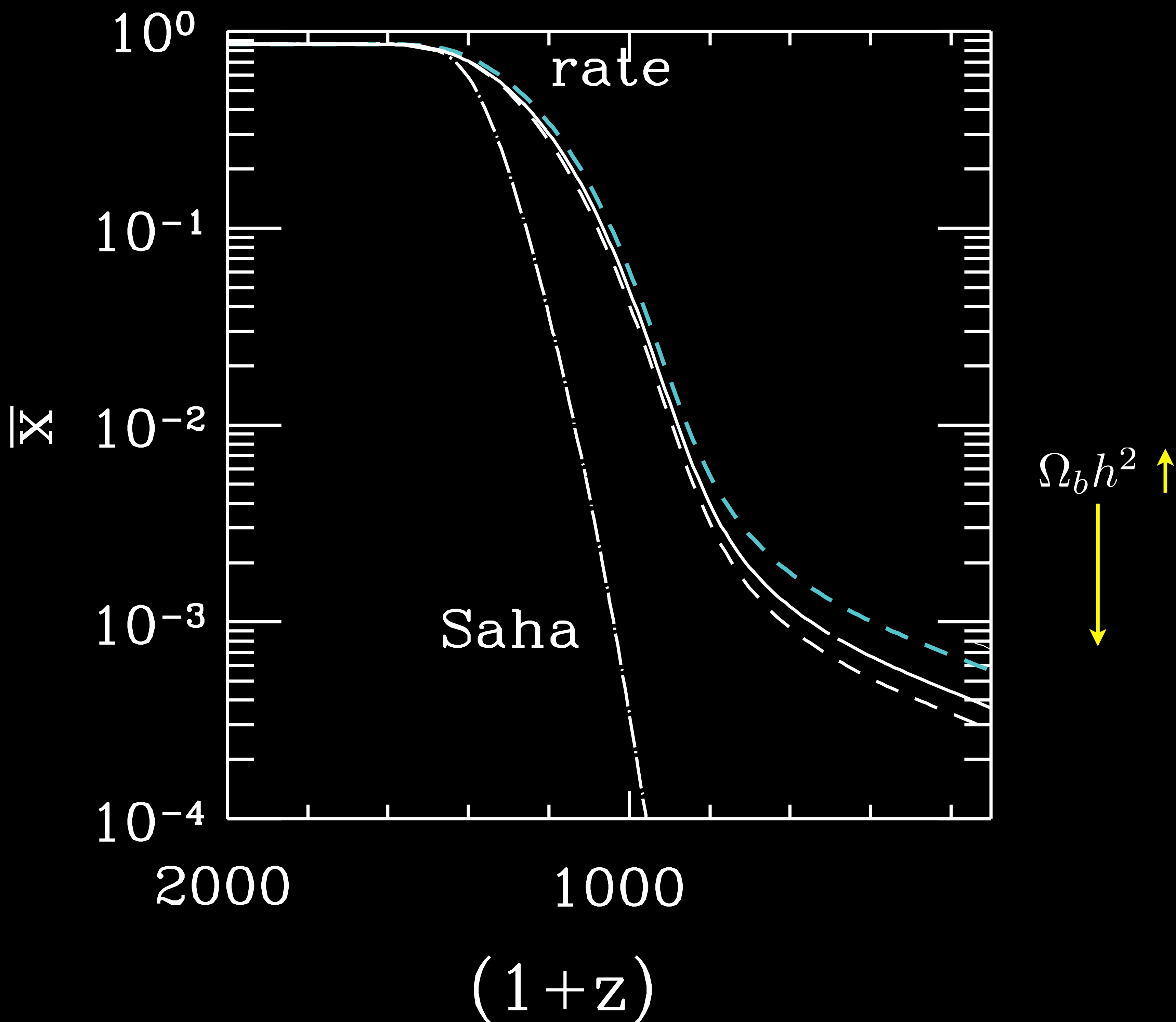

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EQUILIBRIUM ASSUMPTIONS

✳Radiative/collisional eq. between different l

$$\mathcal{N}_{nl} = \mathcal{N}_n \frac{(2l+1)}{n^2}$$

✳ Radiative eq. between different n-states

$$\mathcal{N}_n = \sum_l \mathcal{N}_{nl} = \mathcal{N}_2 e^{-(E_n - E_2)/T}$$

Non-eq rate equations

EQUILIBRIUM ASSUMPTIONS

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Seager/Scott/Sasselov 2000/RECFAST!

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Non-eq rate equations

BREAKING EQUILIBRIUM

- Chluba et al. (2005,6) follow l, n separately, get to $n_{\mathrm{max}} = 100$
- 0.1 %-level corrections to CMB anisotropies at $n_{\mathrm{max}} = 100$
- ~~Equilibrium~~ between *l states:* $\Delta l = \pm 1$ bottleneck
- Beyond this, testing convergence with n_{max} is hard!

$$t_{\mathrm{compute}} \sim \mathcal{O}\,(\mathrm{years}) \;\; \mathrm{for}\;\; \mathrm{n}_{\mathrm{max}} = 300$$

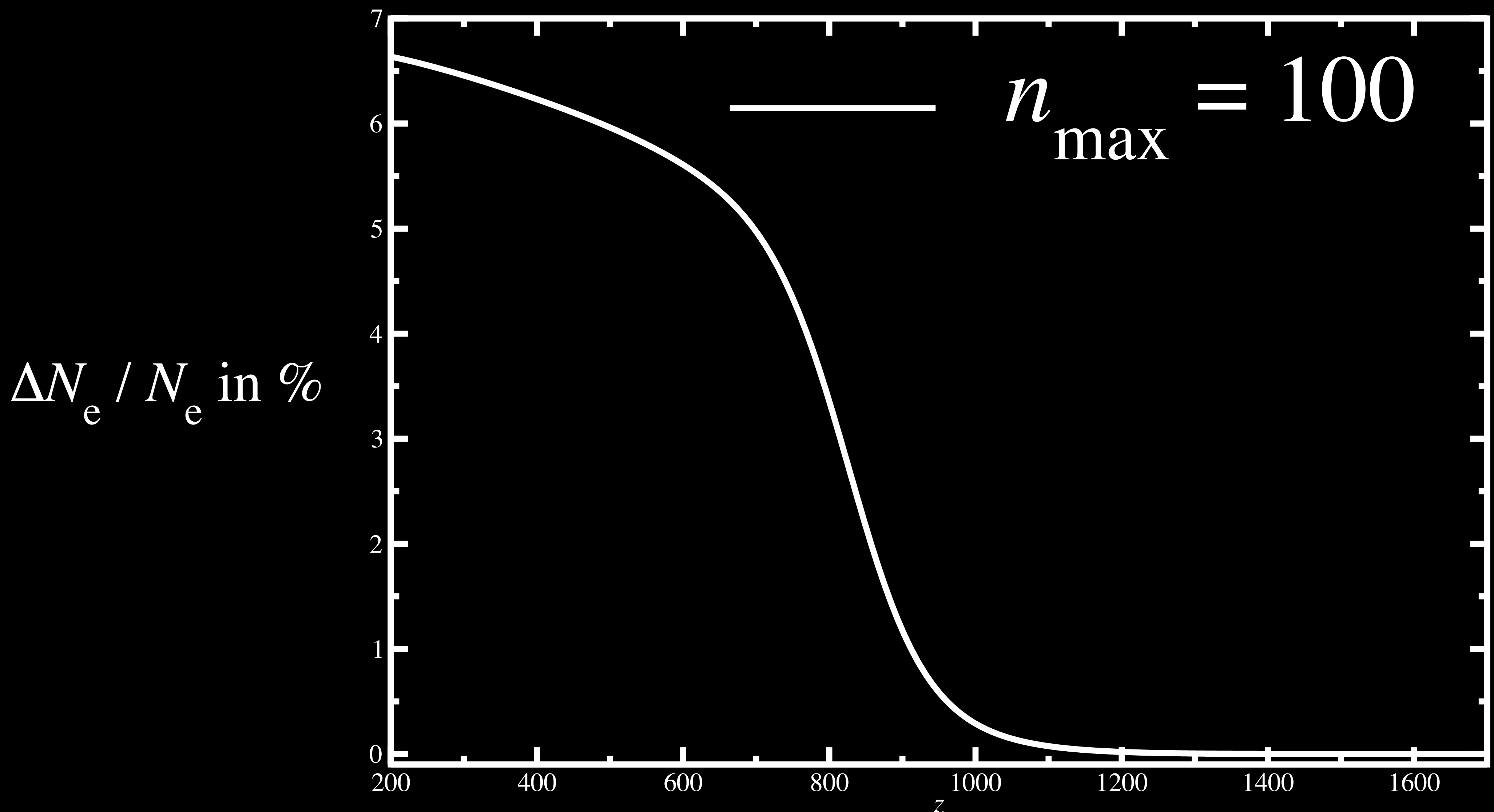
How to proceed if we want $\mathcal{O}\left(1\right) \times 10^{-4}$ accuracy in C_ℓ?

THESE ARE REAL STATES

- Still inside plasma shielding length for n<100000
- $r \sim a_0 n^2$ is as large as 2μm for $n_{\max} = 200$
- $\frac{\Delta E|_{\text{thermal}}}{E} < \frac{2}{n^3}$
- Similarly high n are seen in emission line nebulae

THE EFFECT OF RESOLVING *l*- SUBSTATES

Resolved l vs unresolved l



From Chluba et al. 2006

- ‘Bottlenecked’ l-substates decay slowly to 1s: Recombination is slower; Chluba al. 2006

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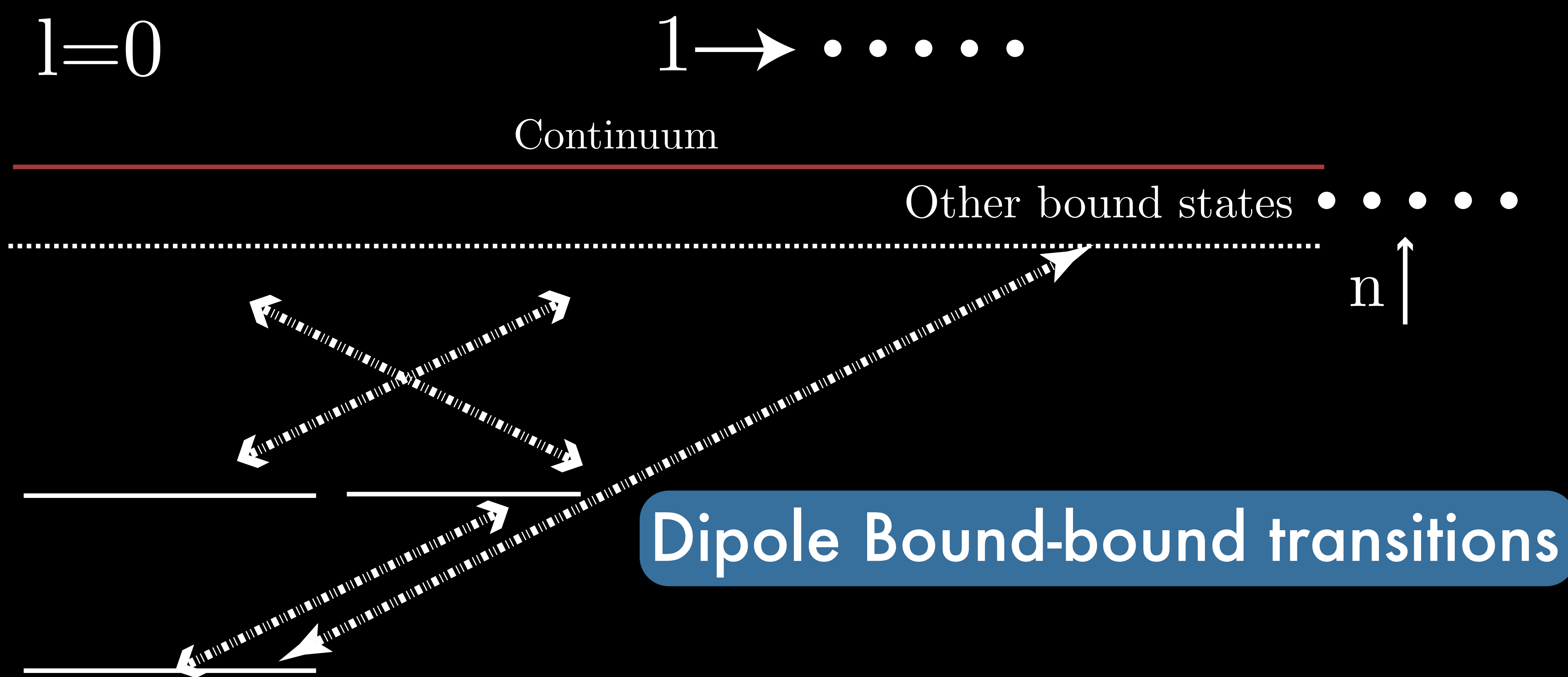
RecSparse and the multi-level atom



- We implement a multi-level atom computation in a new code, RecSparse!
- Boltzmann eq. solved for T_m (T_γ)
- Spontaneous/stimulated emission/absorption included

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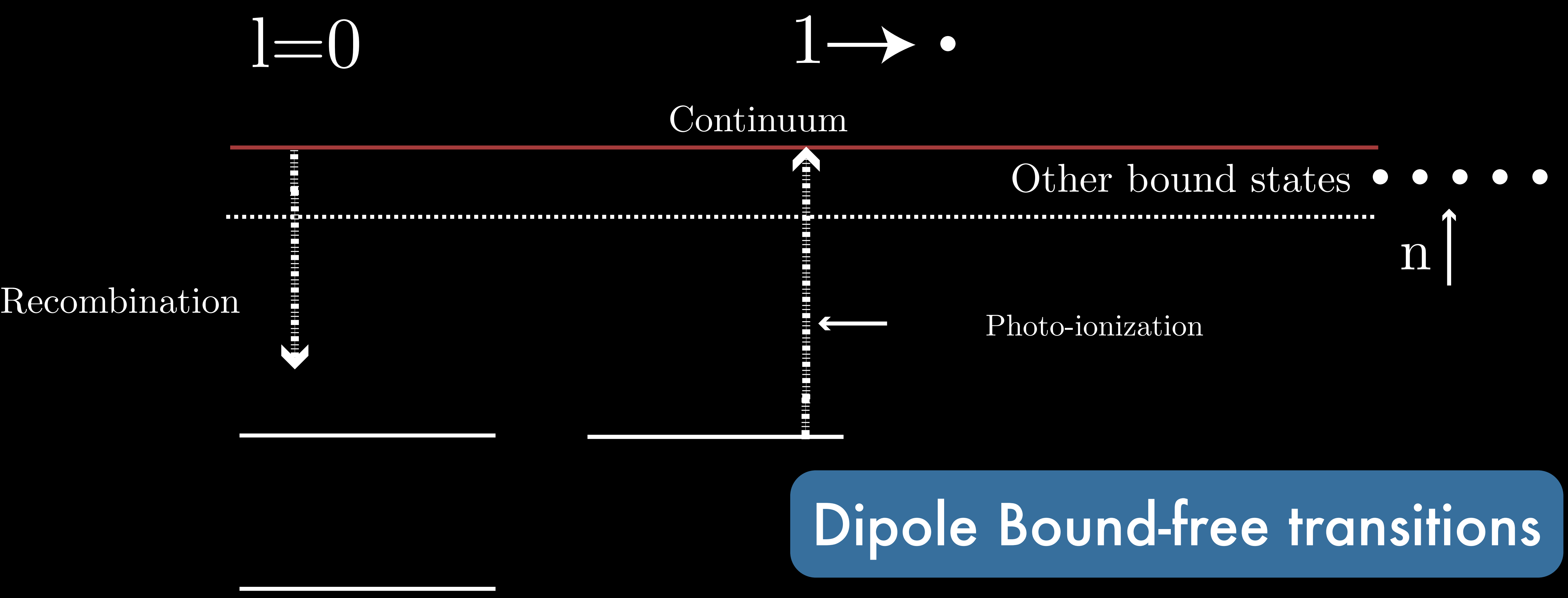

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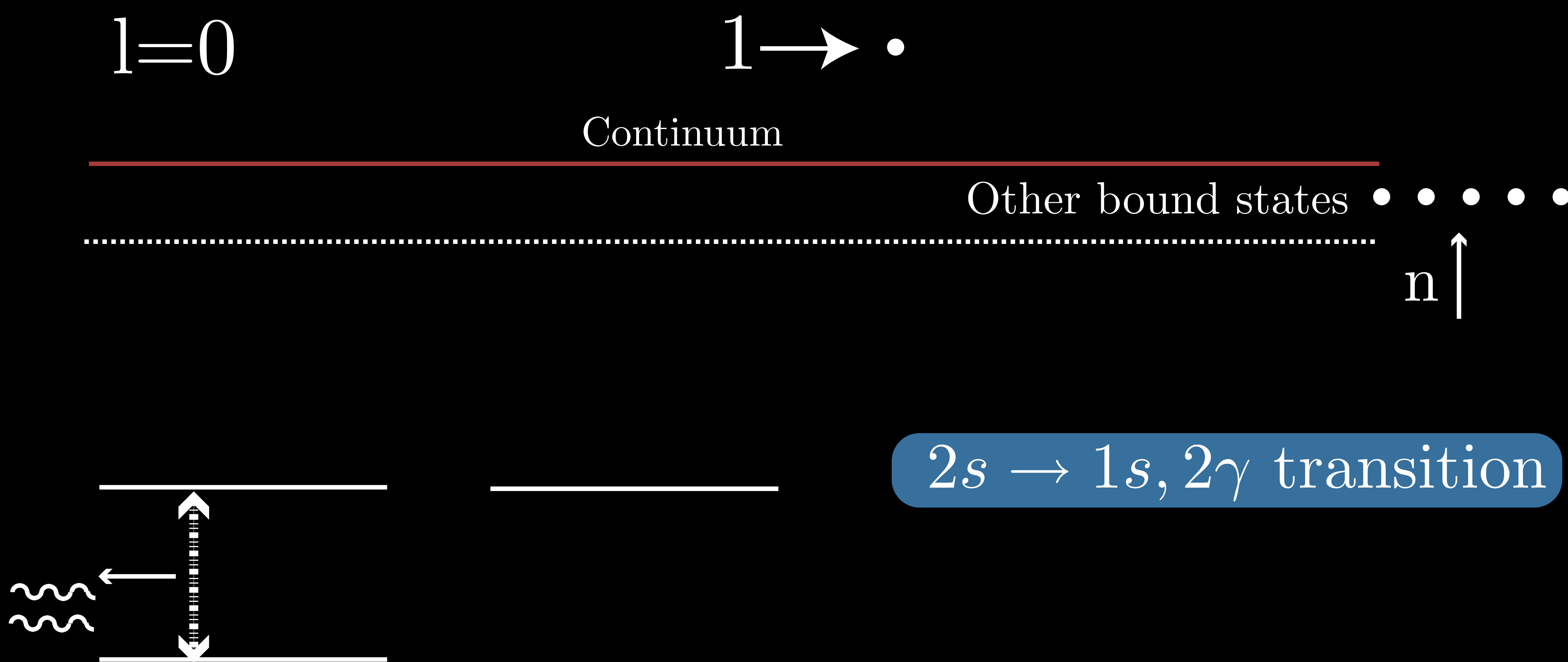

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RECSPARSE AND THE MULTI-LEVEL ATOM

- Free electron fraction evolved according to

$$\begin{aligned}\dot{x}_e &= -\dot{x}_{1s}\\ &= -\Lambda_{2s\rightarrow 1s}\left(x_{2s} - x_{1s}e^{-E_{2s\rightarrow 1s}/T_\gamma}\right) + \sum_{n,l>1s} A_{n1}^{l\,0} P_{n1}^{l0}\left\{g(T,n,l)\right\}\end{aligned}$$

2s-1s decay rate

RecSparse and the multi-level atom

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Lyman series current to ground state

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Einstein coeff.

