COSMOLOGICAL HYDROGEN RECOMBINATION:
The effect of extremely high-n states and forbidden transitions

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University of Pennsylvania Seminar

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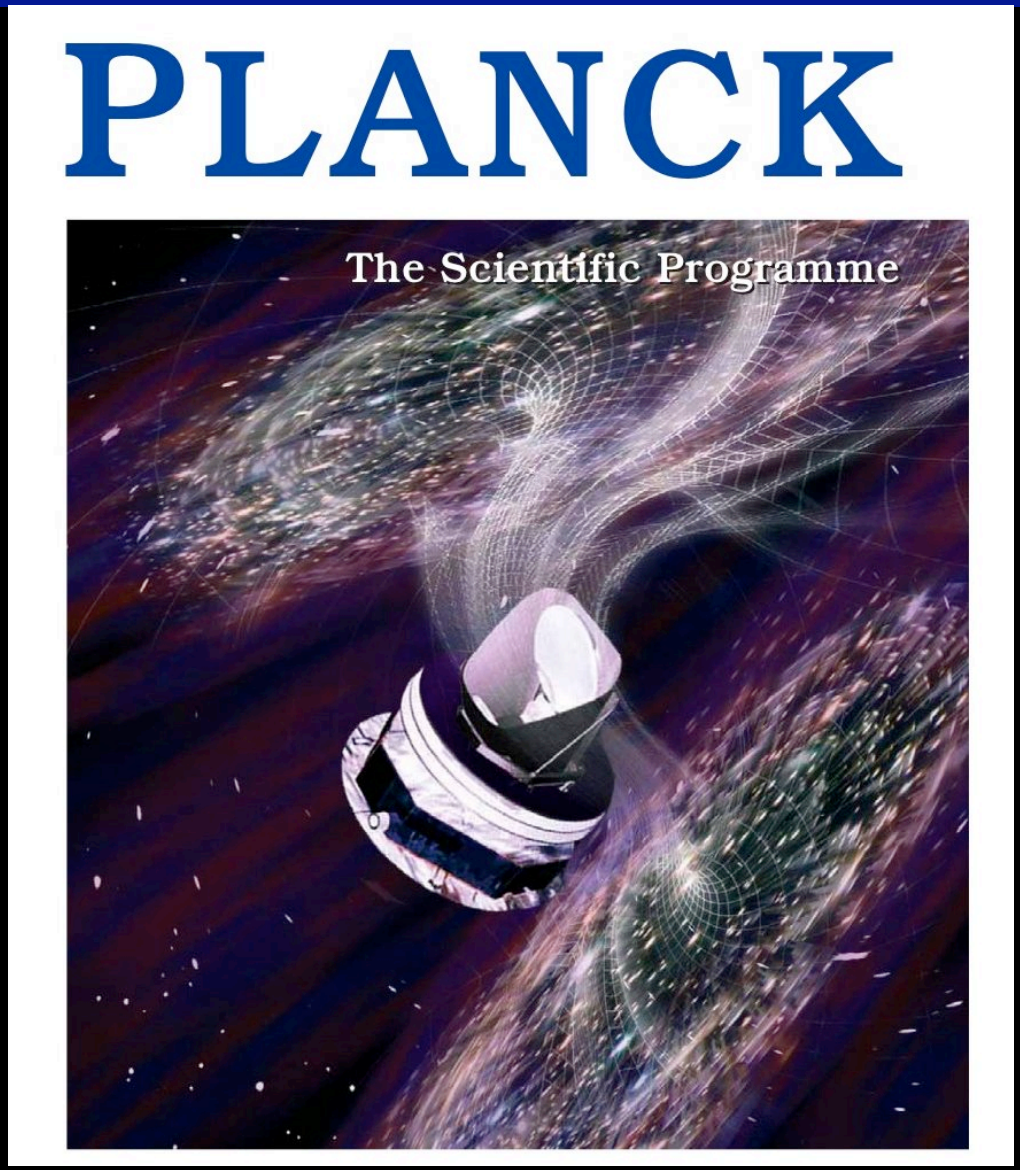
OUTLINE

- Motivation: CMB anisotropies and recombination spectra
- Recombination in a nutshell
- Breaking the Peebles/RecFAST mold
- RecSparse: a new tool for high-n states
- Forbidden transitions
- Results
- Ongoing/future work

WALK THE PLANCK

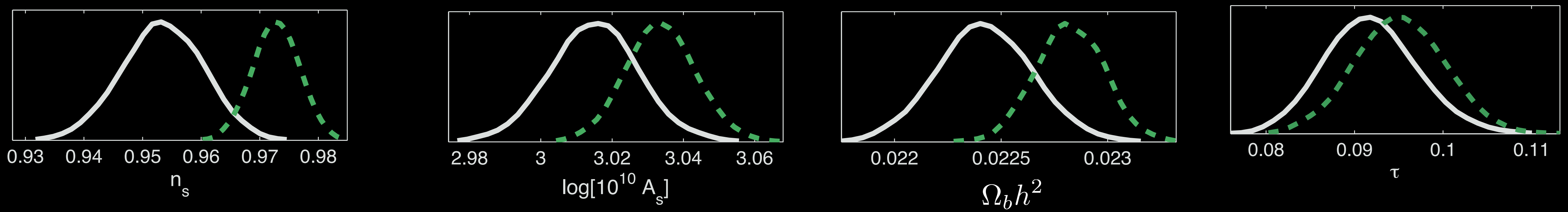


- Planck (launched May 2009) will make cosmic-variance limited CMB anisotropy measurements up to l~2500 (T), and l~1500 (E)
- Wong 2007 and Lewis 2006 show that $x_e(z)$ needs to be predicted to several parts in 10^4 accuracy for Planck data analysis

RECOMBINATION, INFLATION, AND REIONIZATION

$$P(k) = A_s \left(k\eta_0\right)^{n_s}$$

- Planck uncertainty forecasts using MCMC



- Cosmological parameter inferences will be off if recombination is improperly modeled (Wong/Moss/Scott 2007)
- Leverage on new physics comes from high l. Here the details of recombination matter!
- Inferences about inflation will be wrong if recombination is improperly modeled

$$\mathrm{n}_s = 1 - 4\epsilon + 2\eta \qquad \epsilon = \frac{m_{\rm pl}^2}{16\pi}\left[\frac{V'(\phi)}{V(\phi)}\right]^2 \qquad \mathrm{A}_s^2 = \frac{32}{75}\frac{V}{m_{\rm pl}^4\epsilon}\bigg|_{k_{\rm pivot}=aH}$$

CAVEAT EMPTOR: $3 \lesssim\ ?\ \lesssim 16$

Need to do eV physics right to infer anything about $10^?$ GeV physics!