RecSparse and the multi-level atom

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Escape probability

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RADIATION FIELD: BLACK BODY +

- Escape probability treated in Sobolev approx: depends on steady-state and incoherent scattering approximations

$$P_{n,n'}^{l,l'} = \frac{1-e^{-\tau_s}}{\tau_s}$$

$$\boxed{\tau_s \propto \frac{n_{\mathrm{H}} x_n^l A_{nn'}^{ll'}}{H(z)} \qquad n' > n}$$

- RecSparse includes radiative feedback

- Ongoing work in field focuses on corrections to simple radiative transfer picture

- Ultimate goal is to combine all new atomic physics effect in one fast recombination code

RADIATION FIELD: BLACK BODY +

- Escape probability treated in Sobolev approx: depends on steady-state and incoherent scattering approximations

Resonant absorber density

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Einstein coefficient

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Cosmological expansion

$$\tau_s \propto \frac{n_{\mathrm{H}} x_n^l A_{nn'}^{ll'}}{\boxed{H(z)}} \qquad n' > n$$

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OTHER CORRECTIONS TO RECOMBINATION

- Deviations from steady-state approx (Chluba/Sunyaev 2008)
- Coherent scattering (Forbes and Hirata 2009, Switzer/Hirata 2007)
- Atomic recoil (Forbes and Hirata 2009, Dubrovich and Grachev 2008)
- Diffusion near resonance lines
- Line overlap (Ali-Haimoud, Grin, Hirata in progress)
- Feedback from hydrogen/helium (Chluba/Sunyaev 2007)
- Higher-n two-photon processes (Chluba/Sunyaev 2007, Hirata 2008) in hydrogen and Helium (Switzer/Hirata 2007)
- Deuterium
- Additional effects in Helium (Switzer/Hirata 2007)

STEADY-STATE FOR EXCITED LEVELS

- Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

Steady-state for excited levels

✴ Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

$$\vec{x} = \begin{pmatrix} \vec{x}_0 \\ \vec{x}_1 \\ \cdots \\ \vec{x}_l \\ \cdots \\ \vec{x}_{l_{\max}} \end{pmatrix}$$

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Steady-state for excited levels

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$$\vec{x}_l = \begin{pmatrix} x_{l,l+1} \\ \cdots \\ x_{l,n_{\max}} \end{pmatrix}$$

21

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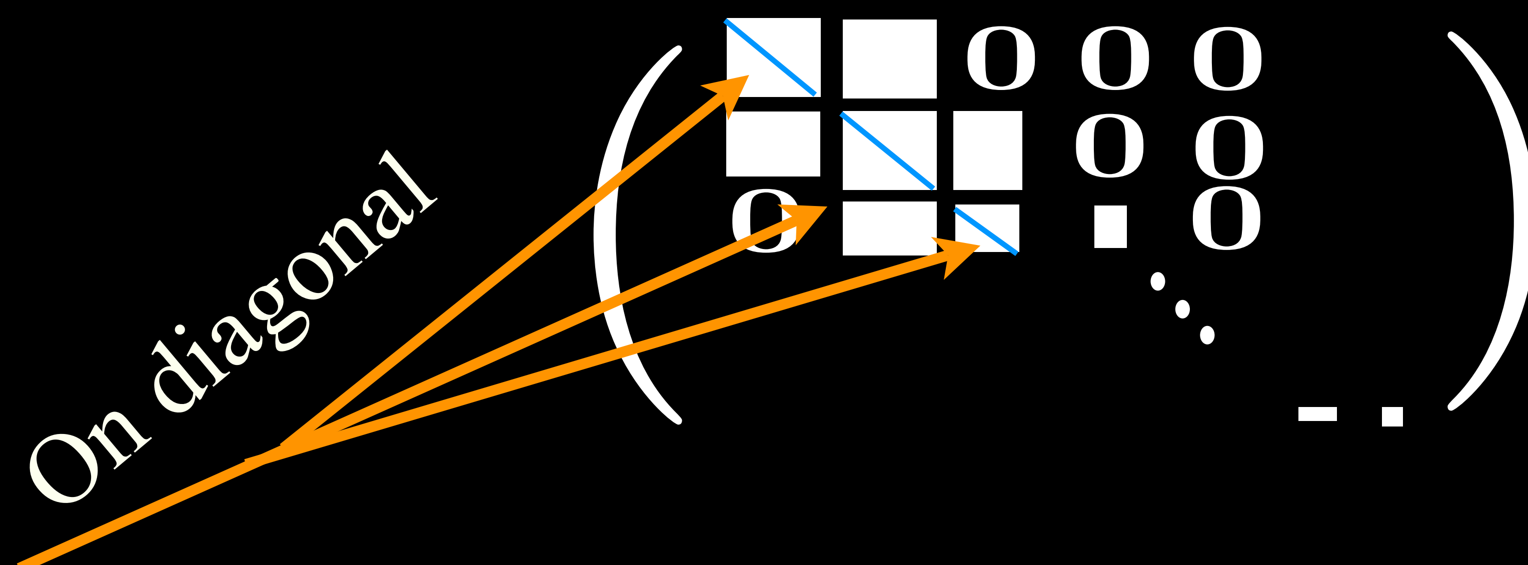

For state l, includes BB transitions out of l to all other l", photo-ionization, 2γ transitions to ground state

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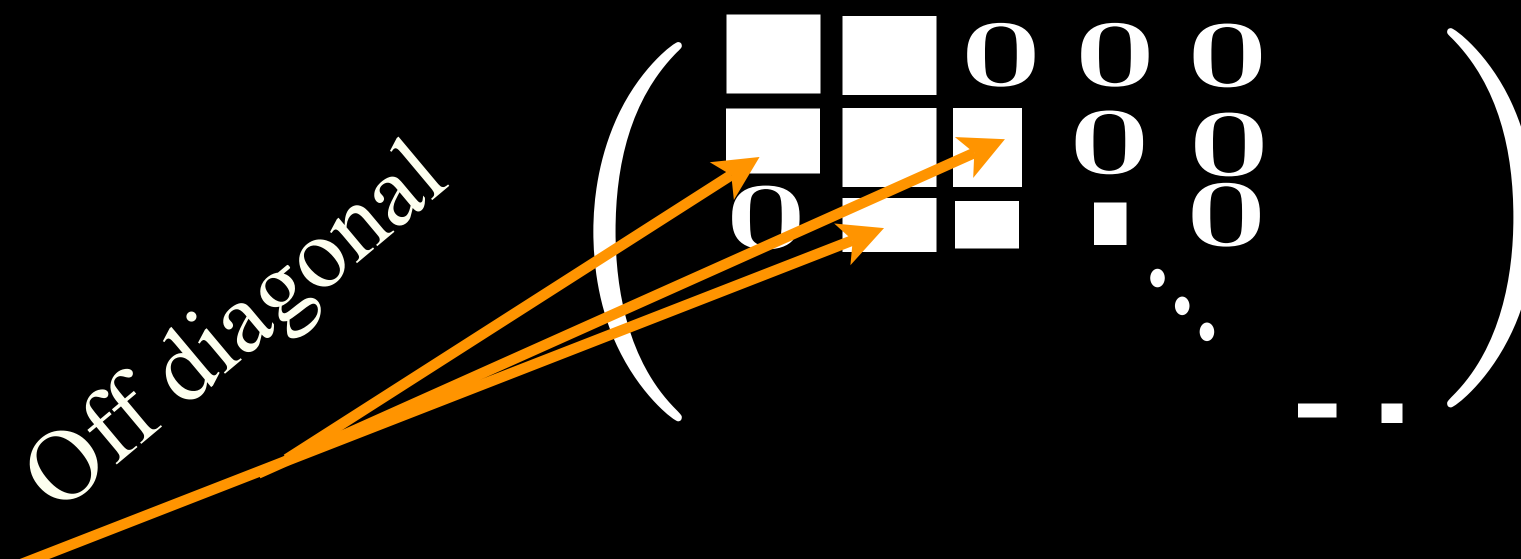

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STEADY-STATE FOR EXCITED LEVELS

- Evolution equations may be re-written in matrix form

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Includes recombination to l,
1 and 2γ transitions from ground state

STEADY-STATE FOR EXCITED LEVELS

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$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

For n>1, $\mathbf{R}, \vec{s} \geq 1\ \mathrm{s}^{-1}$ e.g. Lyman-α

STEADY-STATE FOR EXCITED LEVELS

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$$\boxed{\frac{d\vec{x}}{dt}} = \mathbf{R}\vec{x} + \vec{s}$$

$t_{\rm rec}^{-1} \sim 10^{-12}\ {\rm s}^{-1}$

For n>1, $\mathbf{R}$, $\vec{s} \geq 1\ {\rm s}^{-1}$ e.g. Lyman-α

STEADY-STATE FOR EXCITED LEVELS

✳ Evolution equations may be re-written in matrix form

$$\cancel{\frac{d\vec{x}}{dt}} = \mathbf{R}\vec{x} + \vec{s}$$

LHS $\ll$ RHS

$\vec{x} \simeq -\mathbf{R}^{-1}\vec{s}$

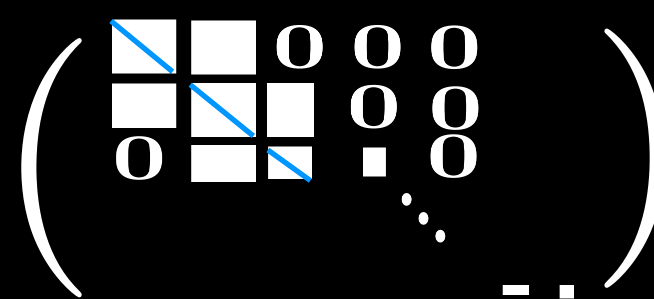
$t_{\rm rec}^{-1} \sim 10^{-12}\ \mathrm{s}^{-1}$

For n>1, $\mathbf{R}$, $\vec{s} \geq 1\ \mathrm{s}^{-1}$ e.g. Lyman-α

Rapid matrix inversion: Sparsity to the rescue

- Matrix is $\sim n_{max}^2 \times n_{max}^2$
- Brute force would require $A n_{max}^6 \sim 10^5$ s for $n_{\max} = 200$ for a single time step
- Dipole selection rules: $\Delta l = \pm 1$

  $\mathrm{M}_{l,\boldsymbol{l-1}}\vec{x}_{\boldsymbol{l-1}} + \mathbf{M}_{l,l}\vec{x}_l + \mathbf{M}_{l,\boldsymbol{l+1}}\vec{x}_{\boldsymbol{l+1}} = \vec{s}_l$



$$\begin{pmatrix} \vec{x}_0 \\ \vec{x}_1 \\ \dots \\ \vec{x}_{n_{\max}-1} \end{pmatrix} = \vec{s}_l$$

- Physics imposes sparseness on the problem. Solved in closed form to yield algebraic $\vec{x}_{l_{\max}}$, then $\vec{x}_l$ in terms of $\vec{x}_{l+1}$

Rapid matrix inversion: Sparsity to the rescue

- Einstein coefficients to states with $n > n_{\max}$ are set $A = 0$: more later!

- RecSparse generates rec. history with computation time ~ $n_{max}^{2.5}$: Huge improvement!

- Case of $n_{\max} = 100$ runs in less than a day, $n_{\max} = 200$ takes ~ 4 days.

FORBIDDEN TRANSITIONS AND RECOMBINATION

- Higher-n 2γ transitions in H important at 7-σ for Planck (TT/EE) data analysis (Hirata 2008, Kholupenko 2006)
- Some forbidden transitions are important in Helium recombination (Dubrovich 2005, Lewis 2006) and would bias cosmological parameter estimation.
- *Are other forbidden transitions in hydrogen important, particularly for Planck data analysis? How about electric quadrupole (E2) transitions?*

QUADRUPOLE TRANSITIONS AND RECOMBINATION

- Ground-state electric quadrupole (E2) lines are optically thick!

$$P_{n,n'}^{l,l'} = \frac{1 - e^{-\tau_s}}{\tau_s}$$

$$R \propto AP \propto A/\tau \text{ if } \tau \gg 1$$

$$\tau \propto A \rightarrow R \rightarrow A/A \rightarrow \text{const}$$

- Coupling to ground state will overwhelmingly dominate:

$$\frac{A_{n,2\rightarrow 1,0}^{\text{quad}}}{A_{n,2\rightarrow m,0}^{\text{quad}}} \propto \frac{\omega_{n1}^5}{\omega_{nm}^5} \geq 10^3 \text{ if } m \geq 2$$

QUADRUPOLE TRANSITIONS AND RECOMBINATION

- Lyman lines are optically thick, so $nd \rightarrow 1s$ immediately followed by $1s \rightarrow np$, so this can be treated as an effective $d \rightarrow p$ process with rate $A_{nd\rightarrow 1s} x_{nd}$.

- Same sparsity pattern of rate matrix, similar to l-changing collisions

- Detailed balance yields net rate

$$R^{\text{quad}}_{nd\rightarrow np} = A_{nd\rightarrow 1s}\left(x_{nd} - \frac{5}{3}x_{np}\right)$$

RESULTS: STATE OF THE GAS

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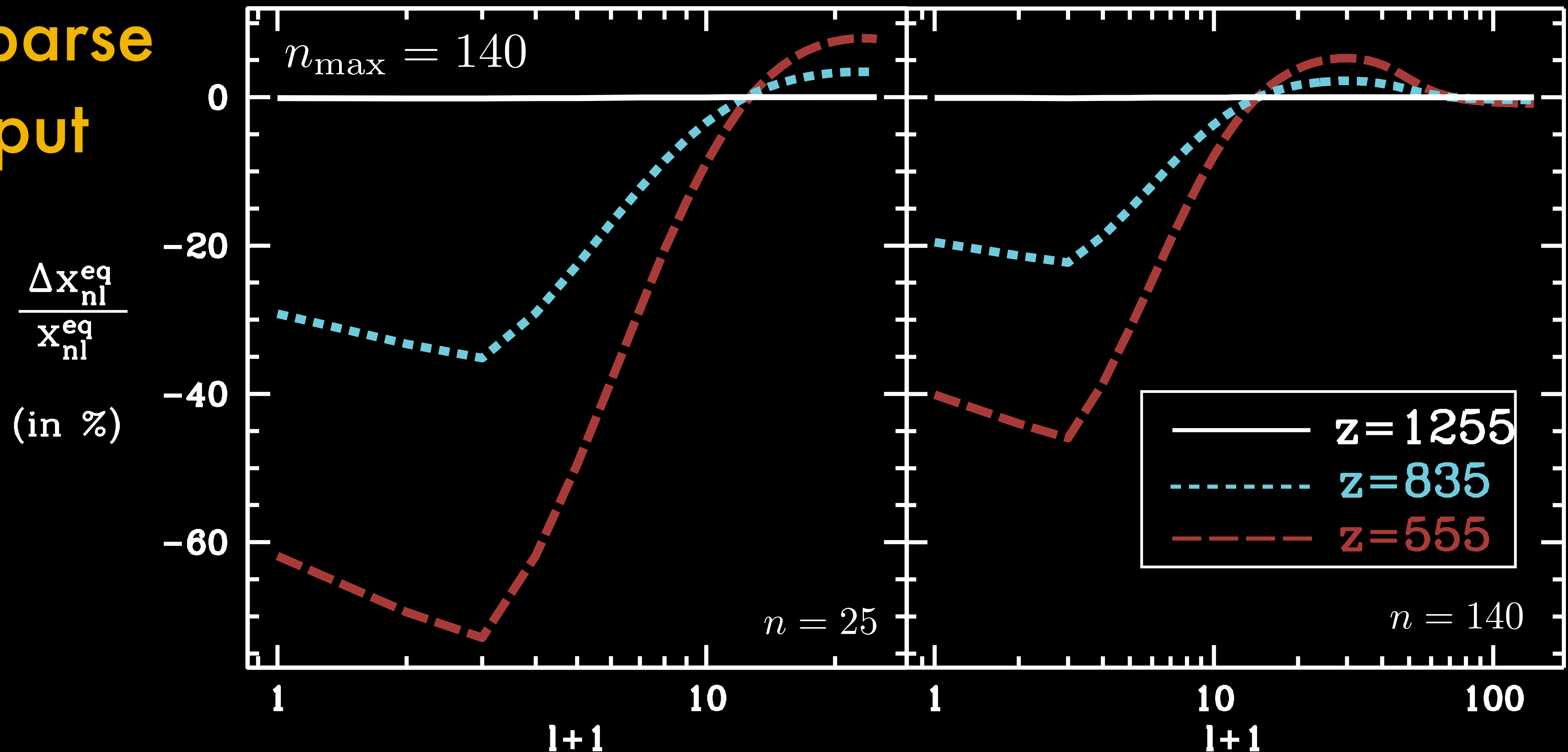
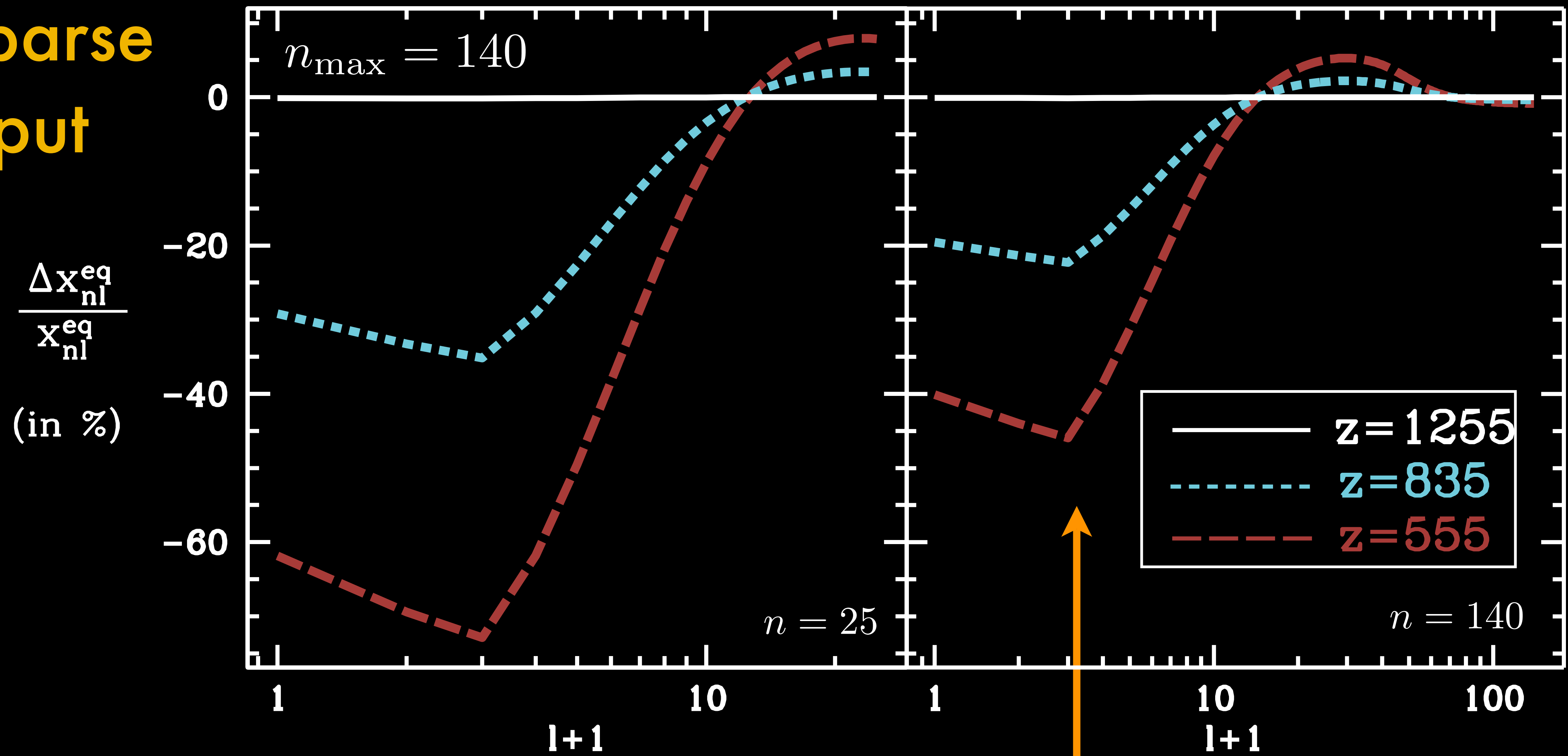
DEVIATIONS FROM BOLTZMANN EQ: l-SUBSTATES

RecSparse output



DEVIATIONS FROM BOLTZMANN EQ: l-SUBSTATES

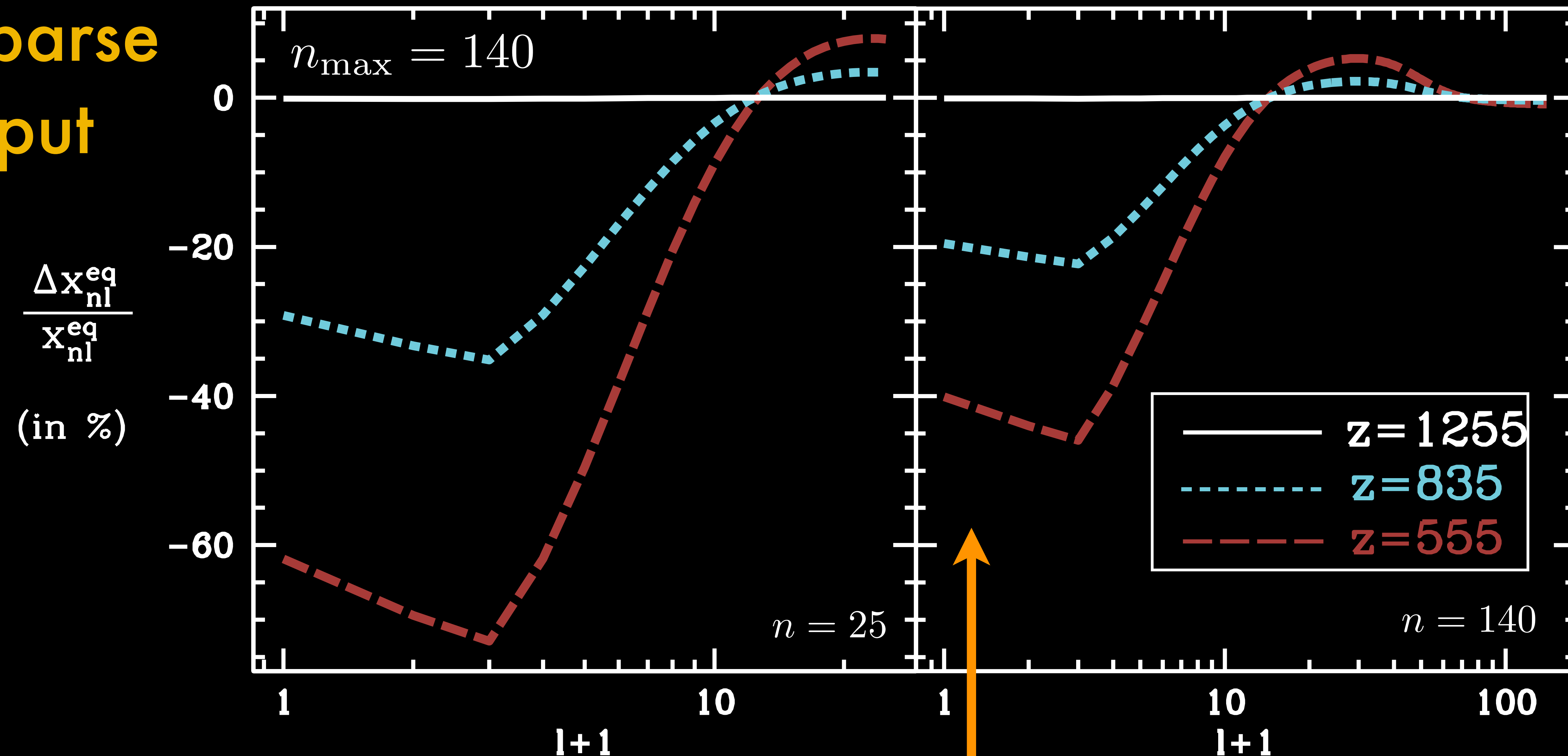
RecSparse output



Lower l states can easily cascade down, and are relatively under-populated

DEVIATIONS FROM BOLTZMANN EQ: l-SUBSTATES

RecSparse output



l=0 can't cascade down, so s states are not as under-populated

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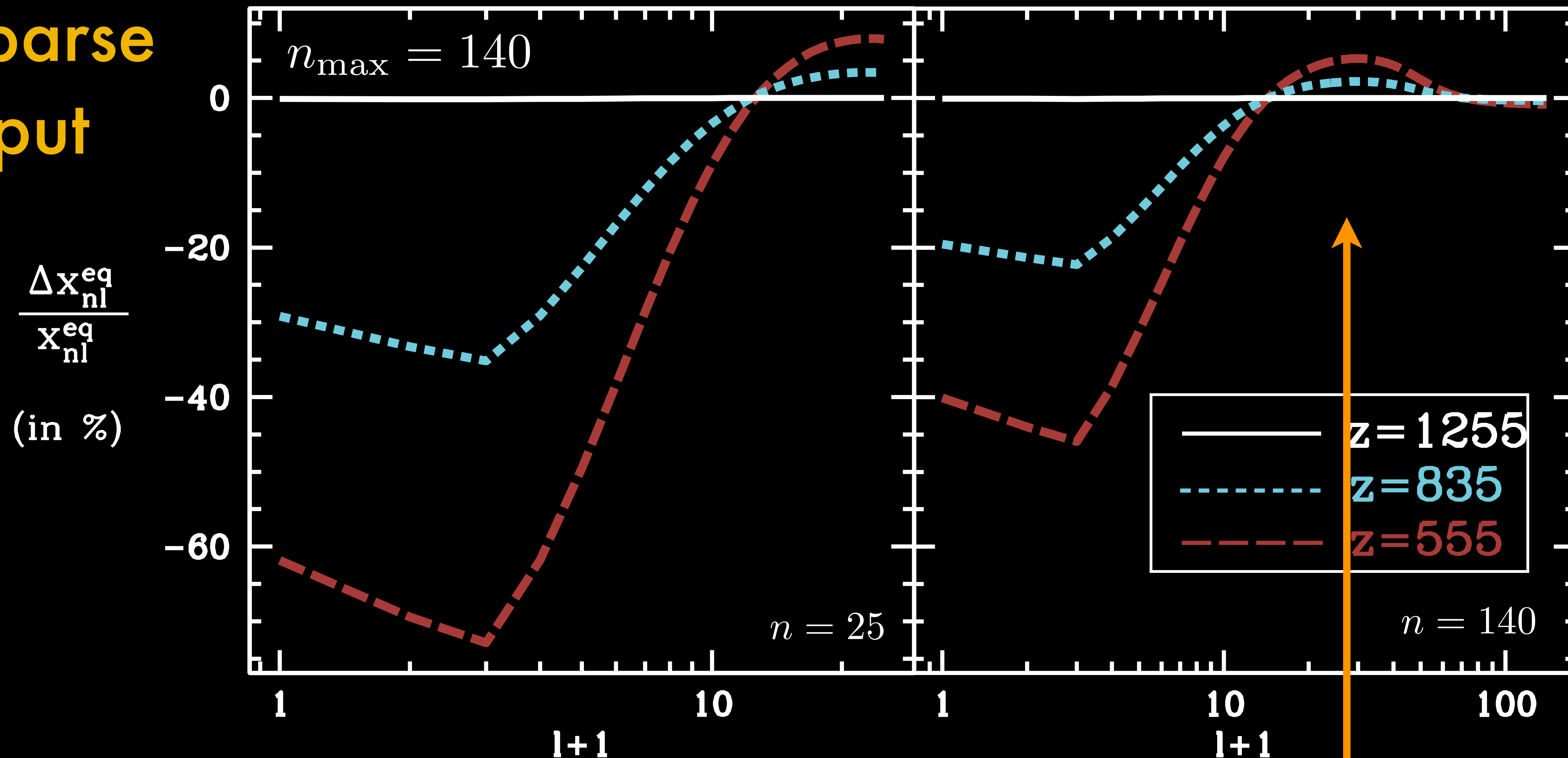
DEVIATIONS FROM BOLTZMANN EQ: l-SUBSTATES

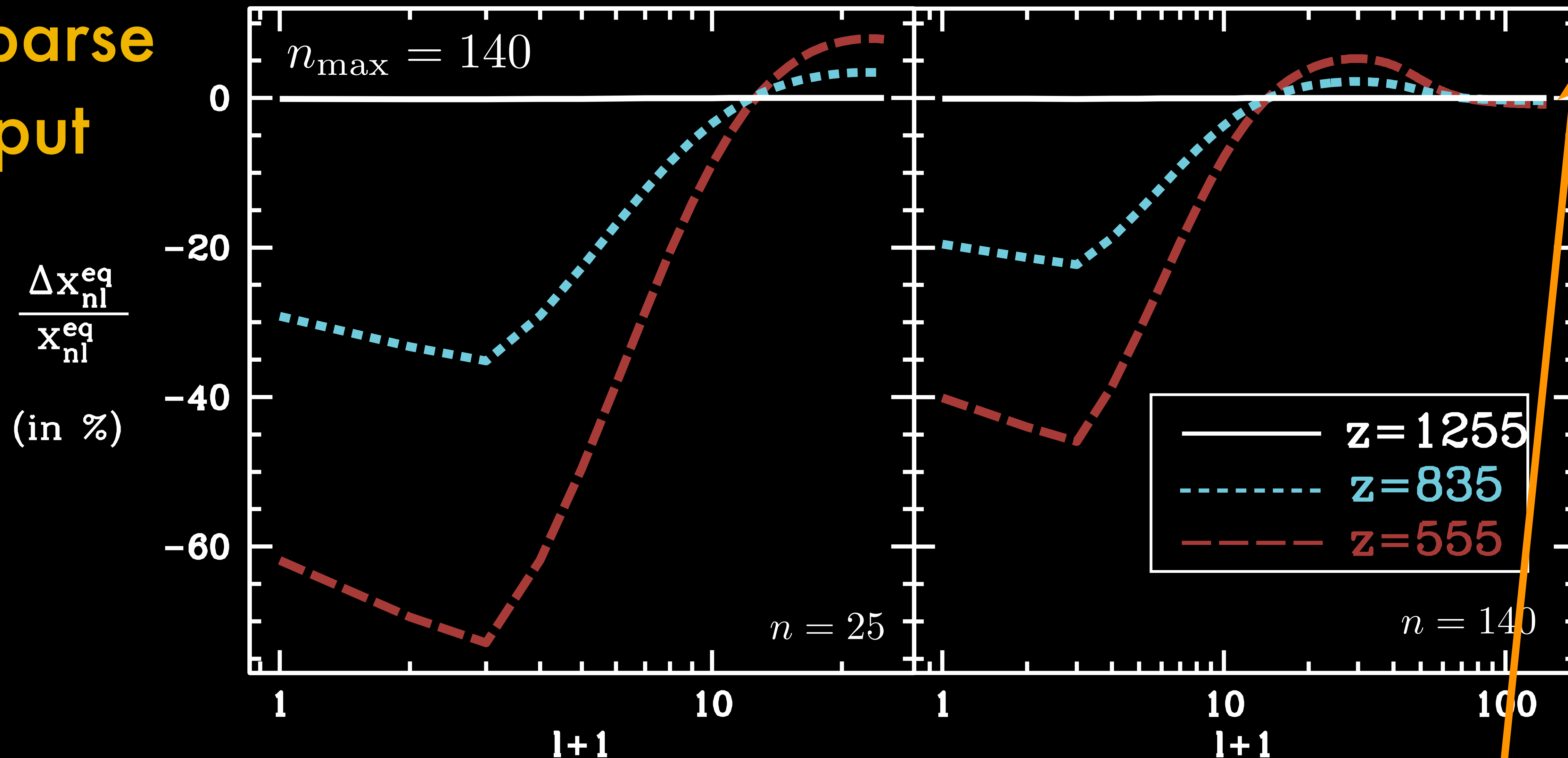
RecSparse output



Higher l are bottlenecked by $\Delta l = \pm 1$ (over-pop)

DEVIATIONS FROM BOLTZMANN EQ: l-SUBSTATES

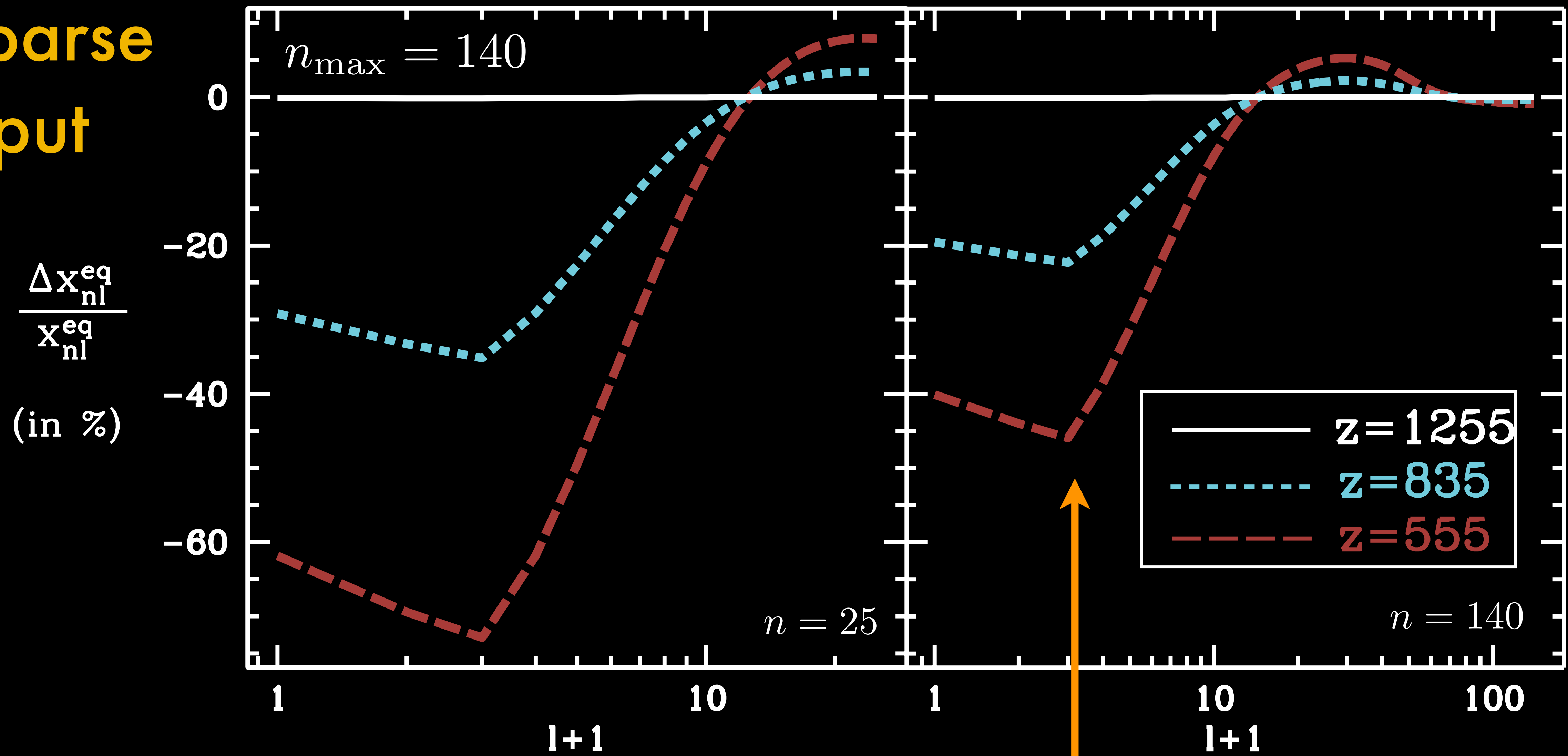
RecSparse output



Highest l states recombine inefficiently, and are under-populated

DEVIATIONS FROM BOLTZMANN EQ: l-SUBSTATES

RecSparse output



l-substates are highly out of Boltzmann eqb'm at late times

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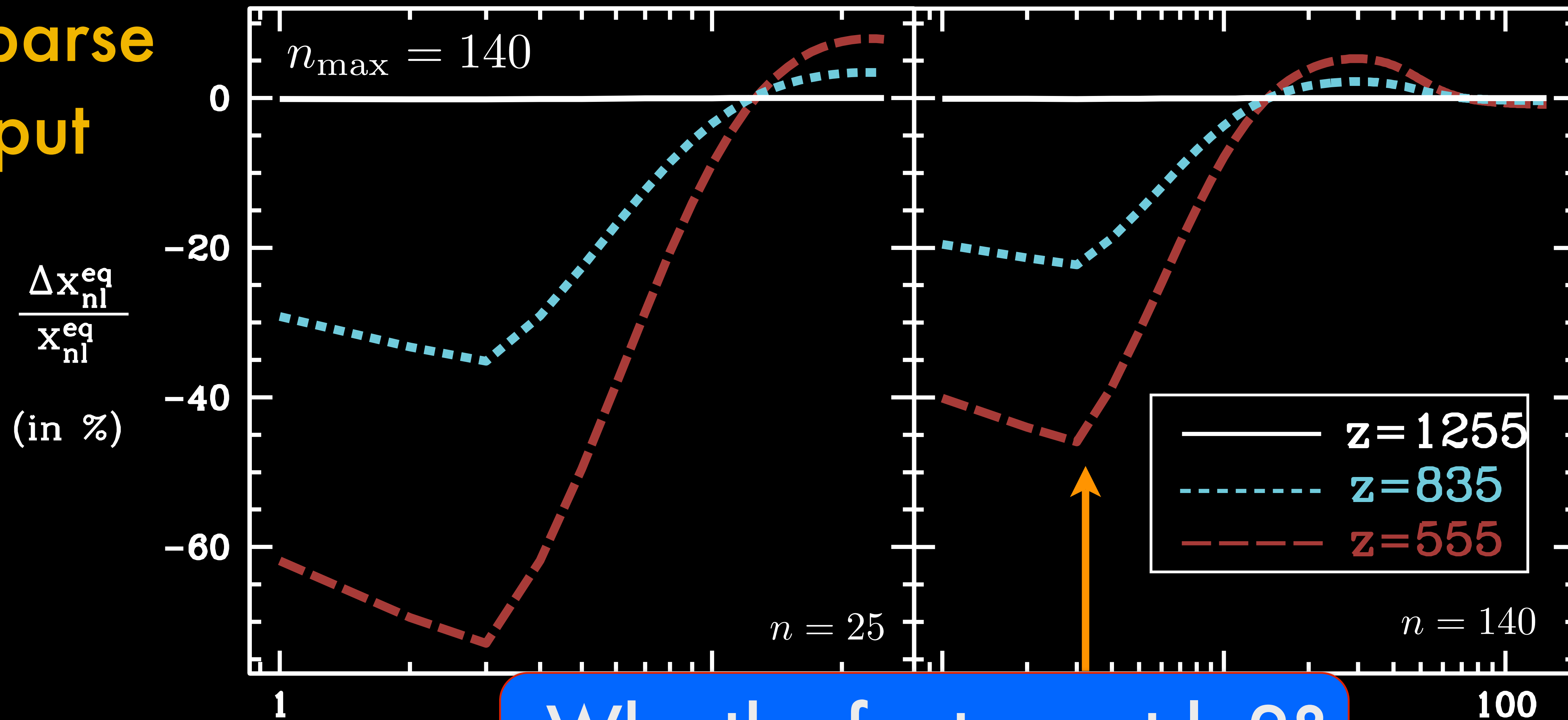
RecSparse output



Why the feature at l=2?