RECOMBINATION, INFLATION, AND REIONIZATION

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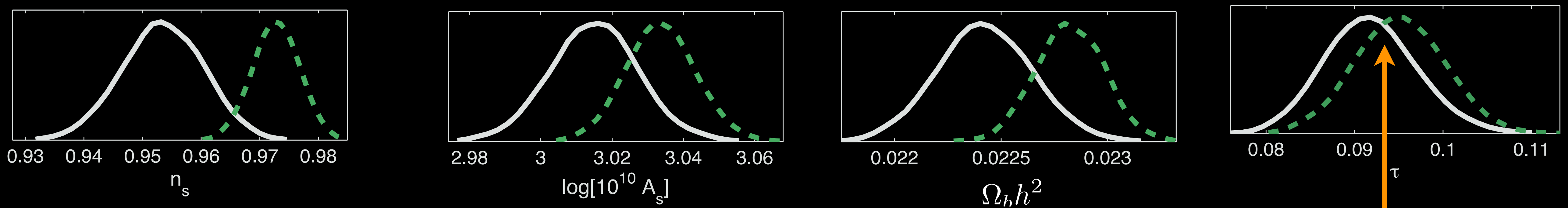

Bad recombination history yields biased inferences about reionization

PHYSICAL RELEVANCE FOR CMB: SMEARING AND MOVING THE SURFACE OF LAST SCATTERING (SLS)

- Photons kin. decouple when Thompson scattering freezes out

$$\gamma + e^- \Leftrightarrow \gamma + e^-$$

- Acoustic mode evolution influenced by visibility function

$$g = \dot{\tau} e^{-\tau} \qquad \qquad \tau(z) = \int_0^{\eta(z)} n_e \sigma_T a(\eta') d\eta'$$