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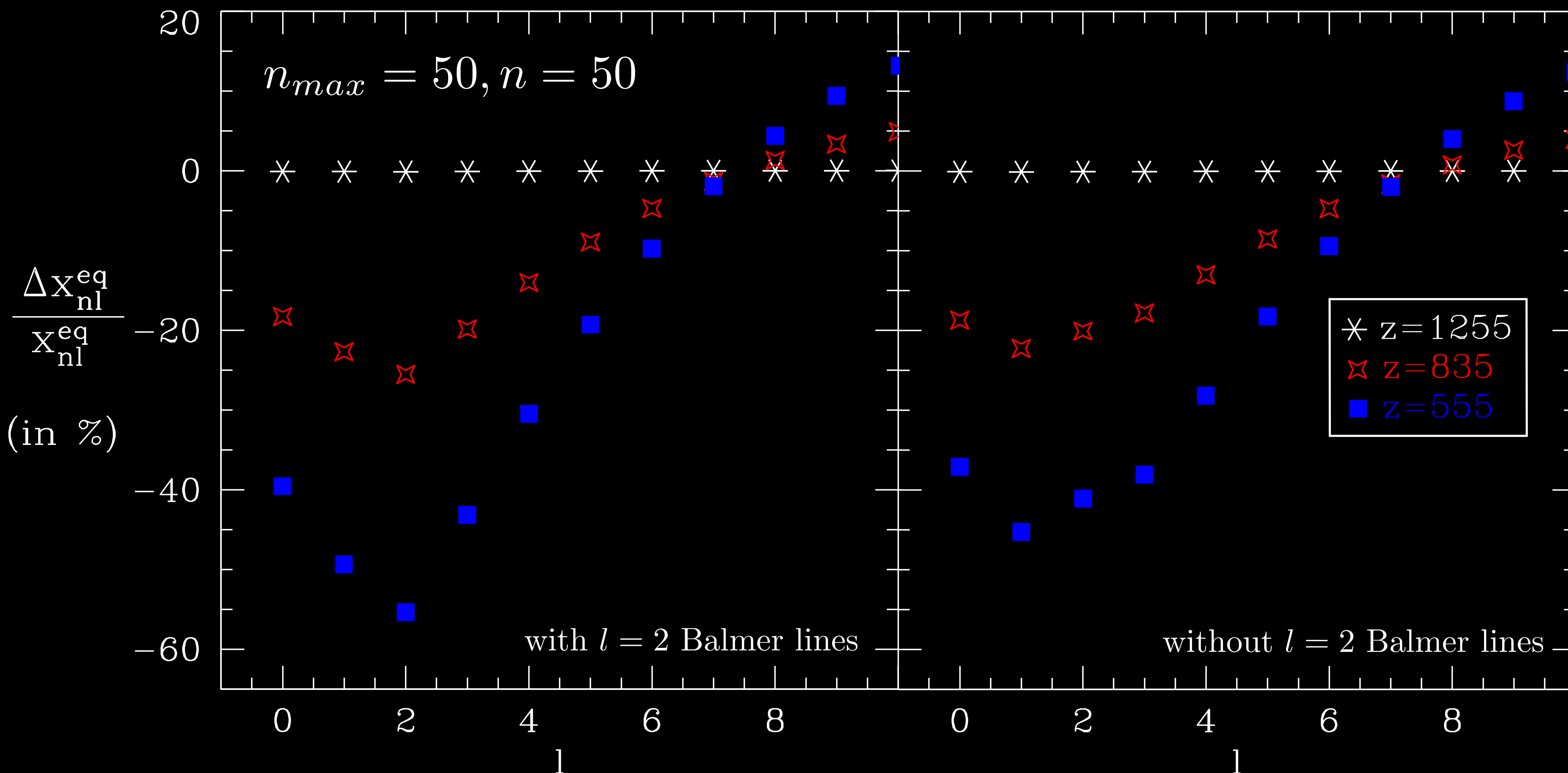
WHAT IS THE ORIGIN OF THE l=2 DIP?

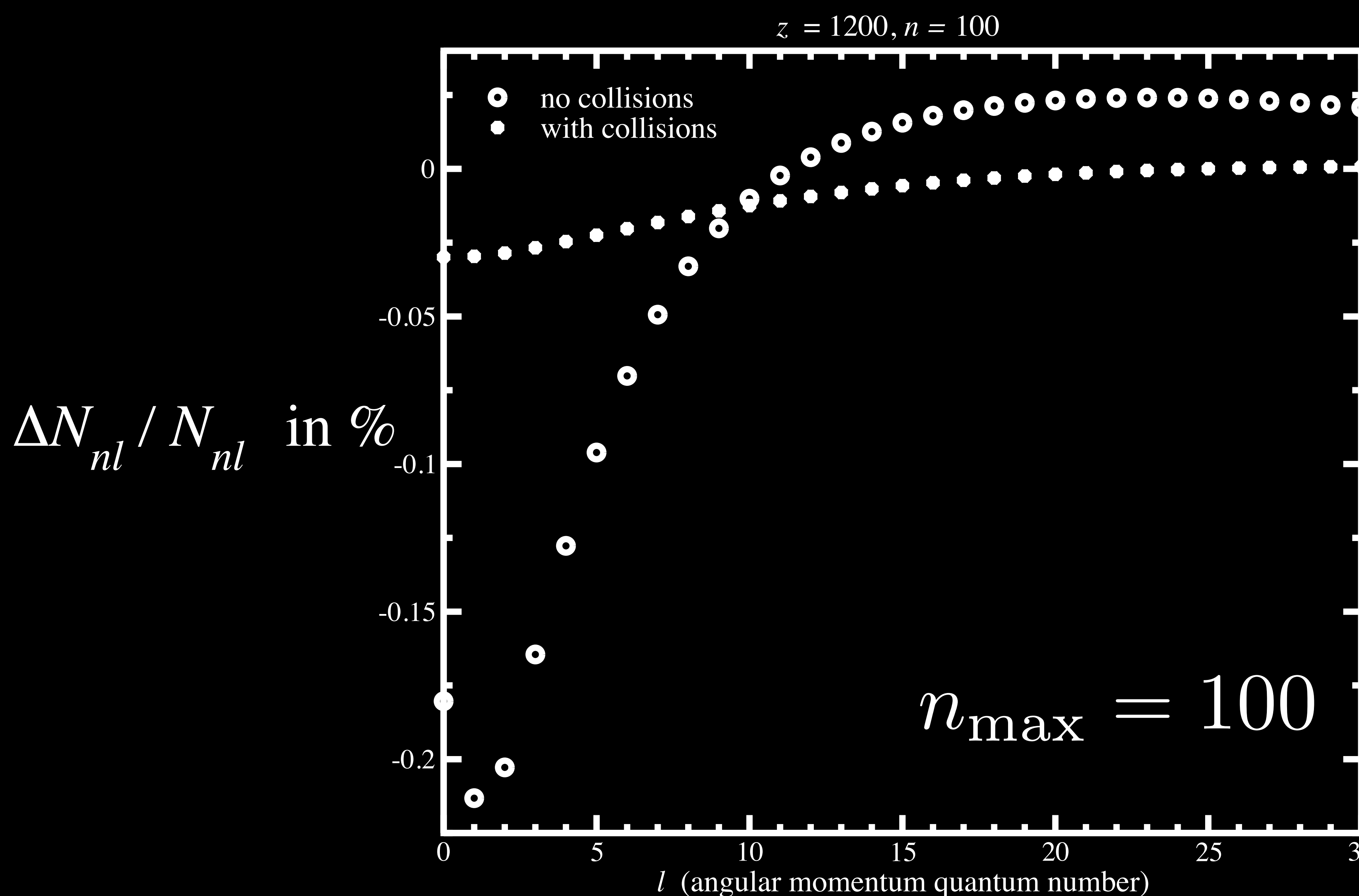
$$A_{\rm nd\rightarrow 2p} > A_{\rm np\rightarrow 2s} > A_{\rm ns\rightarrow 2p}$$

- l=2 depopulates more rapidly than l=1 for higher (n>2) excited states
- We can test if this explains the dip at l=2 by running the code with these Balmer transitions the blip should move to l=1

L-SUBSTATE POPULATIONS, BALMER LINES OFF

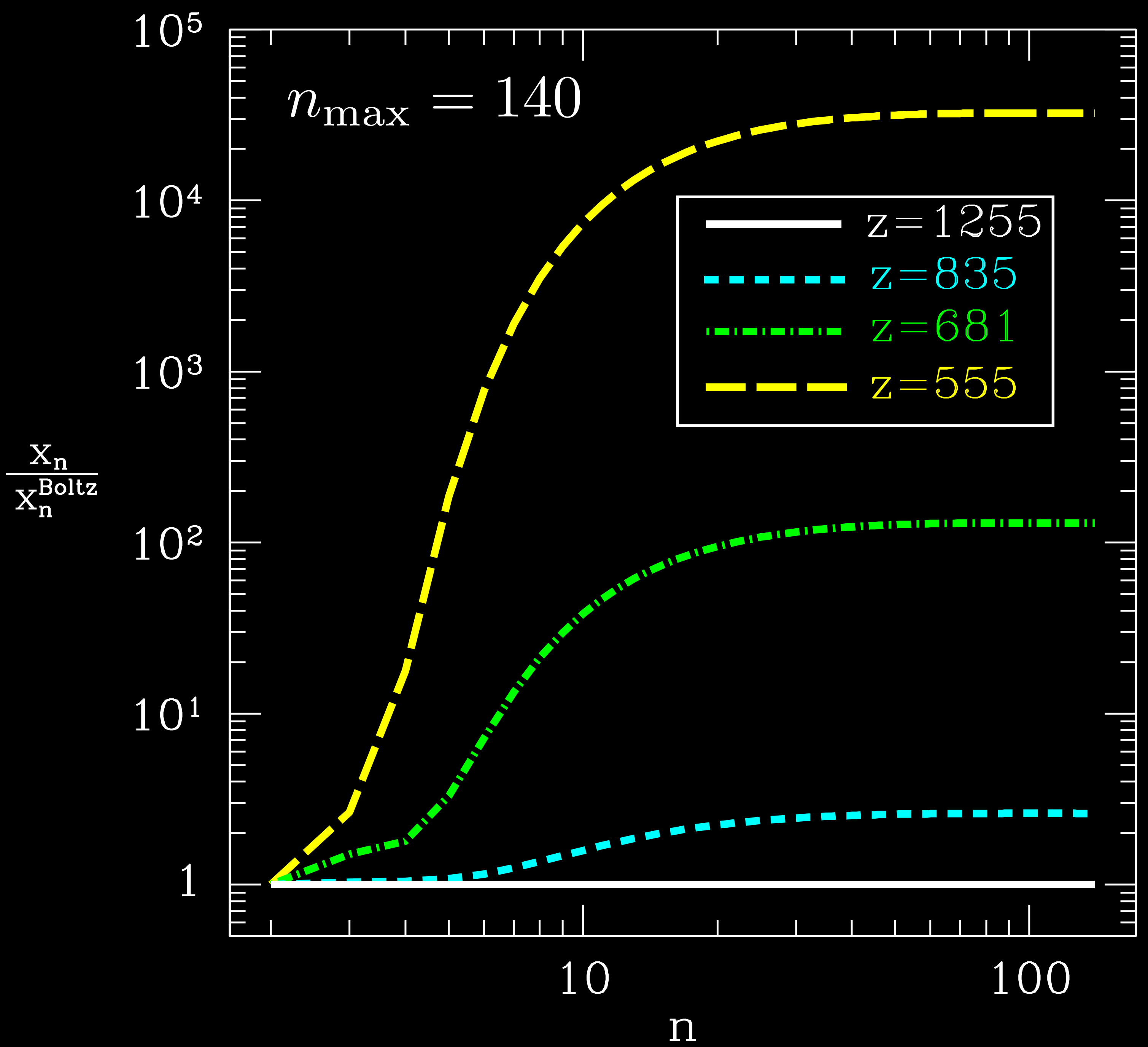


Dip moves as expected when Balmer lines are off!



- l-changing collisions bring l-substates closer to statistical equilibrium (SE) (Chluba, Rubino Martin, Sunyaev 2006)
- Theoretical collision rates unknown to factors of 2!

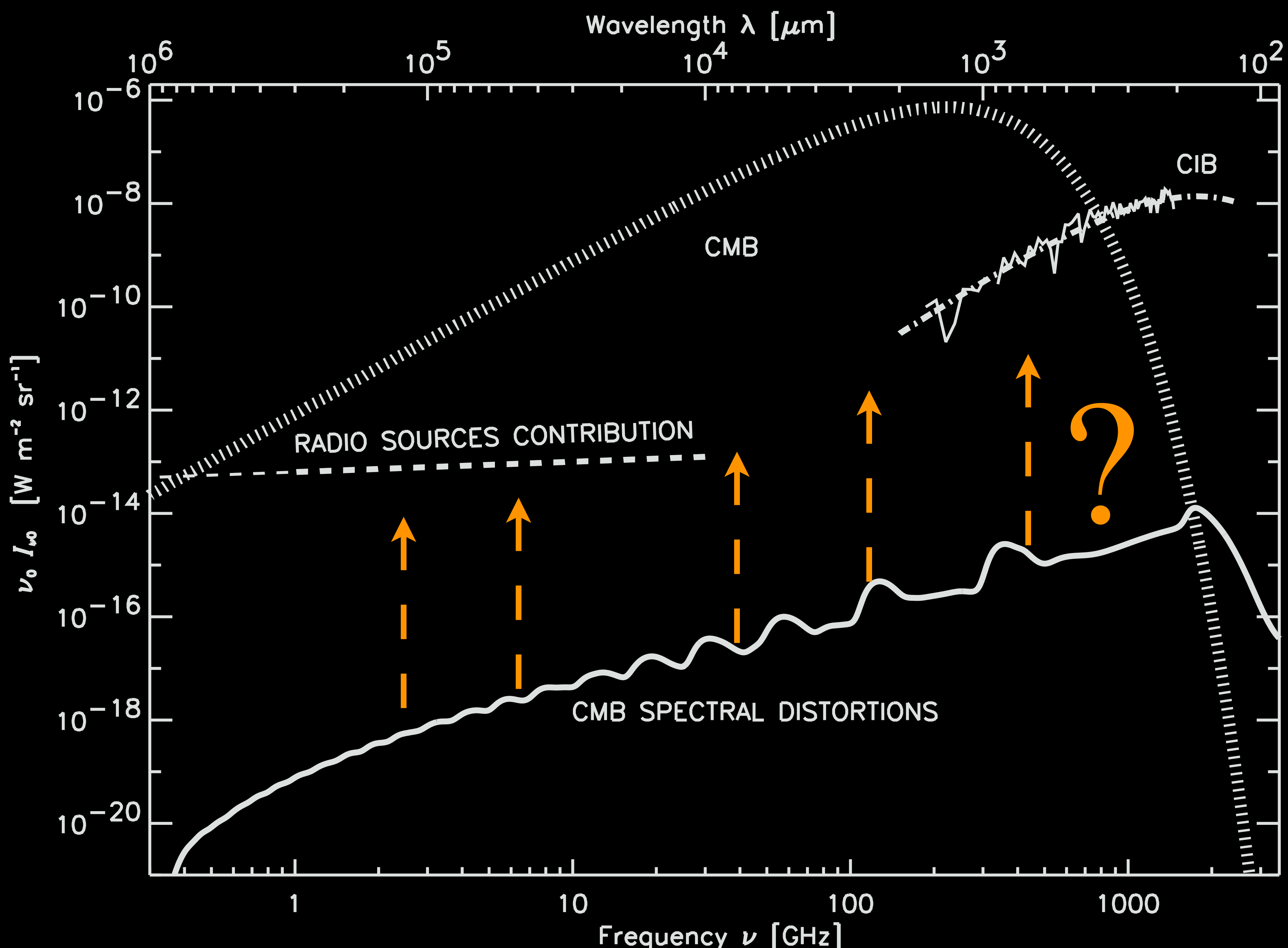
DEVIATIONS FROM BOLTZMANN EQUILIBRIUM: DIFFERENT n-SHELLS



$$\alpha_n n_e > \sum_{n'l}^{n'<n} A_{nn'}^{ll\pm1}$$

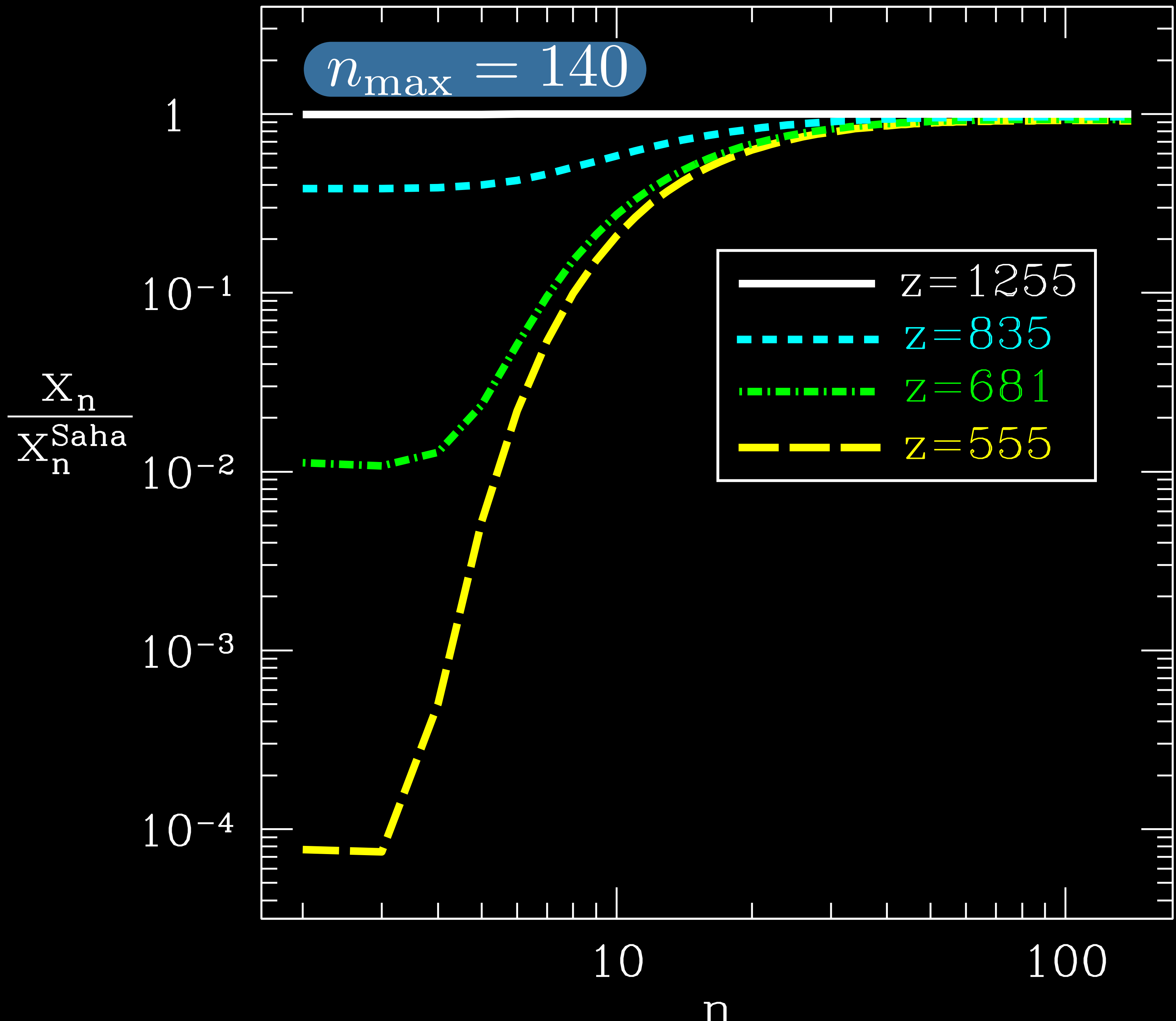
- No inversion relative to n=2 (just over-population)
- Population inversion seen between some excited states: Does radiation stay coherent? Does recombination mase?

DEVIATIONS FROM BOLTZMANN EQUILIBRIUM: DIFFERENT n-SHELLS



Masing could make spectral distortions detectable!

DEVIATIONS FROM SAHA EQUILIBRIUM



HUGE DEVIATIONS FROM SAHA EQ!

- ✳ Effect of states with $n > n_{max}$ could be approximated using asymptotic Einstein coeffs. and Saha eq, but Saha is elusive at high n/late times.
- ✳ At z=200, n_{max}~1000 needed, unless collisions included

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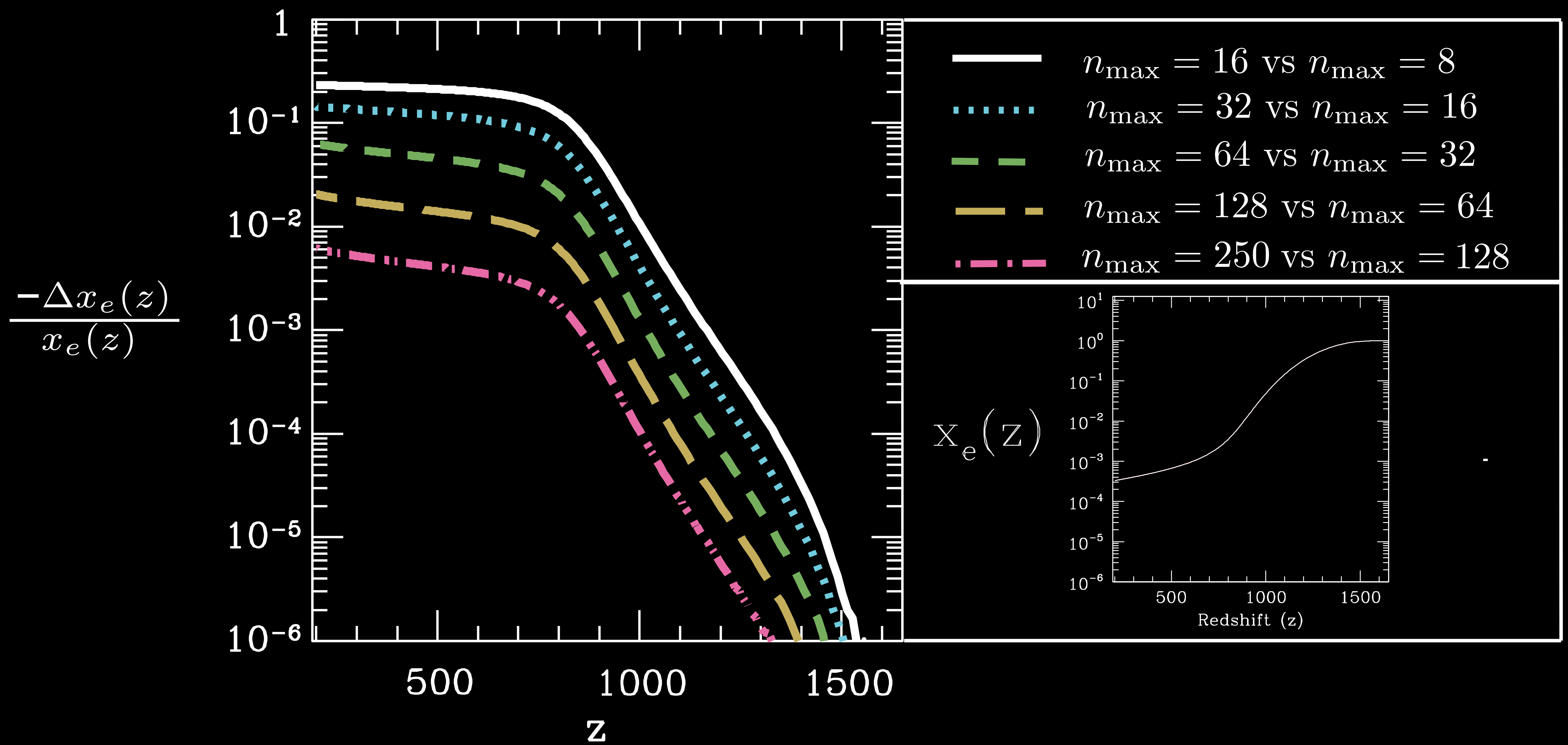
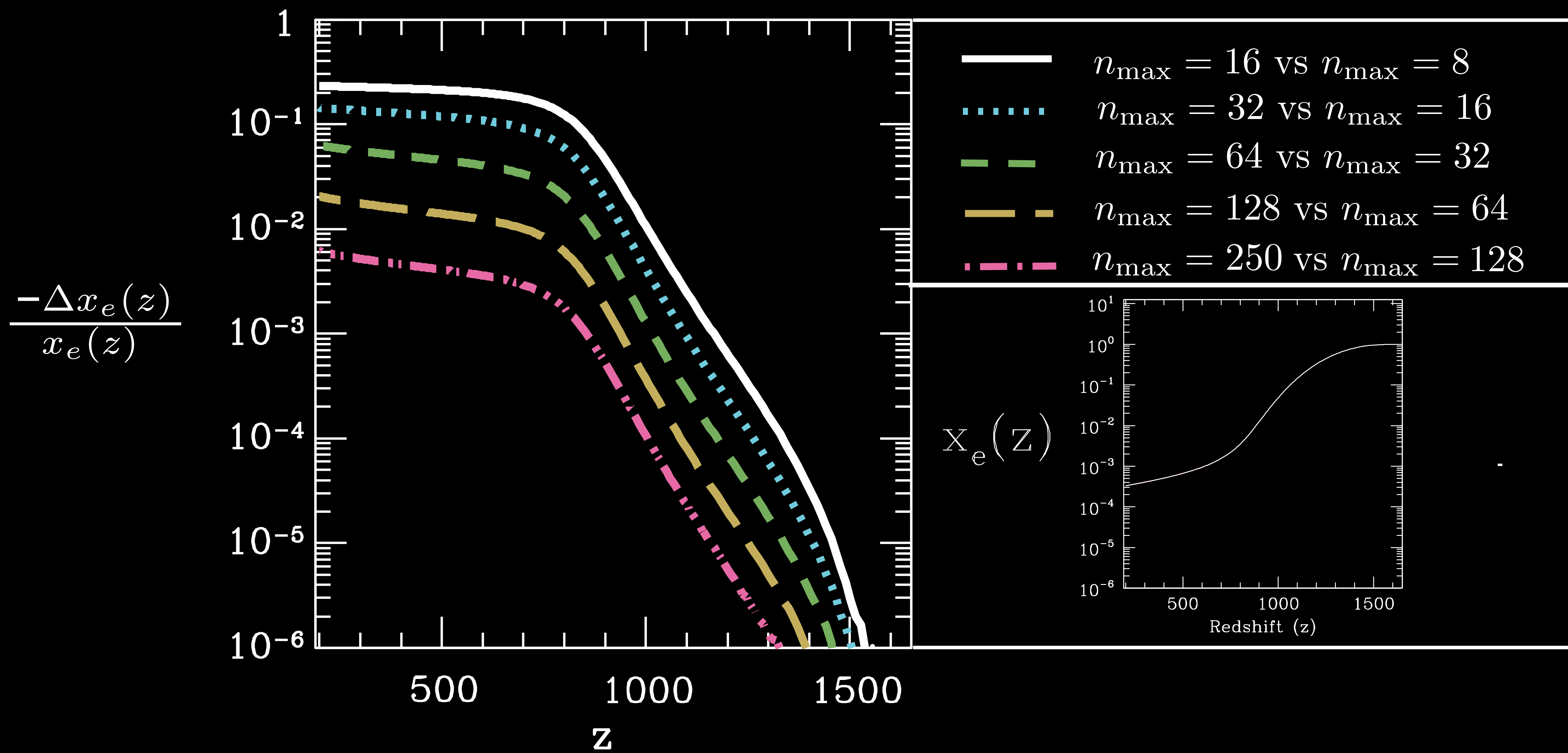
RESULTS: RECOMBINATION HISTORIES

RESULTS: RECOMBINATION HISTORIES INCLUDING HIGH-n



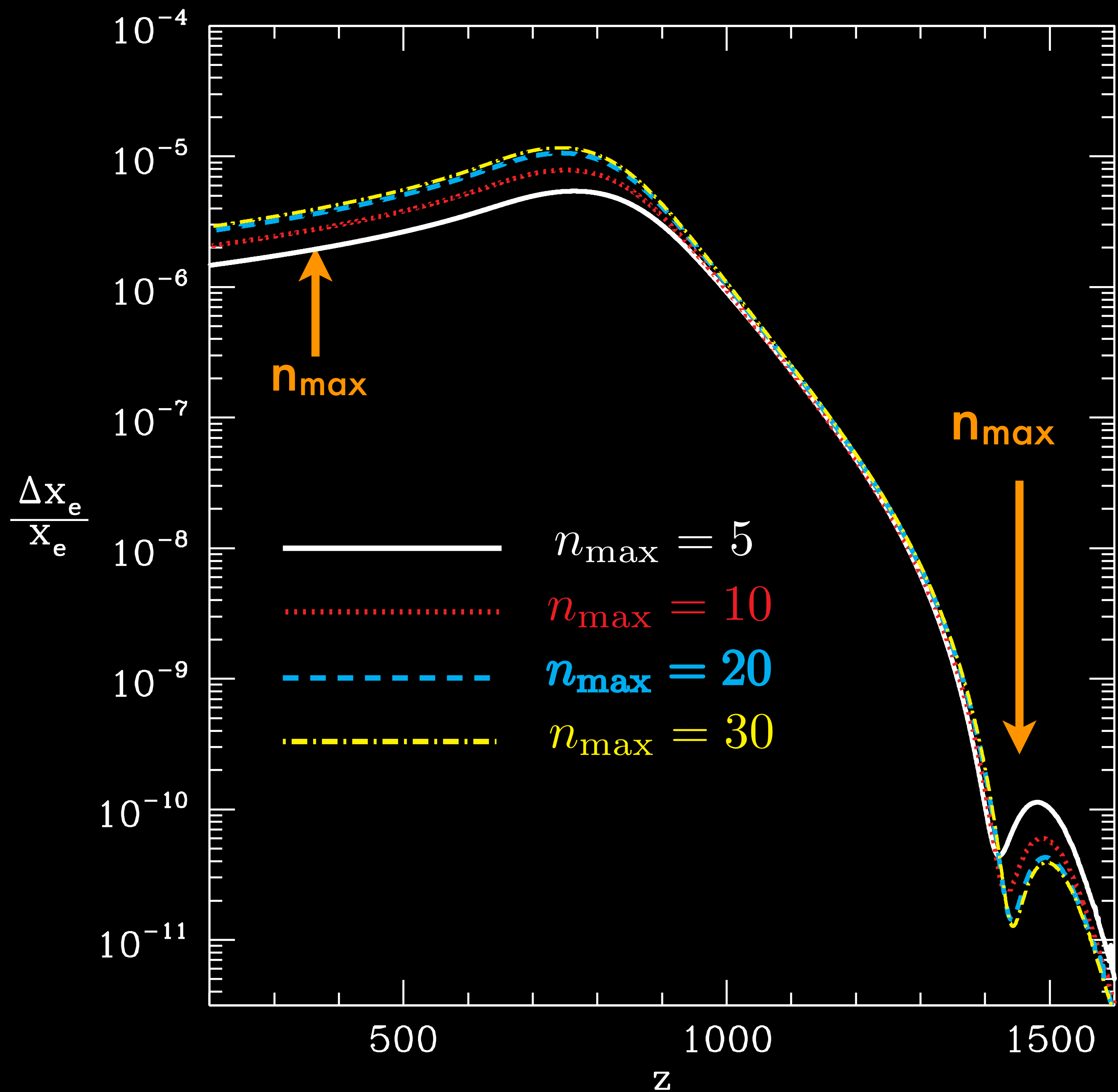
- $x_e(z)$ falls with increasing $n_{\rm max} = 10 \rightarrow 250$, as expected.
- Rec Rate>downward BB Rate> Ionization, upward BB rate
- For $n_{max} = 100$, code computes in only 2 hours

RESULTS: RECOMBINATION HISTORIES INCLUDING HIGH-n



- Relative convergence is not the same thing as absolute convergence: Want to see Saha asymptote and impose well-motivated cutoff! Collisions could help
- These are lower limits to the actual error
- n_{max}=300 just completed

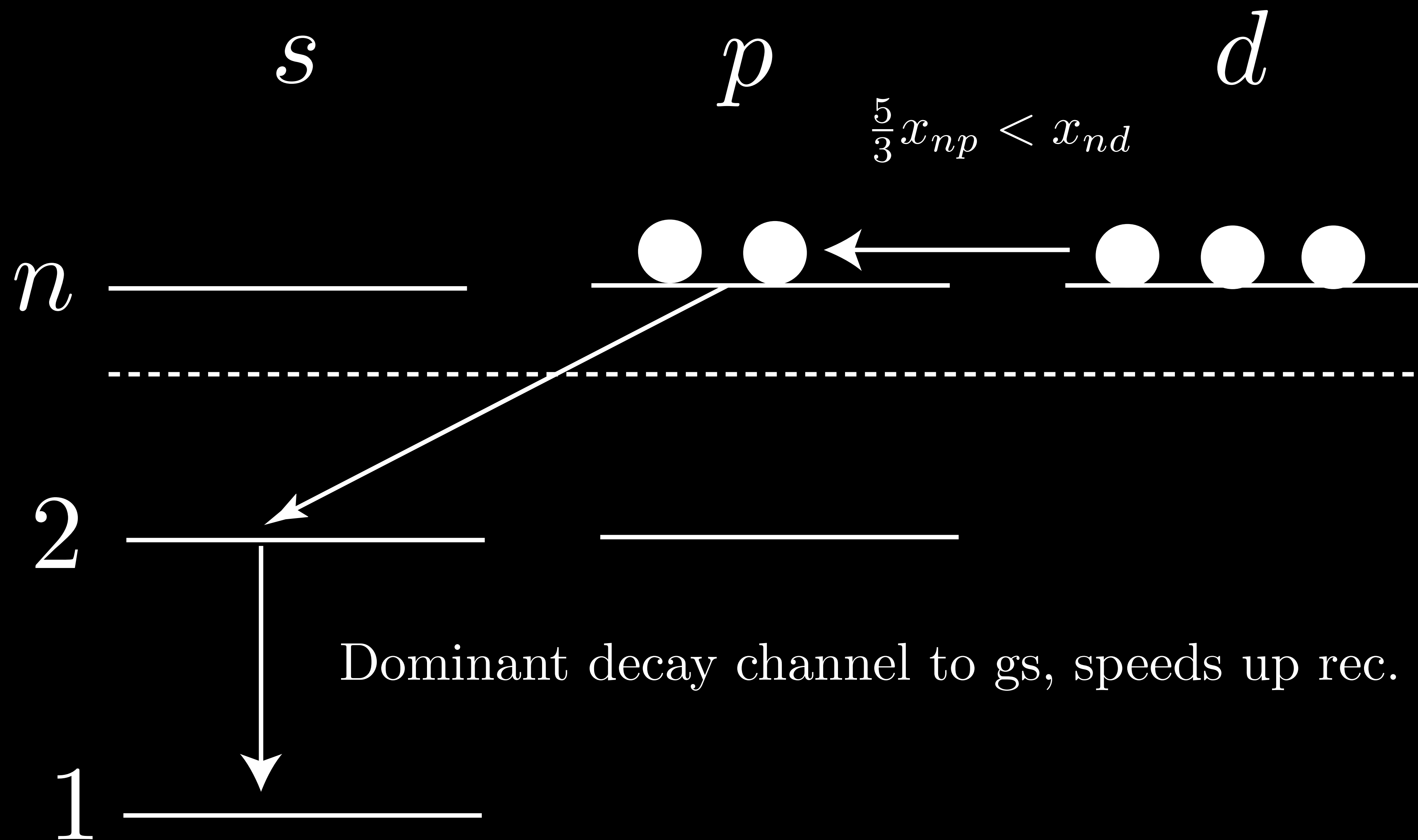
RESULTS: RECOMBINATION WITH HYDROGEN



$$\Delta x_e \equiv x_e\big|_{\text{no } E2 \text{ transitions}} - x_e\big|_{\text{with } E2 \text{ transitions}}$$

Negligible for Planck!