- $z_{\rm dec} \simeq 1100$: Decoupling occurs during recombination

$$C_l \to C_l e^{-2\tau(z)} \text{ if } l > \eta_{\rm dec}/\eta(z)$$

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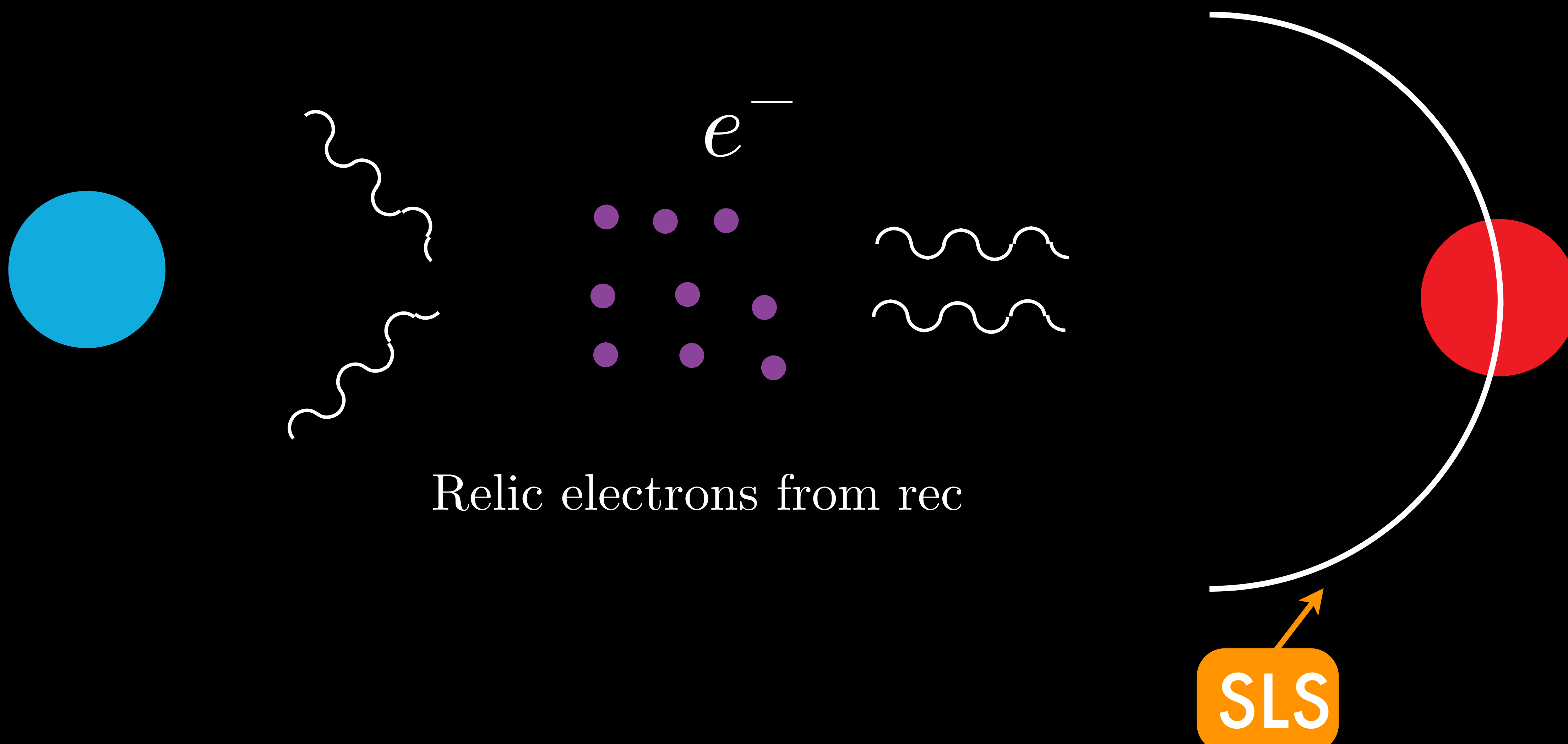
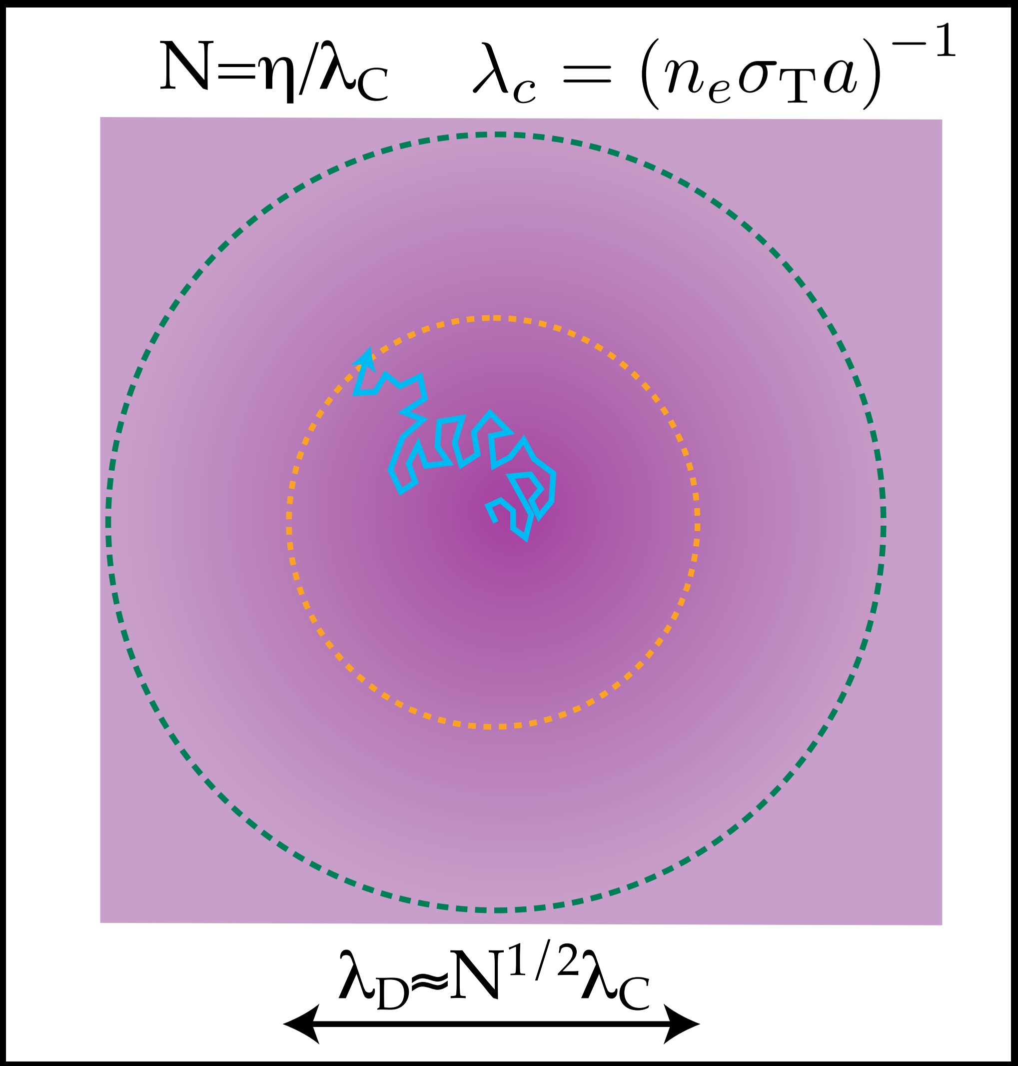
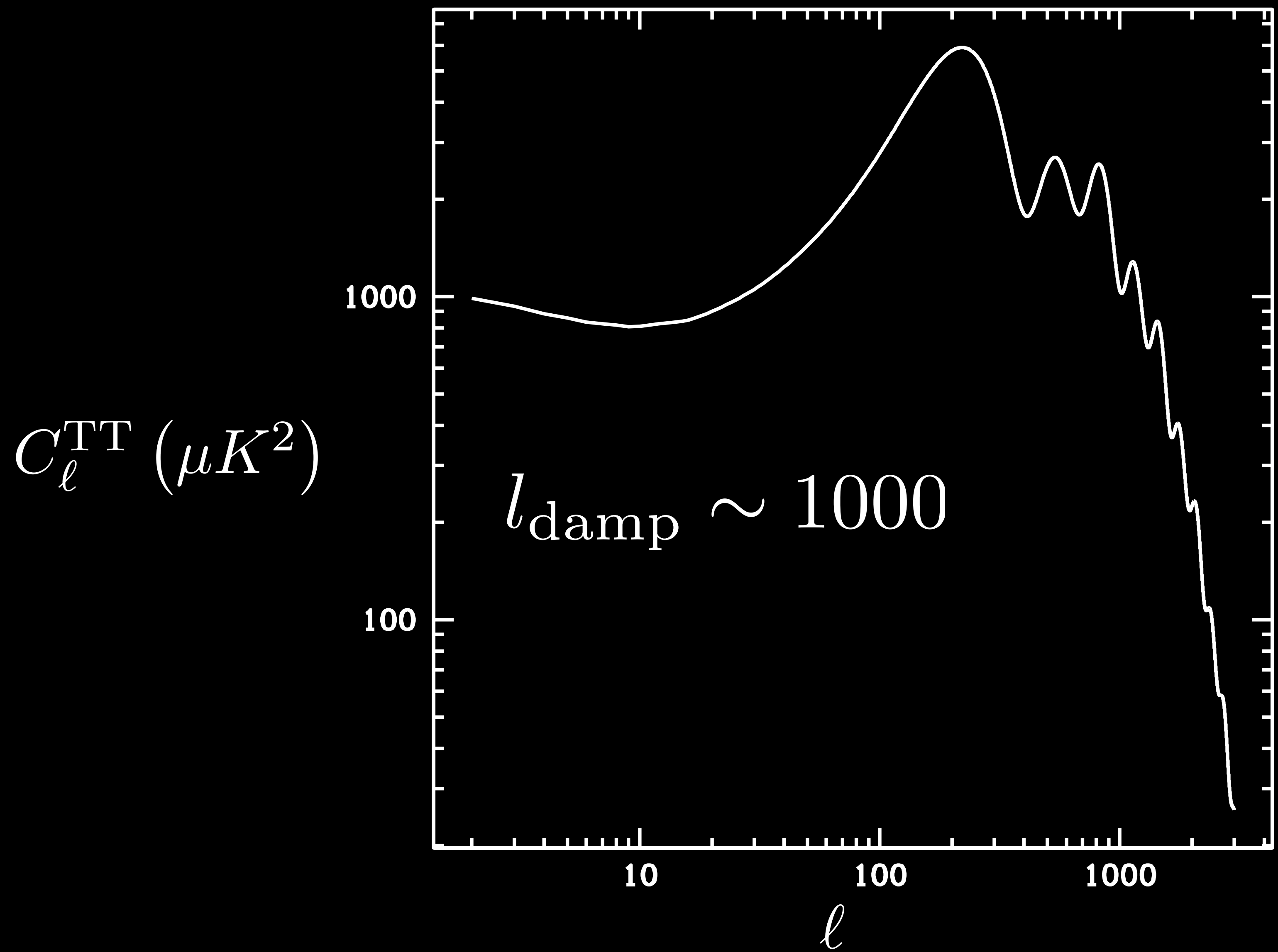