RESULTS: RECOMBINATION WITH HYDROGEN



$$R^{\rm quad}_{nd\rightarrow np} = A_{nd\rightarrow 1s}\left(x_{nd} - \frac{5}{3}x_{np}\right)$$

$n < 5$, early times

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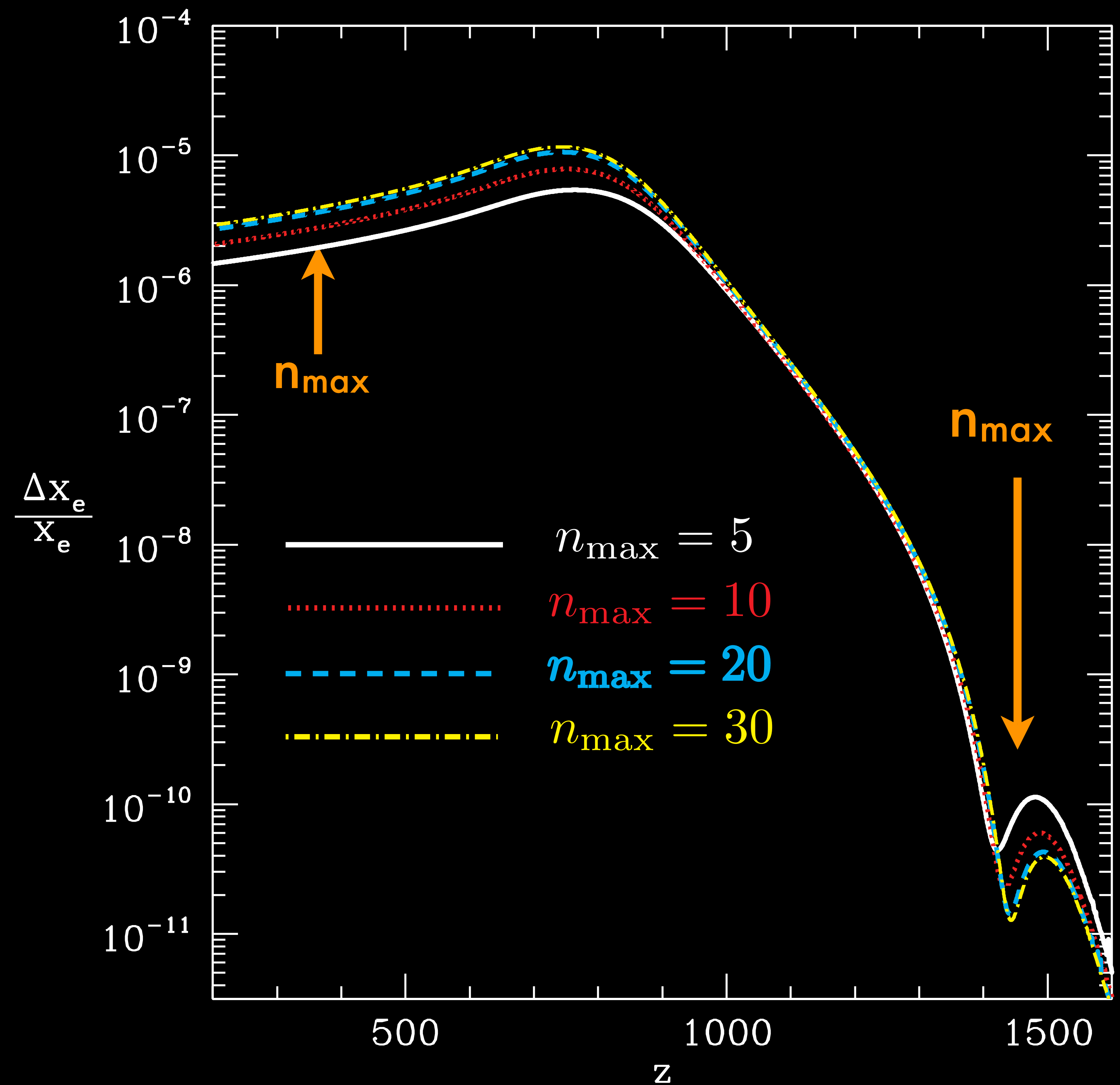
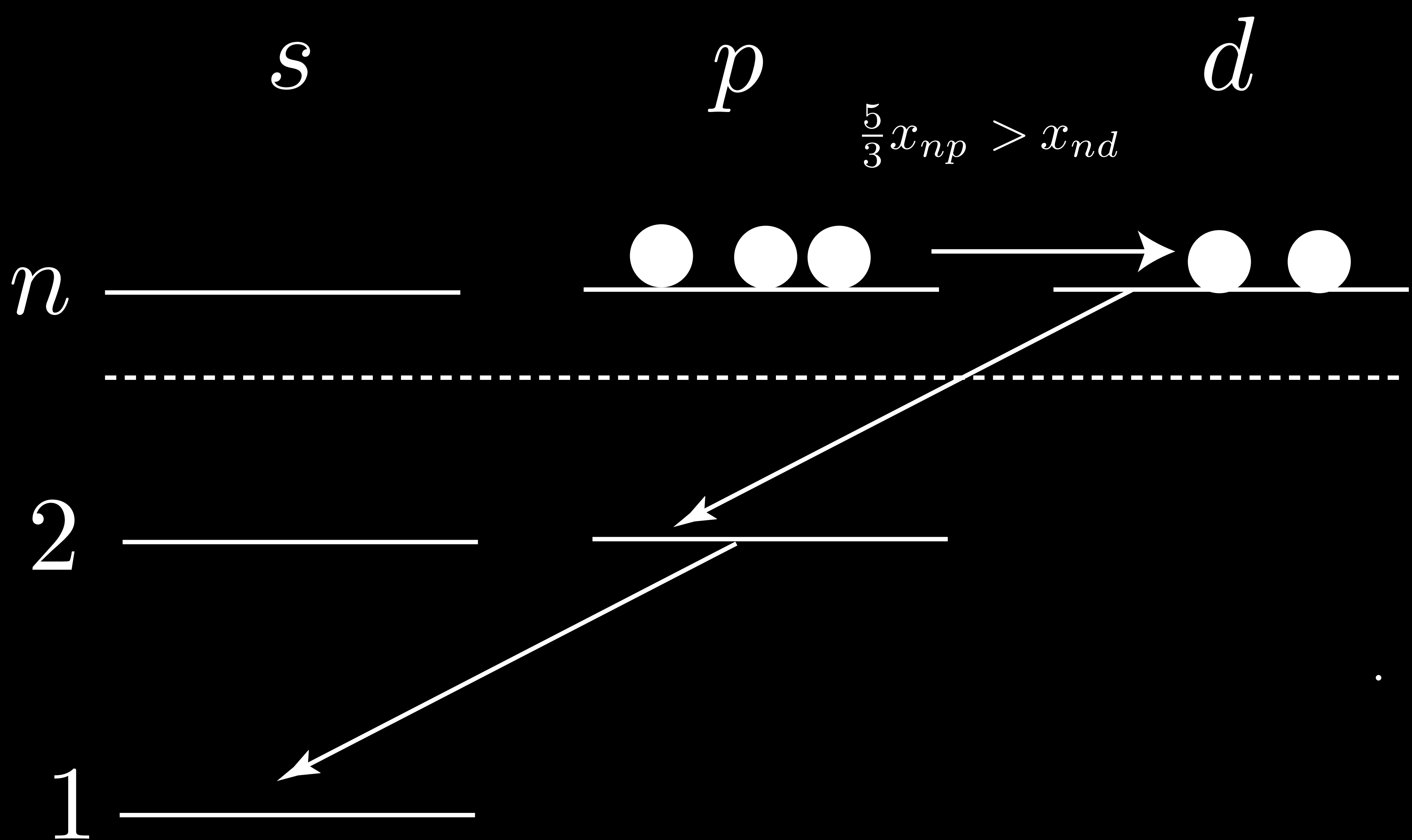
RESULTS: RECOMBINATION WITH HYDROGEN



$$\Delta x_e \equiv x_e\big|_{\text{no } E2 \text{ transitions}} - x_e\big|_{\text{with } E2 \text{ transitions}}$$

Negligible for Planck!

RESULTS: RECOMBINATION WITH HYDROGEN

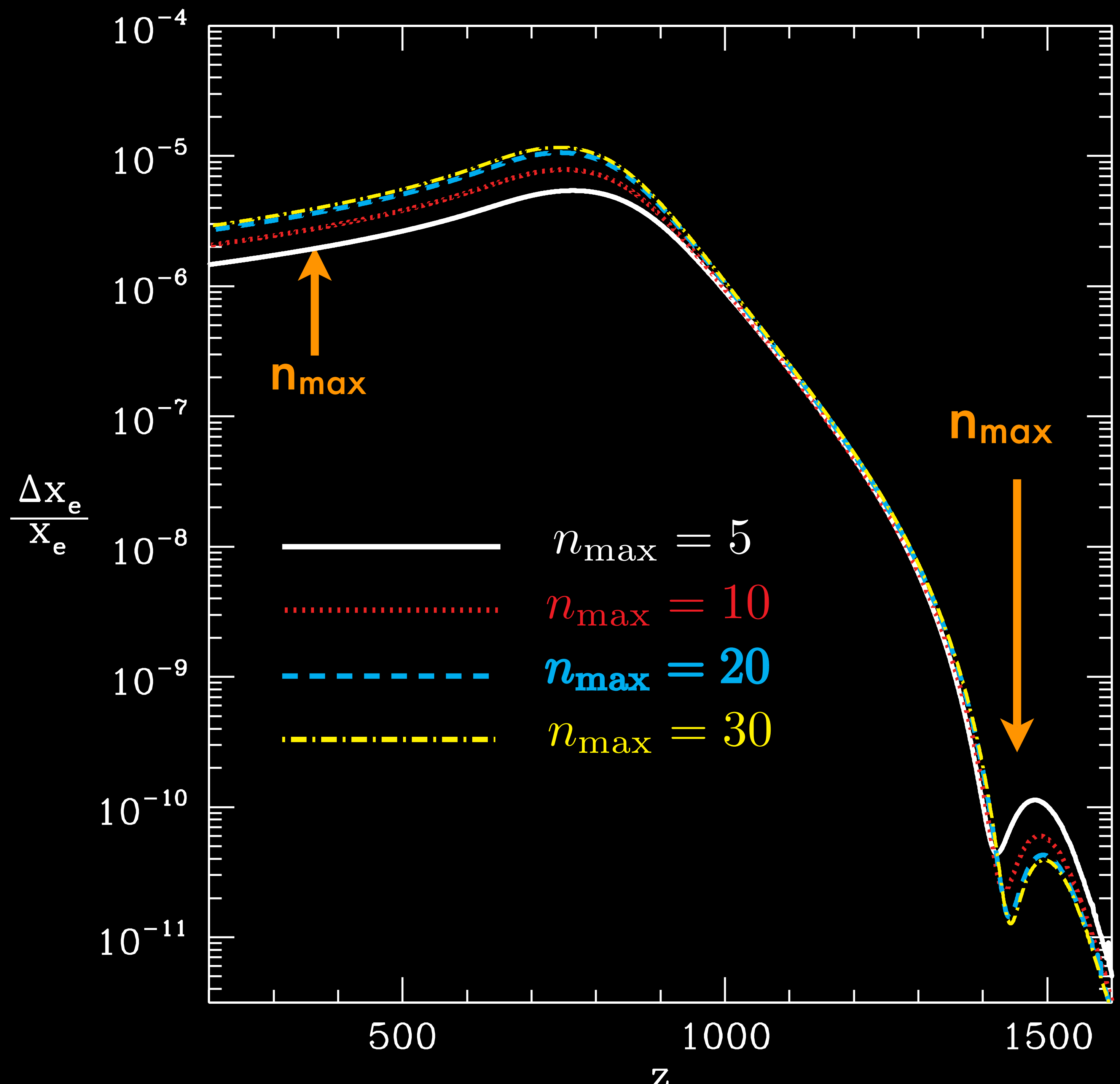


Sub-Dominant decay channel to gs, slows rec down rel. to $n < 5$

$$R^{\text{quad}}_{nd\to np} = A_{nd\to 1s}\left(x_{nd} - \frac{5}{3}x_{np}\right)$$

$n \geq 5$, early times

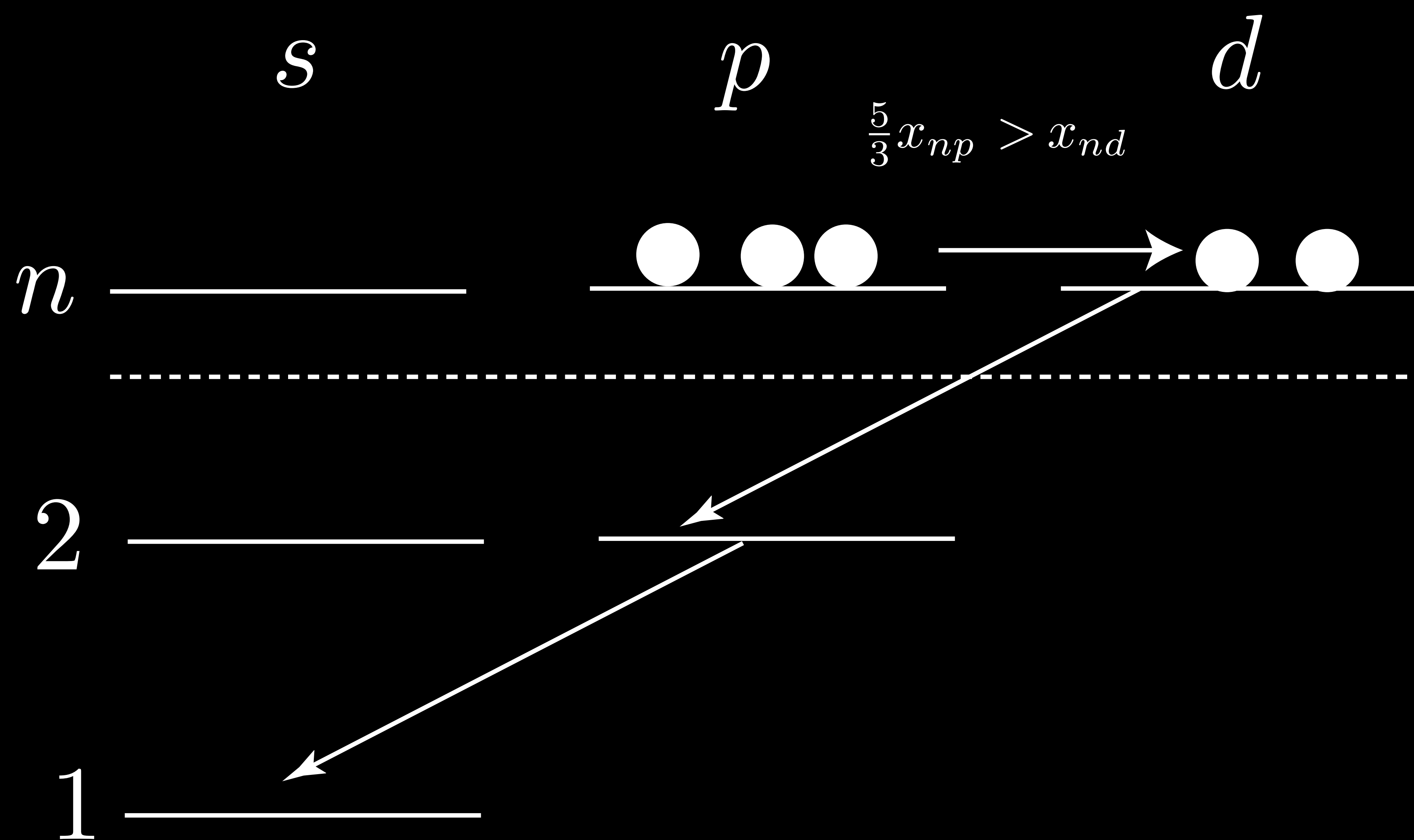
RESULTS: RECOMBINATION WITH HYDROGEN



$$\Delta x_e \equiv x_e\big|_{\text{no } E2 \text{ transitions}} - x_e\big|_{\text{with } E2 \text{ transitions}}$$

Negligible for Planck!

RESULTS: RECOMBINATION WITH HYDROGEN

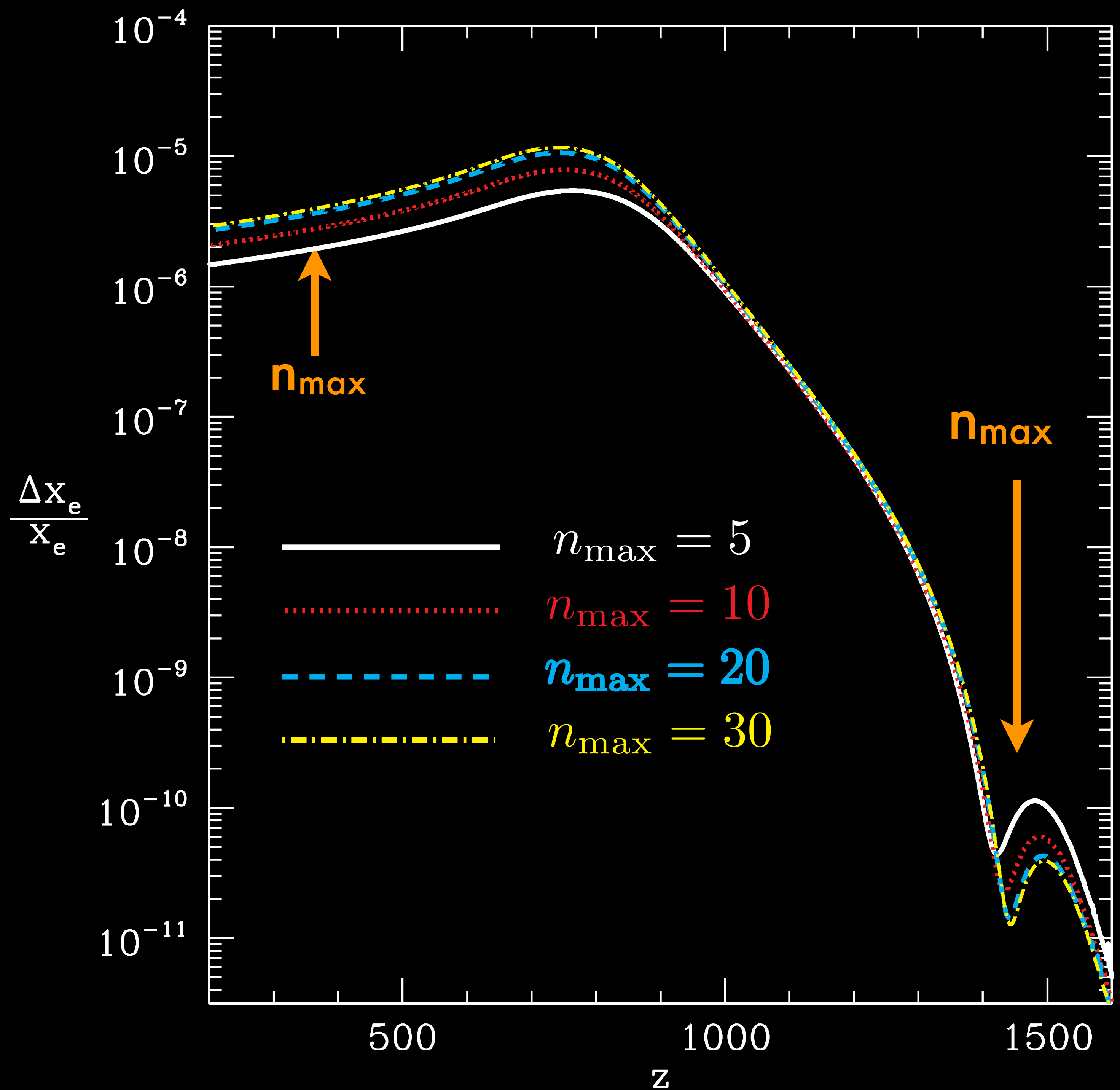


Dominant decay channel to gs, speeds up rec

$$R^{\mathrm{quad}}_{nd\to np} = A_{nd\to 1s}\left(x_{nd} - \frac{5}{3}x_{np}\right)$$

All n, late times

RESULTS: RECOMBINATION WITH HYDROGEN



$$\Delta x_e \equiv x_e\big|_{\text{no } E2 \text{ transitions}} - x_e\big|_{\text{with } E2 \text{ transitions}}$$

Negligible for Planck!

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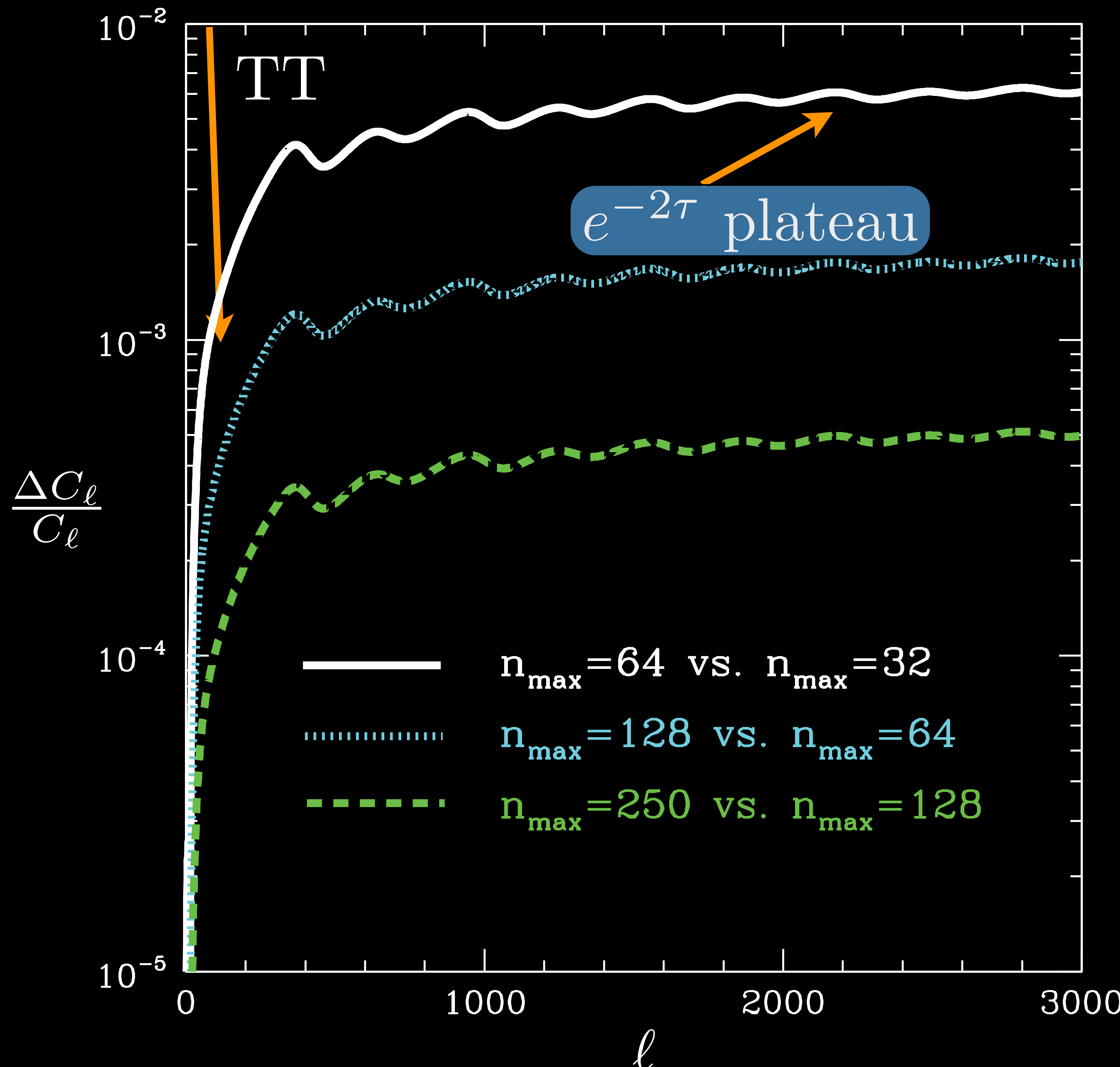
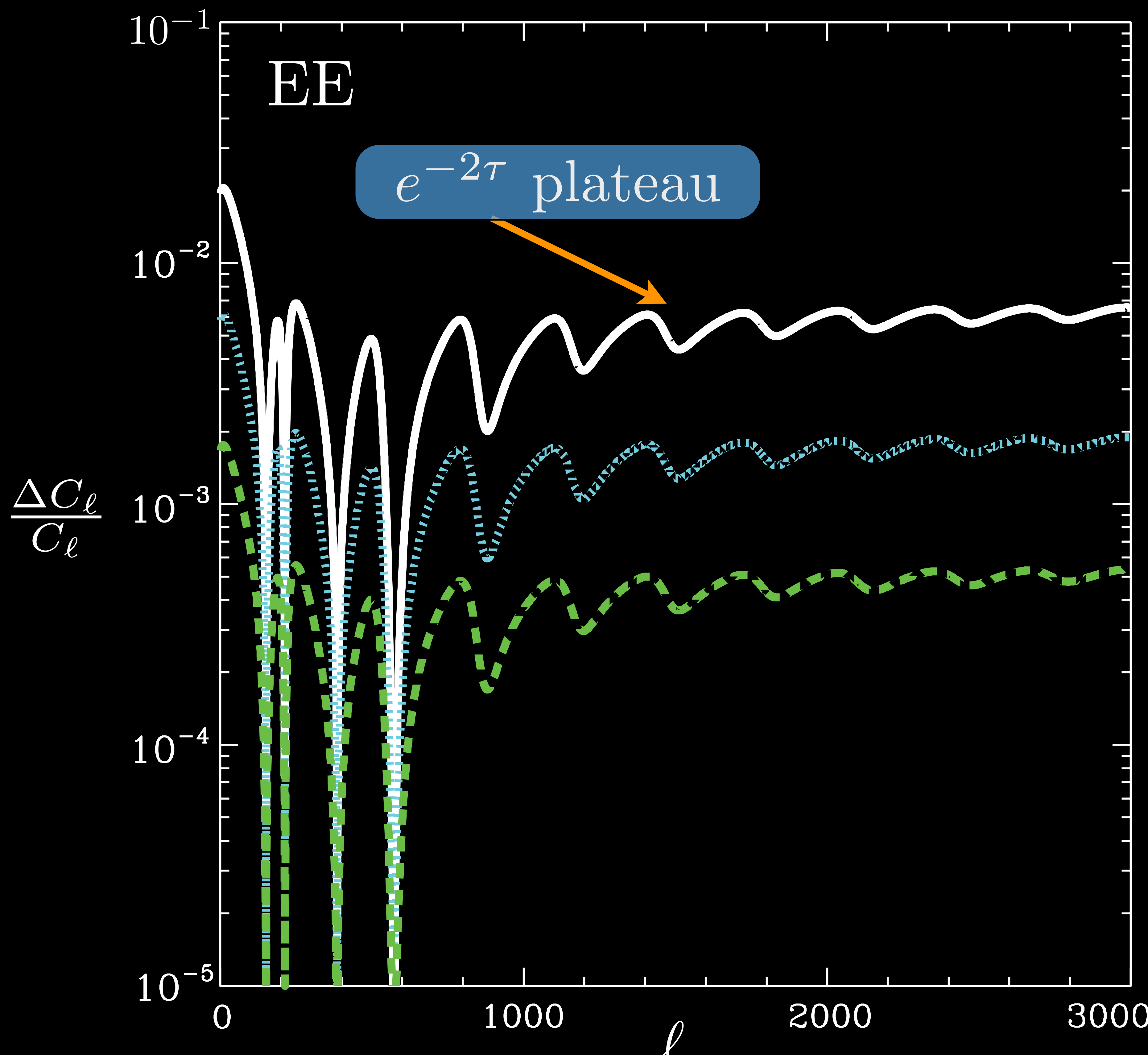
RESULTS: CMB ANISOTROPIES

RESULTS: TT $C_l s$ WITH HIGH-N STATES

Super-horizon scales don't care about recombination

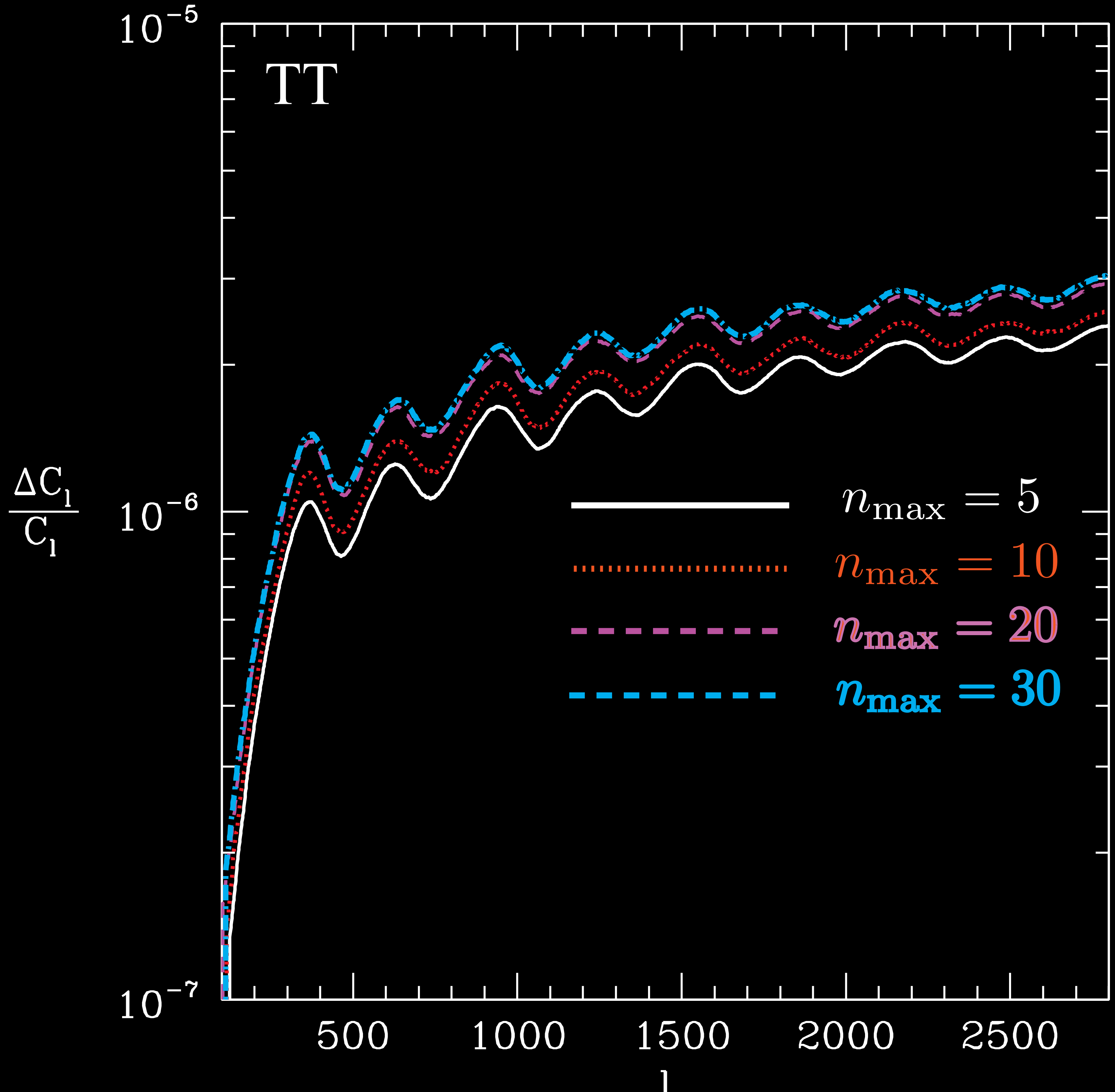


RESULTS: EE $C_l s$ WITH HIGH-N STATES



RESULTS: TEMPERATURE (TT) C_ls WITH HYDROGEN QUADRUPOLES,

Bulk of integral from late times, higher $\mathrm{n_{max}} \rightarrow$ lower $x_e \rightarrow$ lower $\tau \rightarrow$ higher $e^{-2\tau} \rightarrow$ higher C_l



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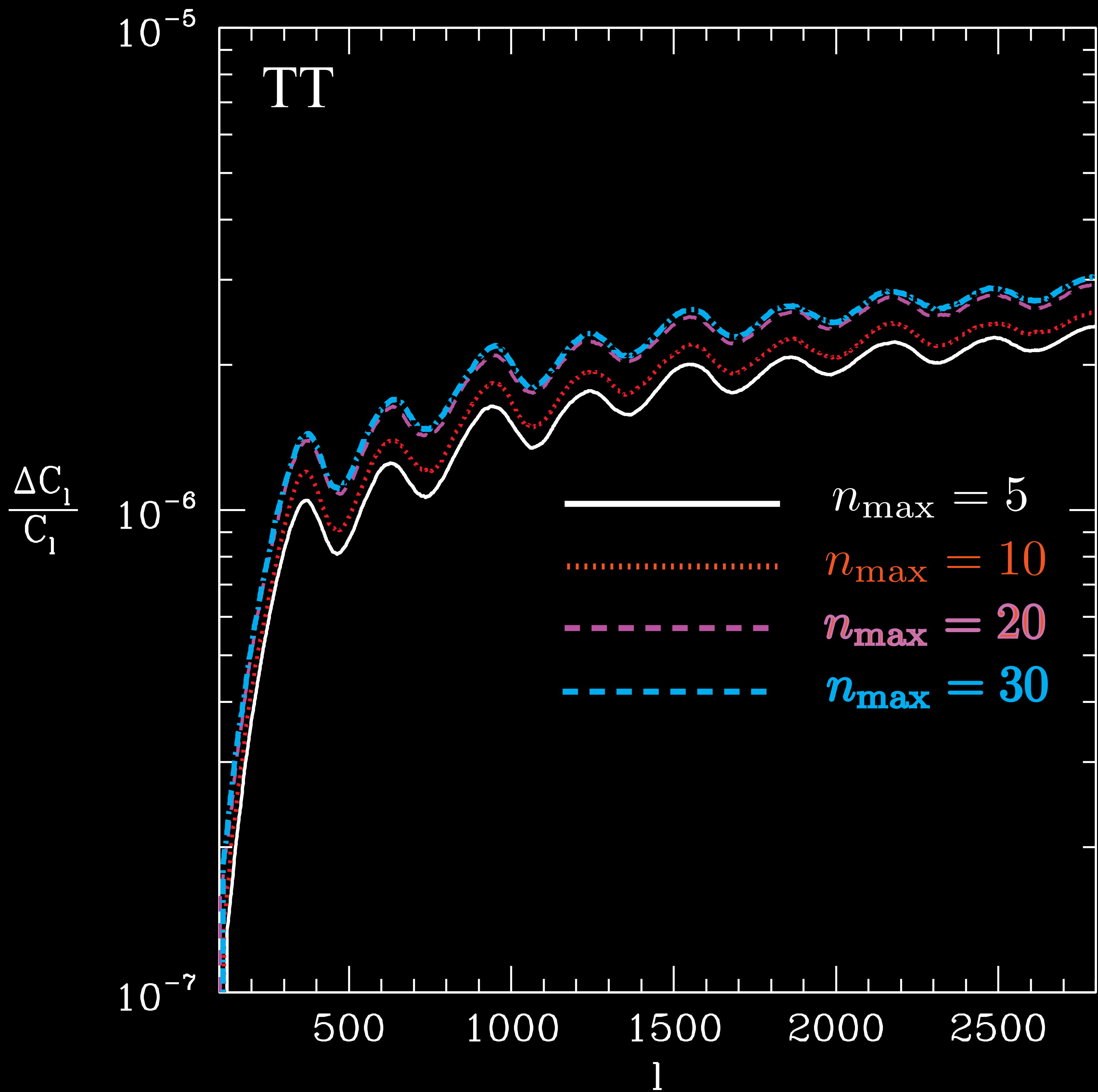
RESULTS: TEMPERATURE (TT) $C_l s$ WITH HYDROGEN QUADRUPOLES,

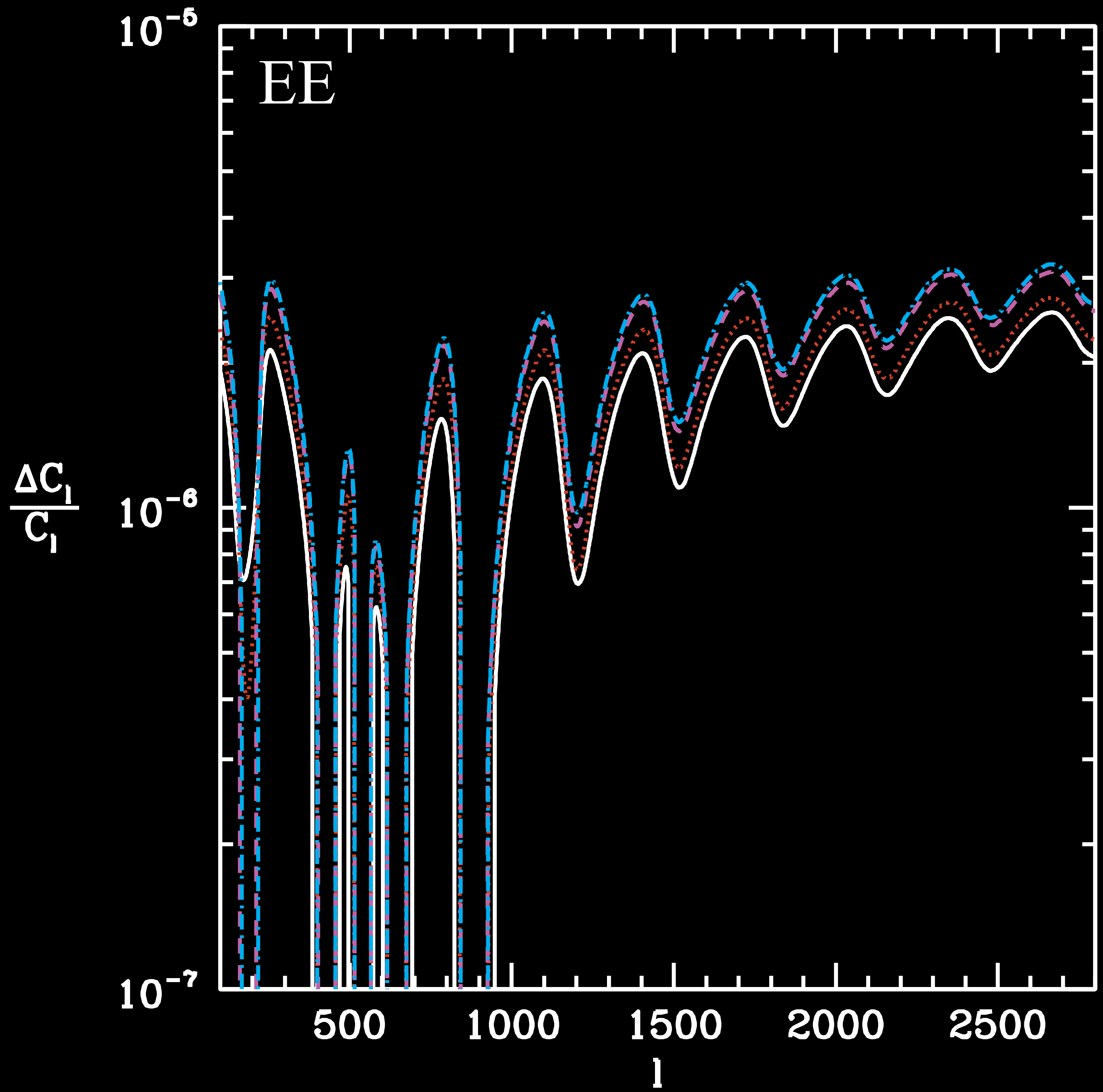
Bulk of integral from late times, higher $n_{max} \rightarrow$ lower $x_e \rightarrow$ lower $\tau \rightarrow$ higher $e^{-2\tau} \rightarrow$ higher C_l



Overall effect is negligible for CMB experiments!