PHYSICAL RELEVANCE FOR CMB: THE SILK DAMPING TAIL





✳ Inhomogeneities are damped for $\lambda < \lambda_D$

PHYSICAL RELEVANCE FOR CMB: POLARIZATION

Isotropic radiation

Quadrupole moment

No polarization

Polarization

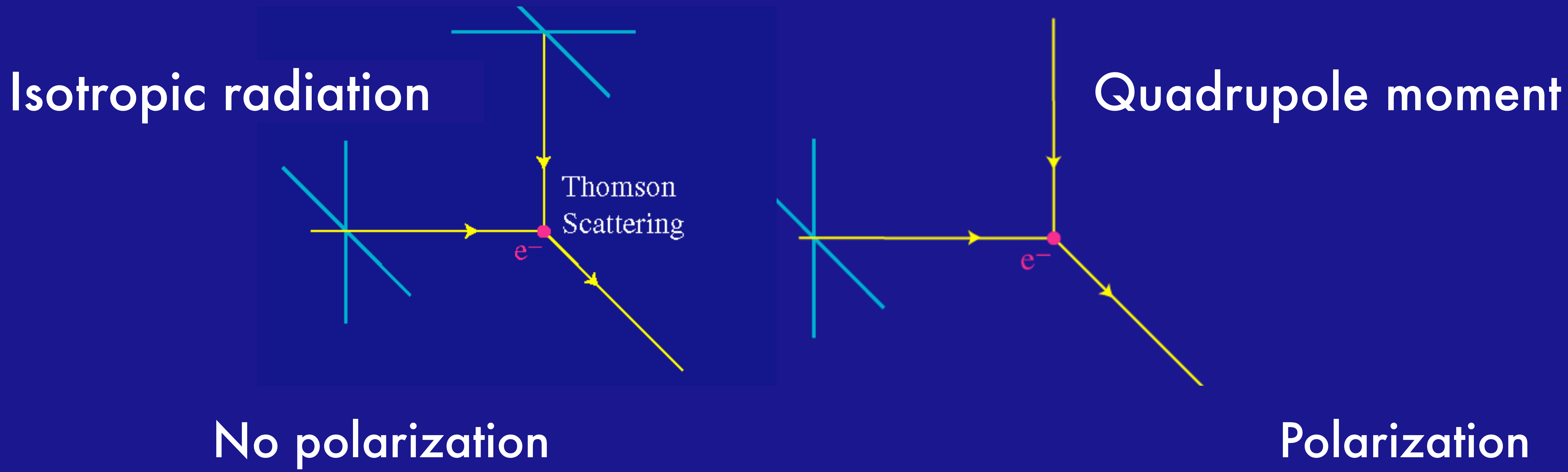
From Wayne Hu's website

- Need time to develop a quadrapole

$$\Theta_l\left(k\eta\right) \sim \frac{k\eta}{2\tau}\Theta_{l+1}\left(k\eta\right) \ll \Theta_{l+1}\left(k\eta\right) \text{ if } l \geq 2, \text{ in tight coupling regime}$$

- Need to scatter quadrapole to polarize CMB

Physical relevance for CMB: Polarization



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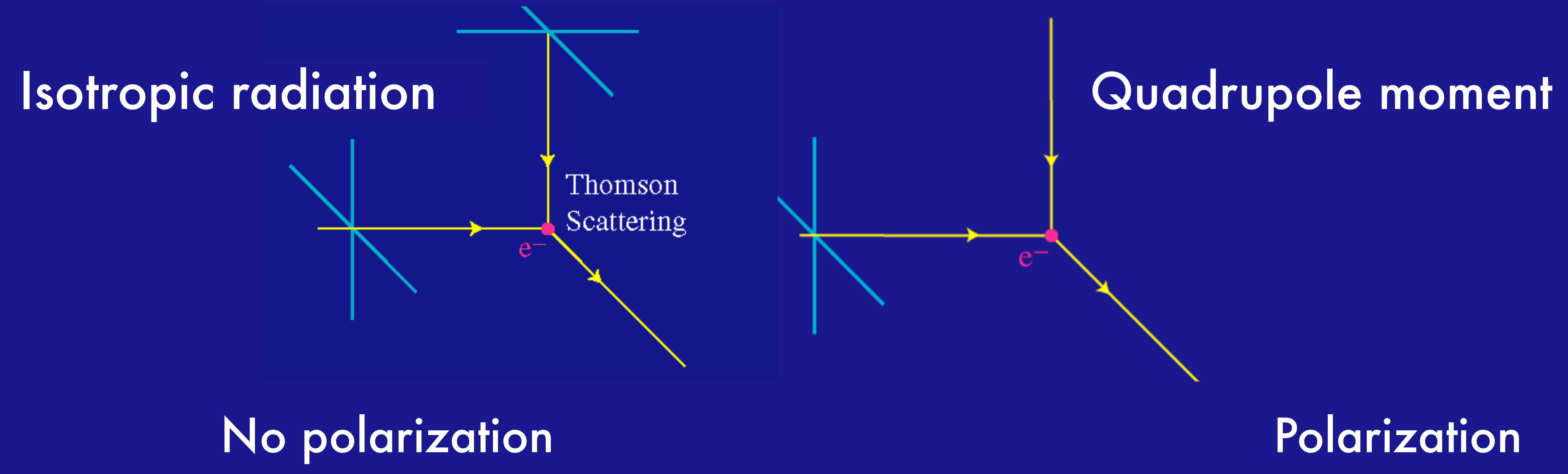

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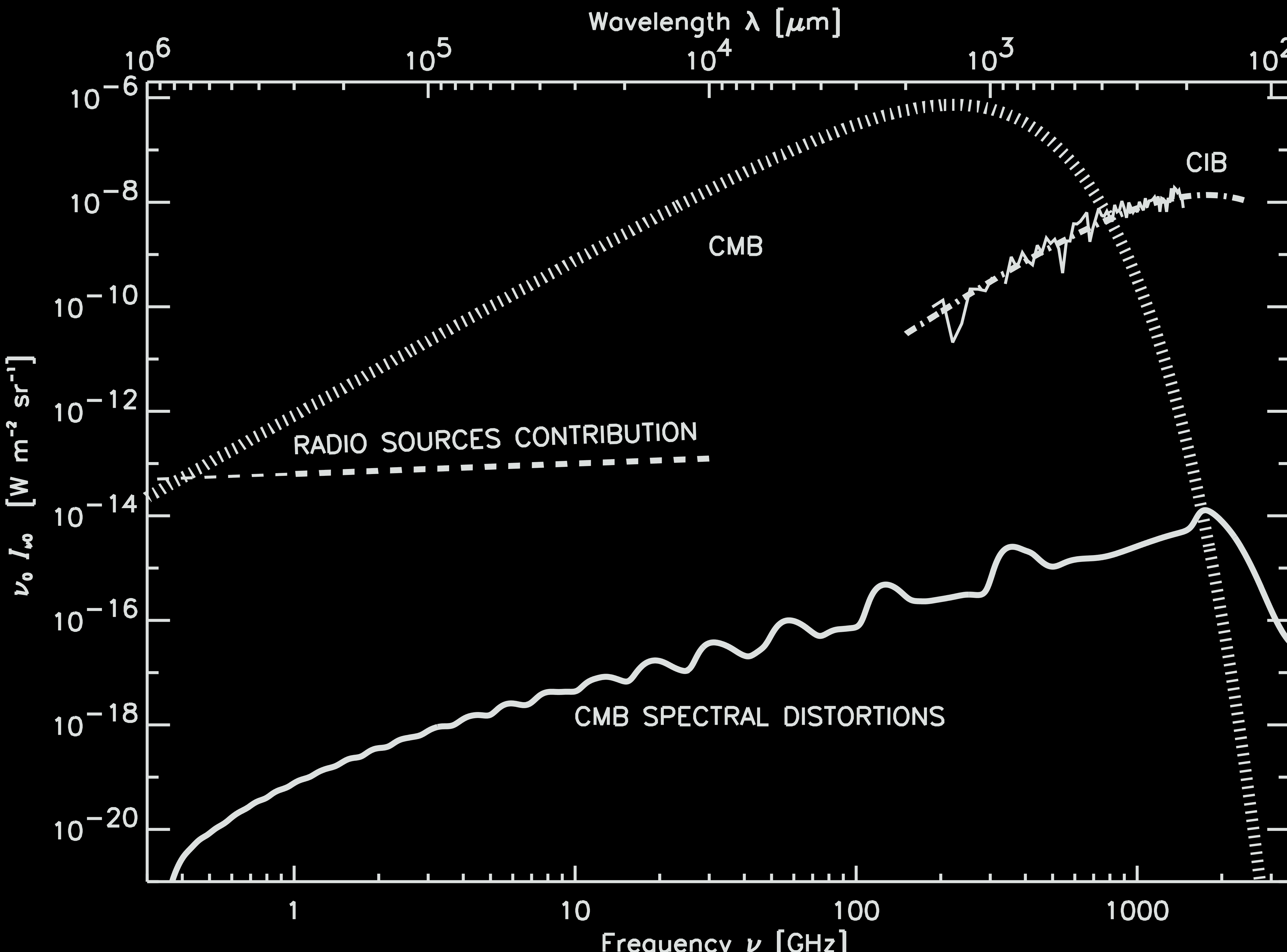
Physical relevance for CMB: Spectral distortions from recombination



SAHA EQUILIBRIUM IS INADEQUATE

$$\boxed{p + e^- \leftrightarrow H^{(n)} + \gamma^{(nc)}}$$

✳ Chemical equilibrium does reasonably well predicting "moment of recombination"

$$\frac{x_e^2}{1-x_e} = \left(\frac{13.6}{T_{\rm eV}}\right)^{3/2} e^{35.9-13.6/T_{\rm eV}}$$

$$\boxed{x_e = 0.5 \text{ when } T = T_{\rm rec} \simeq 0.3 \text{ eV}} \qquad z_{\rm rec} \simeq 1300$$

✳Further evolution falls prey to reaction freeze-out

$$\boxed{\Gamma < H \text{ when } T < T_{\rm F} \simeq 0.25 \text{ eV}}$$

BOTTLENECKS/ESCAPE ROUTES

BOTTLENECKS

✳ Ground state recombinations are ineffective

$$\Gamma_{c\to 1s} = 10^{-1}\ \mathrm{s}^{-1} \gg H \simeq 10^{-12}\ \mathrm{s}^{-1}$$

✳Resonance photons are re-captured, e.g. Lyman α

$$\Gamma_{2p\to 1s} = 10^{-2}\ \mathrm{s}^{-1} \gg H \simeq 10^{-12}\ \mathrm{s}^{-1}$$

ESCAPE ROUTES (e.g. n=2)

✳ Two-photon processes

$$H^{2s} \to H^{1s} + \gamma + \gamma \qquad \boldsymbol{\Lambda}_{2\mathrm{s}\to 1\mathrm{s}} = \mathbf{8.22}\ \mathrm{s}^{-1}$$

✳ Redshifting off resonance

$$R \sim \left(n_{\mathrm{H}}\lambda_{\alpha}^{3}\right)^{-1} H$$

THE PEEBLES PUNCHLINE

✴ Only n=2 bottlenecks are treated

✴Net Rate is suppressed by bottleneck vs. escape factor

$$-\frac{dx_e}{dt} = \mathcal{S} \sum_{n,l>1s} \alpha_{nl}(T) \left\{ n x_e^2 - x_{1s} f(T) \right\}$$

11

The peebles punchline

- Only n=2 bottlenecks are treated

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$$-\frac{dx_e}{dt} = \mathcal{S} \sum_{n,l>1s} \alpha_{nl}(T)\left\{nx_e^2 - x_{1s} f(T)\right\}$$

Recombination rate

11

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Ionization rate

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THE PEEBLES MODEL

- Net Rate is suppressed by bottleneck vs. escape factor

$$\mathcal{S} = \frac{\frac{8\pi}{\lambda_\alpha^3 n_{1s}} H + \Lambda_{2s\to 1s}}{\frac{8\pi}{\lambda_\alpha^3 n_{1s}} H + (\Lambda_{2s\to 1s} + \beta_c)}$$

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Redshifting term

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2γ term

12

The Peebles Model

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Ionization Term

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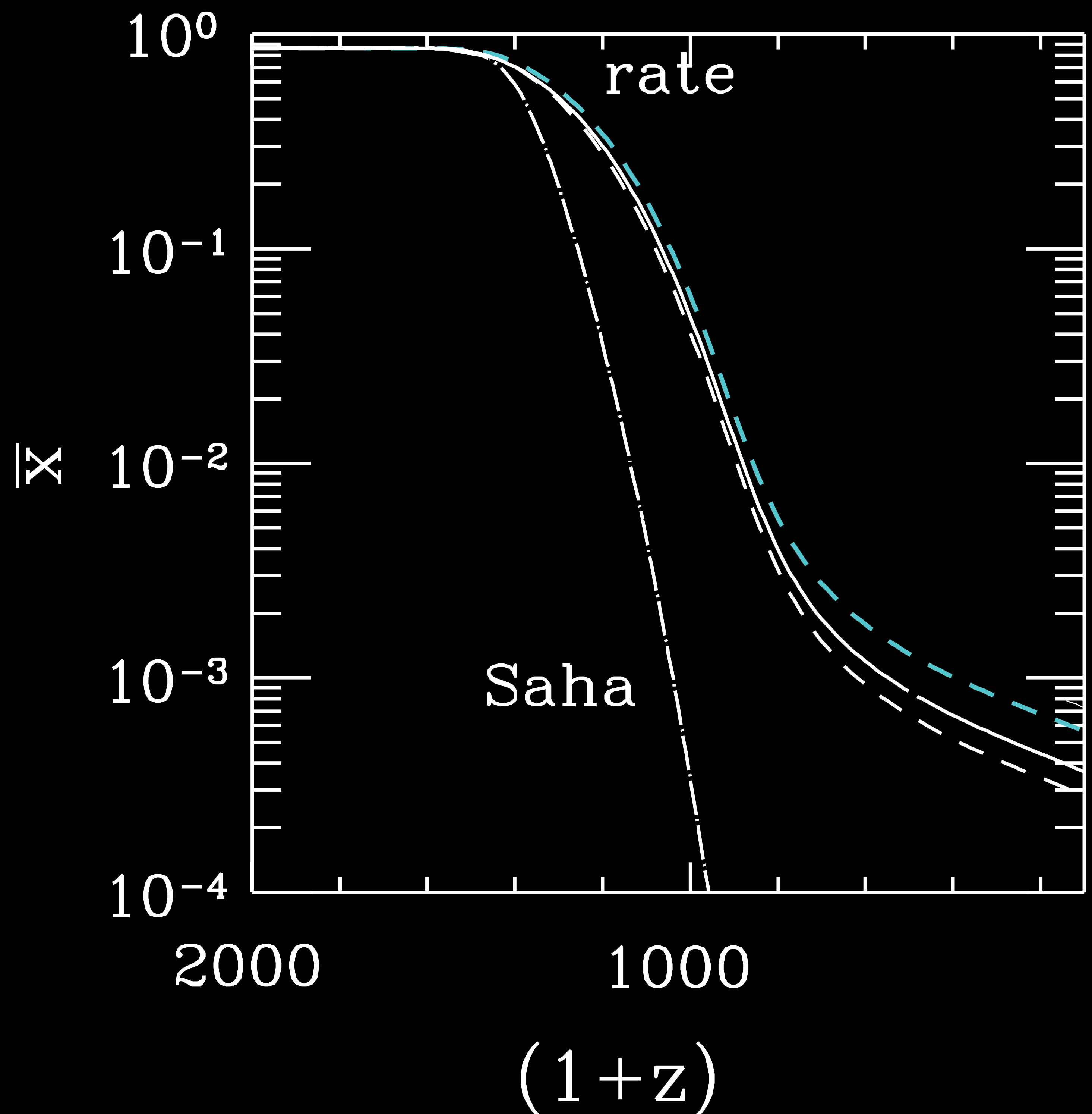
$\beta_c \longrightarrow$ Ionization Term

$$\frac{\text{redshift term}}{2\gamma \text{ term}} \simeq 0.02 \frac{\Omega_m^{1/2}}{(1 - x_e[z]) \left(\frac{1+z}{1100}\right)^{3/2}}$$

2γ process dominates until late times $(z \lesssim 850)$

THE PEEBLES MODEL

- Peebles 1967: State of the Art for 30 years!