40

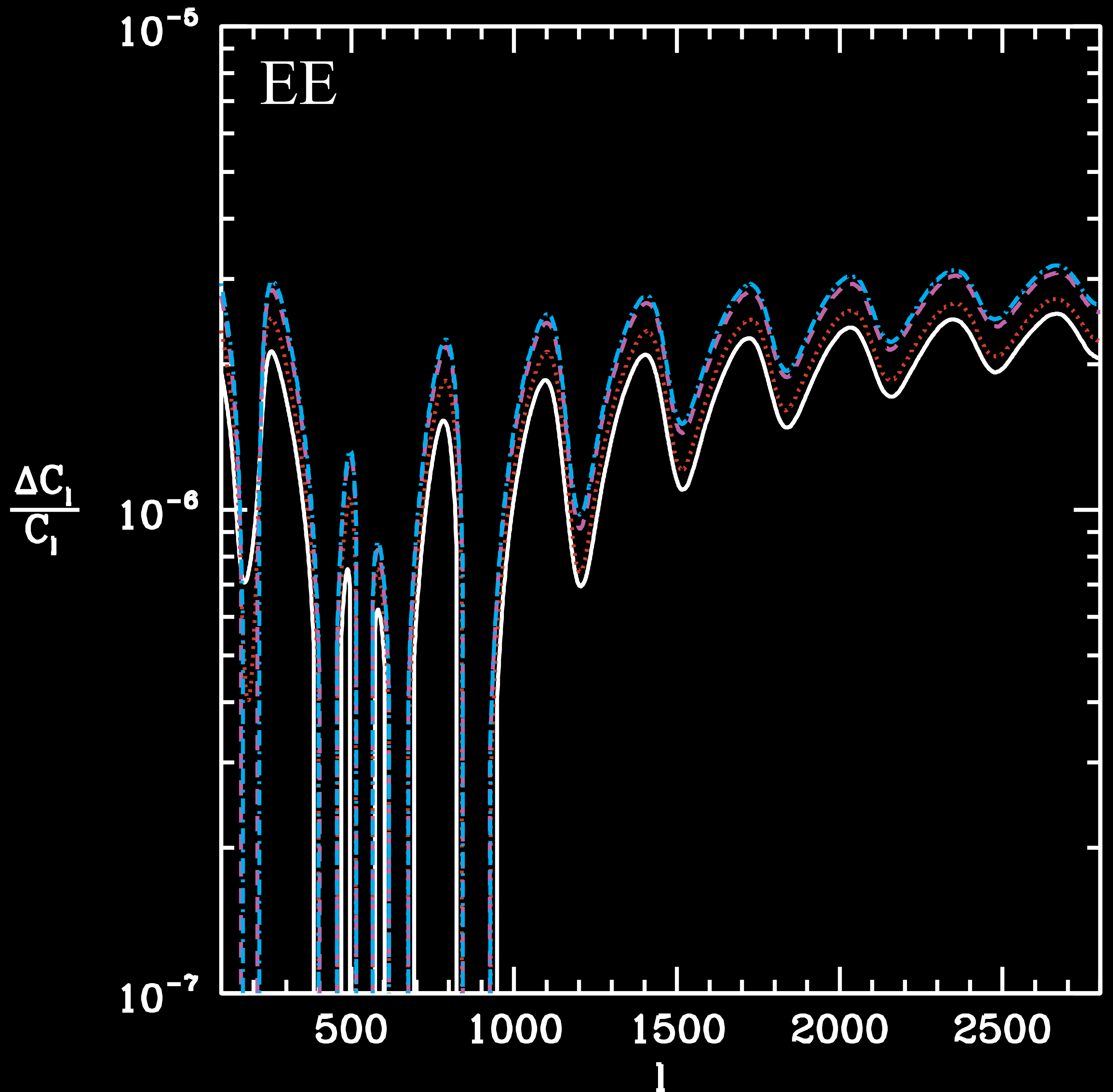
RESULTS: POLARIZATION (EE) C_ls WITH HYDROGEN QUADRUPOLES



$$\Delta C_l \equiv C_l|_{\text{with } E2 \text{ transitions}} - x_e|_{\text{no } E2 \text{ transitions}}.$$

Bulk of integral from late times, higher n_{max} $\rightarrow$ lower x_e $\rightarrow$ lower τ $\rightarrow$ higher $e^{-2\tau}$ $\rightarrow$ higher C_l

RESULTS: POLARIZATION (EE) C_ls WITH HYDROGEN QUADRUPOLES

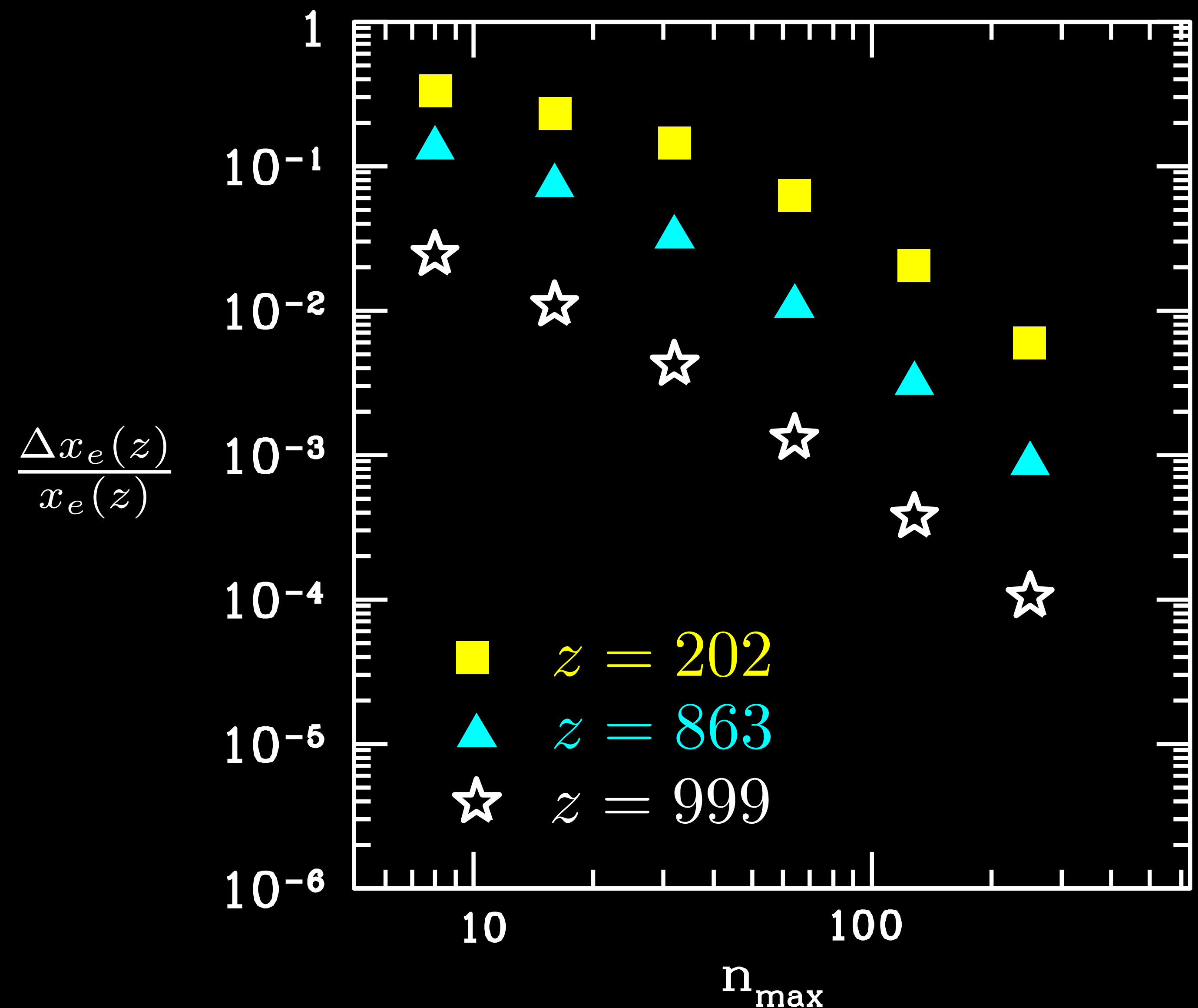


$$\Delta C_l \equiv C_l|_{\text{with } E2 \text{ transitions}} - x_e|_{\text{no } E2 \text{ transitions}}.$$

Overall effect is negligible for upcoming CMB experiments!

Bulk of integral from late times, higher $n_{\max} \rightarrow$ lower x_e $\rightarrow$ lower $\tau \rightarrow$ higher $e^{-2\tau} \rightarrow$ higher C_l

CONVERGENCE



- ✳ Relative error well described by power law at high n_{max}

$$\Delta x_e / x_e \propto n_{\mathrm{max}}^{-1.9}$$

- ✳ Can extrapolate to absolute error

THE UPSHOT FOR COSMOLOGY

- Can explore effect on overall Planck likelihood analysis

$$Z^2 = \sum_{ll',X,Y} F_{ll'} \Delta C_l^{\mathrm{X}} \Delta C_l^{\mathrm{Y}}$$

$$Z = 1.8 \text{ if } n_{\max} = 64,$$
$$Z = 0.50 \text{ if } n_{\max} = 128,$$
$$Z = 0.14 \text{ if } n_{\max} = 250.$$

CONCLUSIONS

- RecSparse: a new tool for MLA recombination calculations (***arXiv:0911.1359***)
- Highly excited levels (n~64 and higher) are relevant for Planck CMB data analysis
- E2 transitions in H are not relevant for Planck CMB data analysis

FUTURE WORK

- ✳ Include line-overlap
- ✳ Develop cutoff method for excluded levels
- ✳ Generalize RecSparse to calc. rec. line. spectra
- ✳ Compute and include collisional rates
- ✳ Monte-Carlo analyses
- ✳ Cosmological masers

Bound-free rates

- Using continuum wave functions, bound-free rates are obtained (Burgess 1957)
- Bound-free matrix elements satisfy a convenient recursion relation:

- Matrix elements compared with Burgess 1965 (5 digits) and with WKB approximation (5%):
- At each temperature, thermal recombination/ionization rates obtained using 11-point Newton-Cotes formula, agreement with Burgess to 4 published digits

BB Rate coefficients: verification

- WKB estimate of matrix elements

$$\rho(n'l', nl) = a_0 n^2 \int_{-\pi}^{\pi} d\tau e^{i\Omega\tau}\,(1 + cos\eta)$$

$$\Omega = \omega_n - \omega_{n'}$$

$$r = r_{\max}\,(1 + \cos\eta)\,/2$$

$$\tau = \eta + \sin\eta$$

Fourier transform of classical orbit! Application of correspondence principle!

$$\rho^{\mathrm{dipole}}(n, l, n', l') = \frac{n_c^2}{s}\left\{J_{s-1}(s\epsilon) - \frac{1 \mp \sqrt{1-\epsilon^2}}{\epsilon} J_s(s\epsilon)\right\}$$

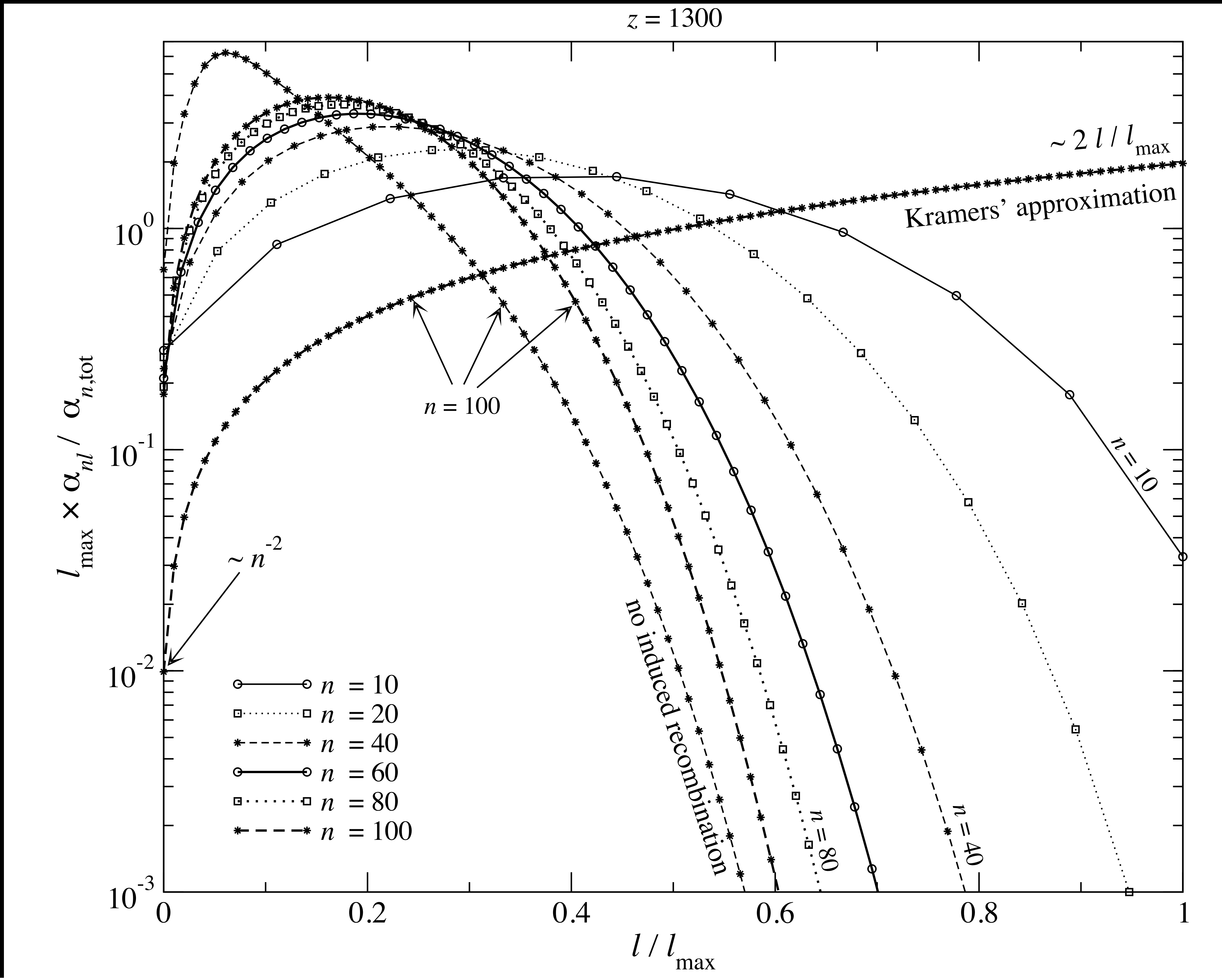
$$\epsilon = \left(1 - \frac{l(l+1)}{n^2}\right)^{1/2}$$

$$s = n - n'$$

- Radial matrix elements checked against WKB (10%), published rates of Brocklehurst (1971), Green, Rush, and Chandler (1967) (agreement to their published 4 digits)

DEVIATIONS FROM BOLTZMANN EQ: L-SUBSTATES

Chluba/Rubino-Martin/Sunyaev 2006



Quadrupole rates: basic formalism

$$A^{\text{quad}}_{n_a,l_a\to n_b,l_b} = \frac{\alpha}{15}\frac{1}{2l_a+1}\frac{\omega_{ab}^5}{c^4}\left\langle l_a||C^{(2)}||l_b\right\rangle^2 \left({}^2R^{n_a l_a}_{n_b l_b}\right)^2$$

- Reduced matrix element evaluated using Wigner 3J symbols:

$$\left\langle l_a||C^{(2)}||l_b\right\rangle = (-1)^{l_a}\sqrt{(2l_a+1)(2l_b+1)}\begin{pmatrix} l_a & 2 & l_b \\ 0 & 0 & 0 \end{pmatrix}$$

- Radial matrix element evaluated using operator methods

$${}^2R^{n_a l_a}_{n_b l_b} \equiv \int_0^\infty r^4 R_{n_a l_a}(r) R_{n_b l_b}(r)\,dr$$

Quadrapole rates: Operator algebra

- Radial Schrödinger equation can be factored to yield:

$$^{-}\Omega_{nl} = \frac{1}{lA_{nl}}\left[1 - l\left(\frac{d}{dr} + \frac{l+1}{r}\right)\right] \qquad ^{+}\Omega_{nl} = \frac{1}{lA_{nl}}\left[1 + l\left(\frac{d}{dr} - \frac{l-1}{r}\right)\right]$$

$$^{-}\Omega_{nl}R_{nl}(r) = R_{n\ l-1}(r)$$
$$^{+}\Omega_{n\ l-1}R_{nl}(r) = R_{nl}(r)$$

$$A_{nl} = \frac{\sqrt{n^2 - l^2}}{nl}$$

- This algebra can be applied to radial matrix elements:

Quadrapole rates: Operator algebra

- Radial Schrödinger equation can be factored to yield:

$$ {}^{-}\Omega_{nl} = \frac{1}{l A_{nl}} \left[1 - l \left(\frac{d}{dr} + \frac{l+1}{r} \right) \right] \qquad {}^{+}\Omega_{nl} = \frac{1}{l A_{nl}} \left[1 + l \left(\frac{d}{dr} - \frac{l-1}{r} \right) \right] $$

$$ {}^{-}\Omega_{nl} R_{nl}(r) = R_{n\ l-1}(r) \qquad A_{nl} = \frac{\sqrt{n^2 - l^2}}{nl} $$

$$ {}^{+}\Omega_{n\ l-1} R_{nl}(r) = R_{nl}(r) $$

- This algebra can be applied to radial matrix elements:

$$ {}^{2}R^{n\ l-1}_{n'\ l-1} = \frac{1}{A_{nl}} \left\{ A_{n'l}\, {}^{2}R^{nl}_{n'l} + 2\, {}^{(1)}R^{nl}_{n'\ l-1} \right\} \qquad {}^{(2)}R^{n\ n'-1}_{n'\ n'-1} = \frac{2nn'}{\sqrt{n^2 - n'^2}}\, {}^{(1)}R^{nn'}_{n\ n'-1} $$

Diagonal!

Quadrapole rates: Operator algebra

- Radial Schrödinger equation can be factored to yield:

$$ {}^{-}\Omega_{nl} = \frac{1}{lA_{nl}}\left[1 - l\left(\frac{d}{dr} + \frac{l+1}{r}\right)\right] \qquad {}^{+}\Omega_{nl} = \frac{1}{lA_{nl}}\left[1 + l\left(\frac{d}{dr} - \frac{l-1}{r}\right)\right] $$

$$ {}^{-}\Omega_{nl} R_{nl}(r) = R_{n\ l-1}(r) \qquad A_{nl} = \frac{\sqrt{n^2 - l^2}}{nl} $$
$$ {}^{+}\Omega_{n\ l-1} R_{nl}(r) = R_{nl}(r) $$

- This algebra can be applied to radial matrix elements:

$$ l(2l{+}3)A_{n'l}{}^{(2)}R^{n\ l+1}_{n'\ l-1} = (2l{+}1)(l{+}2)A_{n\ l+2}{}^{(2)}R^{n\ l+2}_{n'l} + 2(l{+}1)A_{n'\ l+1}{}^{(2)}R^{n\ l+1}_{n'\ l+1} + 2(2l+1)(3l+5)^{(1)}R^{n\ l+1}_{n'l} \quad (1 \le l \le\ n'-1) $$

$$ {}^{(2)}R^{n\ n'-1}_{n'\ n'+1} = 0 $$

$$ {}^{(2)}R^{n\ n'+1}_{n'\ n'-1} = (-1)^{n-n'} 2^{2n'+4}\left[\frac{(n{+}n'{+}1)!}{(n{-}n'{-}2)!(2n'{-}1)!}\right]^{1/2} n'\,(nn')^{n'+3}\,\frac{(n{-}n')^{n-n'-3}}{(n{+}n')^{n+n'+3}} $$

Off-diagonal!