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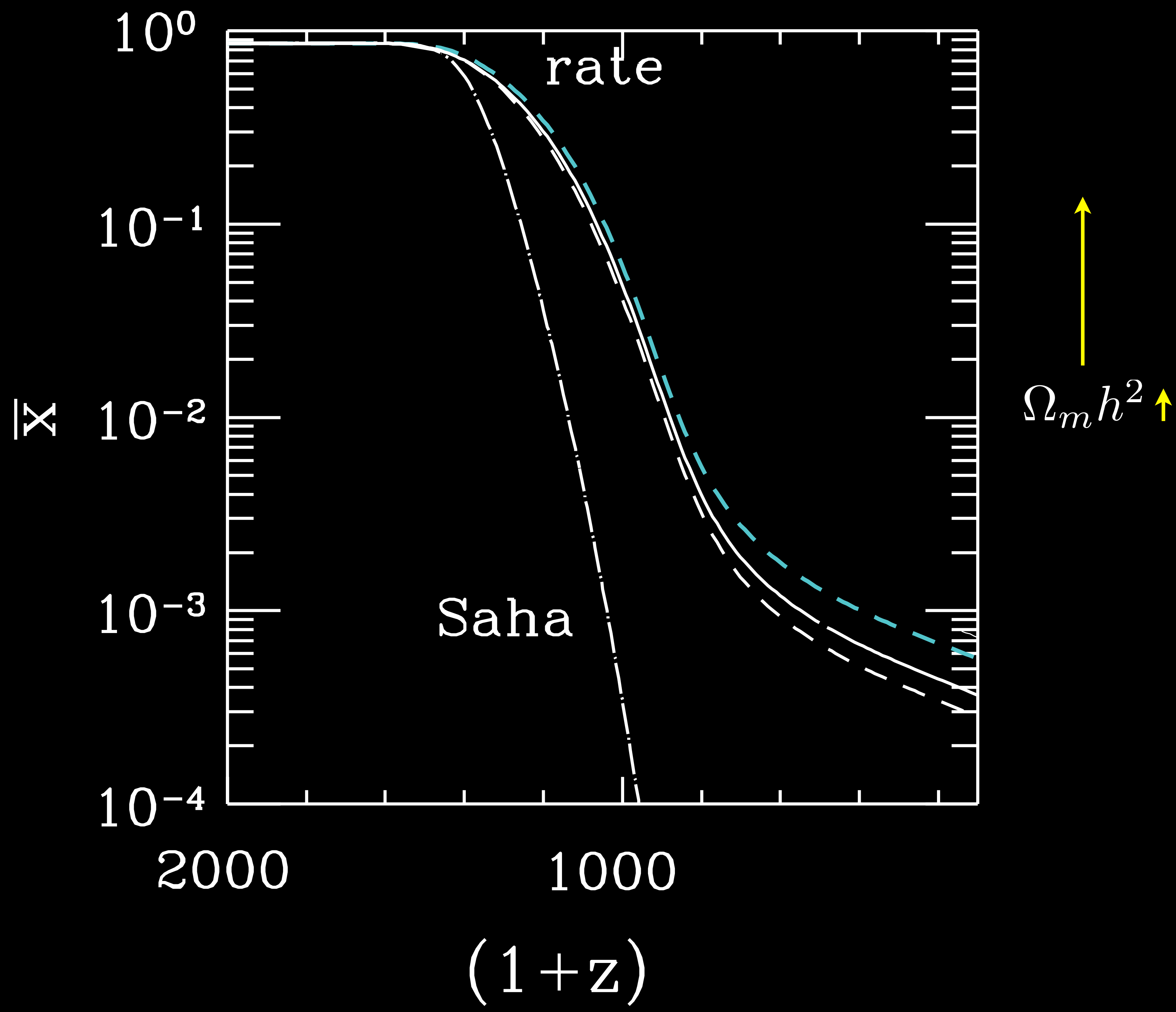

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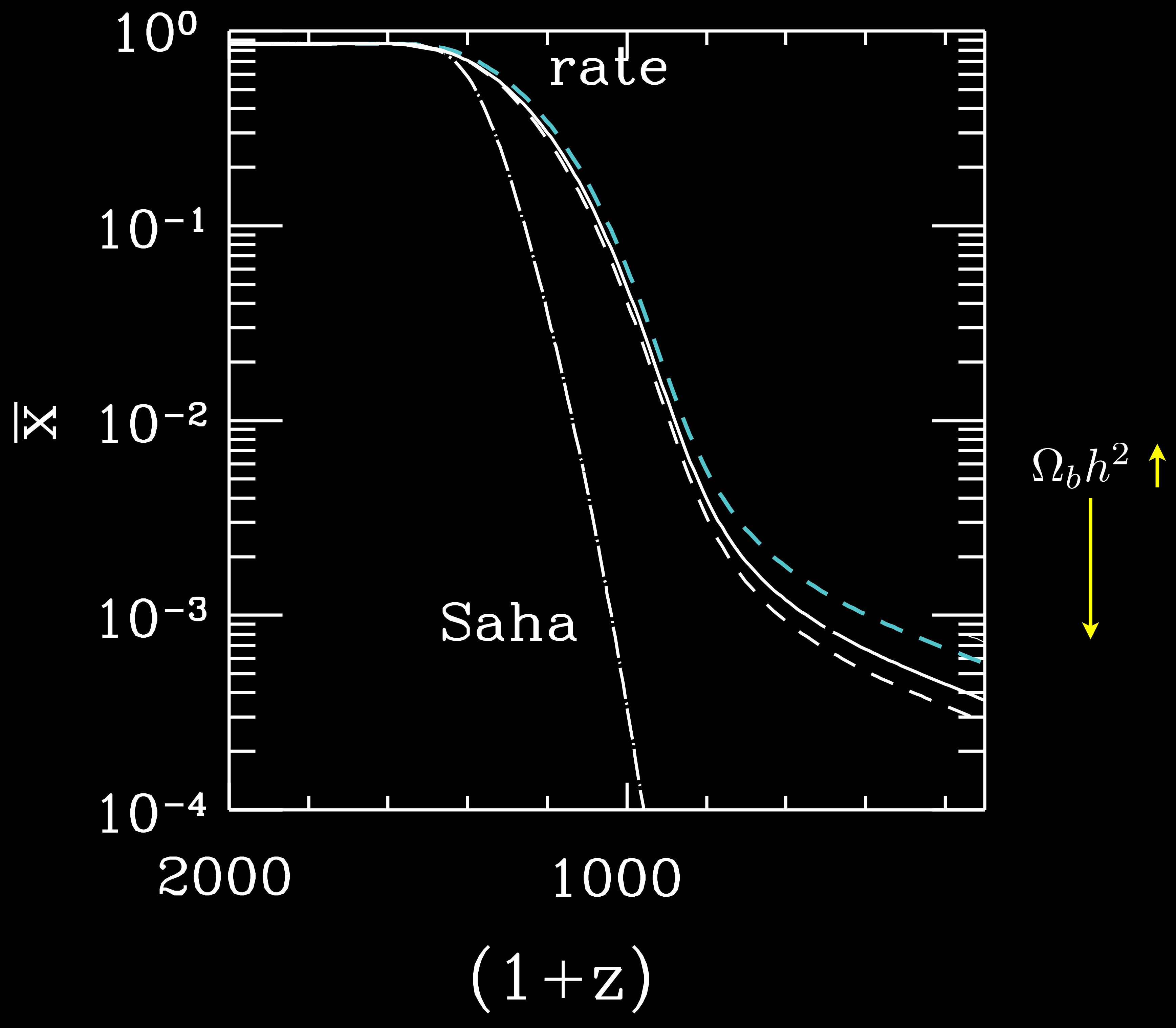

13

EQUILIBRIUM ASSUMPTIONS

- ✳Radiative/collisional eq. between different l

$$\mathcal{N}_{nl} = \mathcal{N}_n \frac{(2l+1)}{n^2}$$

- ✳ Radiative eq. between different n-states

$$\mathcal{N}_n = \sum_l \mathcal{N}_{nl} = \mathcal{N}_2 e^{-(E_n - E_2)/T}$$

Non-eq rate equations

- ✳Matter in eq. with radiation due to Thompson scattering

$$T_m = T_\gamma \text{ since } \frac{\sigma_{\mathrm{T}} a T_\gamma^4 c}{m_e c^2} < H(T)$$

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Seager/Scott/Sasselov 2000/RECFAST!

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BREAKING EQUILIBRIUM

- ✳ Chluba et al. (2005,6) follow l, n separately, get to $n_{\mathrm{max}} = 100$
- ✳ 0.1 %-level corrections to CMB anisotropies at $n_{\mathrm{max}} = 100$
- ✳ ~~Equilibrium~~ between *l states:* $\Delta l = \pm 1$ bottleneck
- ✳ Beyond this, testing convergence with n_{max} is hard!

$$t_{\mathrm{compute}} \sim \mathcal{O}\left(\mathrm{years}\right) \text{ for } \mathrm{n}_{\mathrm{max}} = 300$$

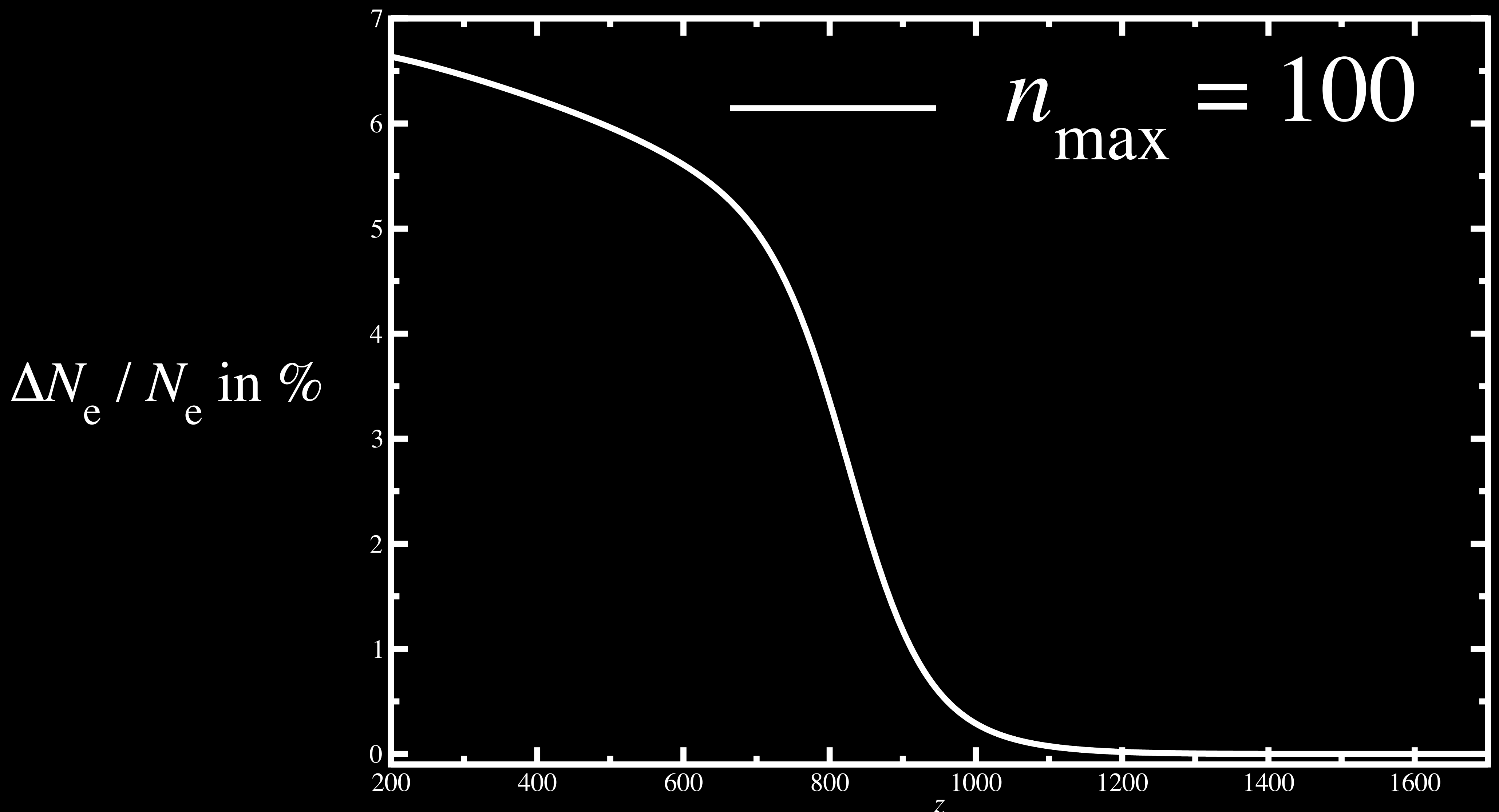
How to proceed if we want $\mathcal{O}\left(1\right) \times 10^{-4}$ accuracy in C_{ℓ}?

THESE ARE REAL STATES

- Still inside plasma shielding length for n<100000
- $r \sim a_0 n^2$ is as large as 2μm for $n_{\text{max}} = 200$
- $\frac{\Delta E|_{\text{thermal}}}{E} < \frac{2}{n^3}$
- Similarly high n are seen in emission line nebulae

THE EFFECT OF RESOLVING *l*- SUBSTATES

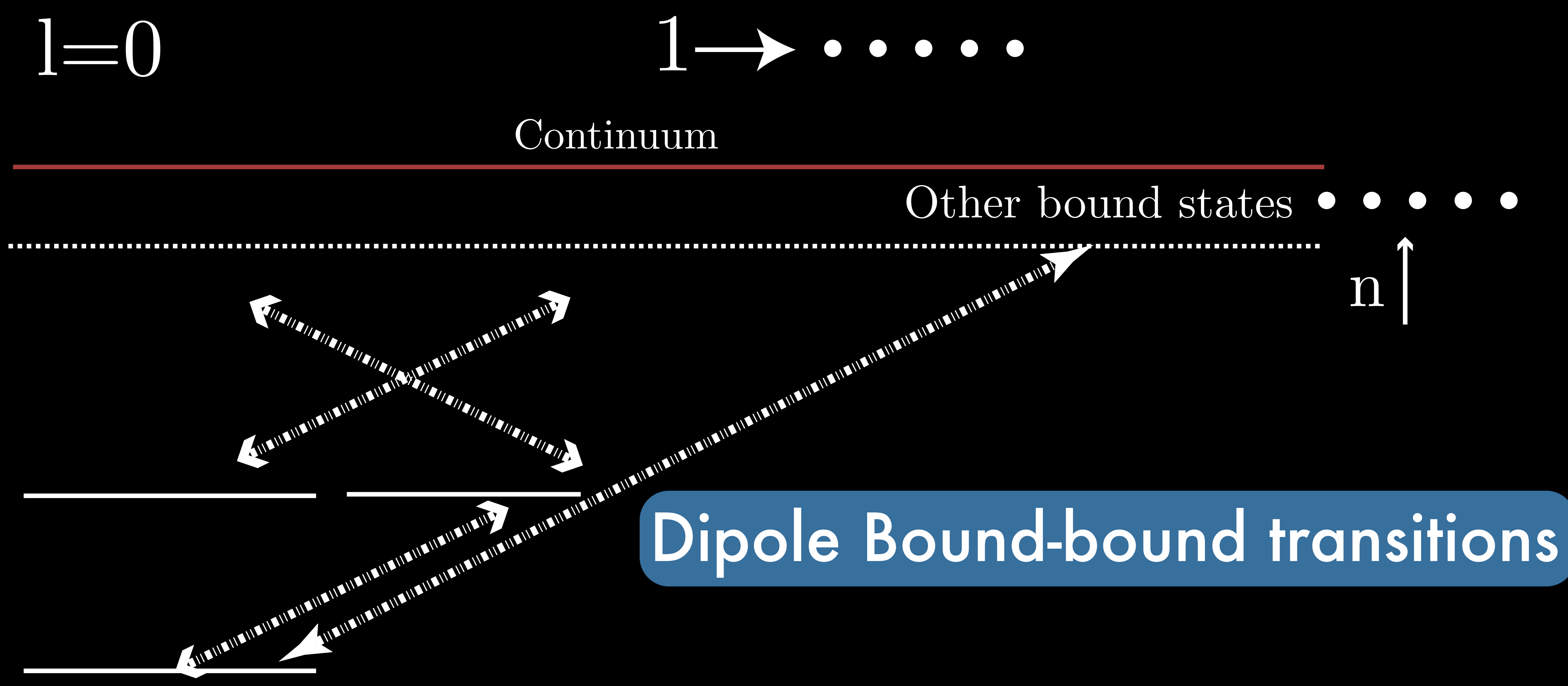
Resolved l vs unresolved l



From Chluba et al. 2006

- ‘Bottlenecked’ l-substates decay slowly to 1s: Recombination is slower; Chluba al. 2006

RecSparse and the multi-level atom



- We implement a multi-level atom computation in a new code, RecSparse!
- Boltzmann eq. solved for $\mathrm{T}_m \ (T_\gamma)$
- Spontaneous/stimulated emission/absorption included

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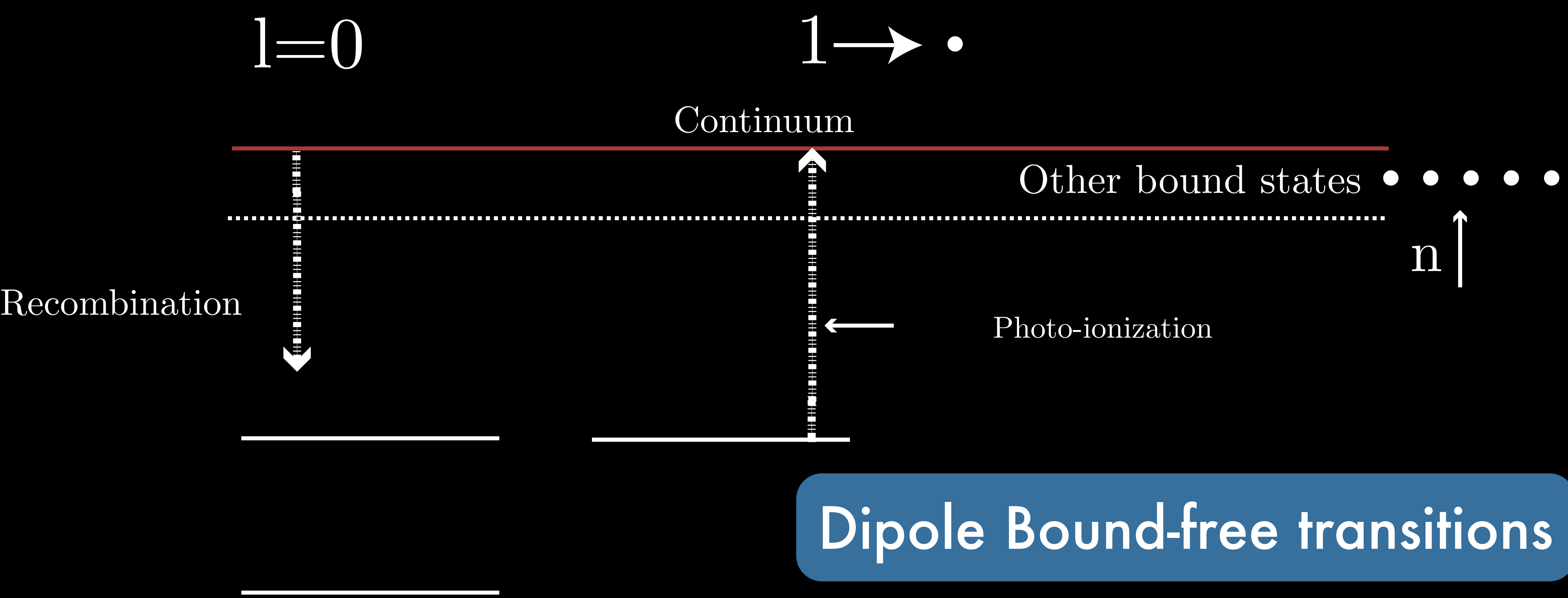

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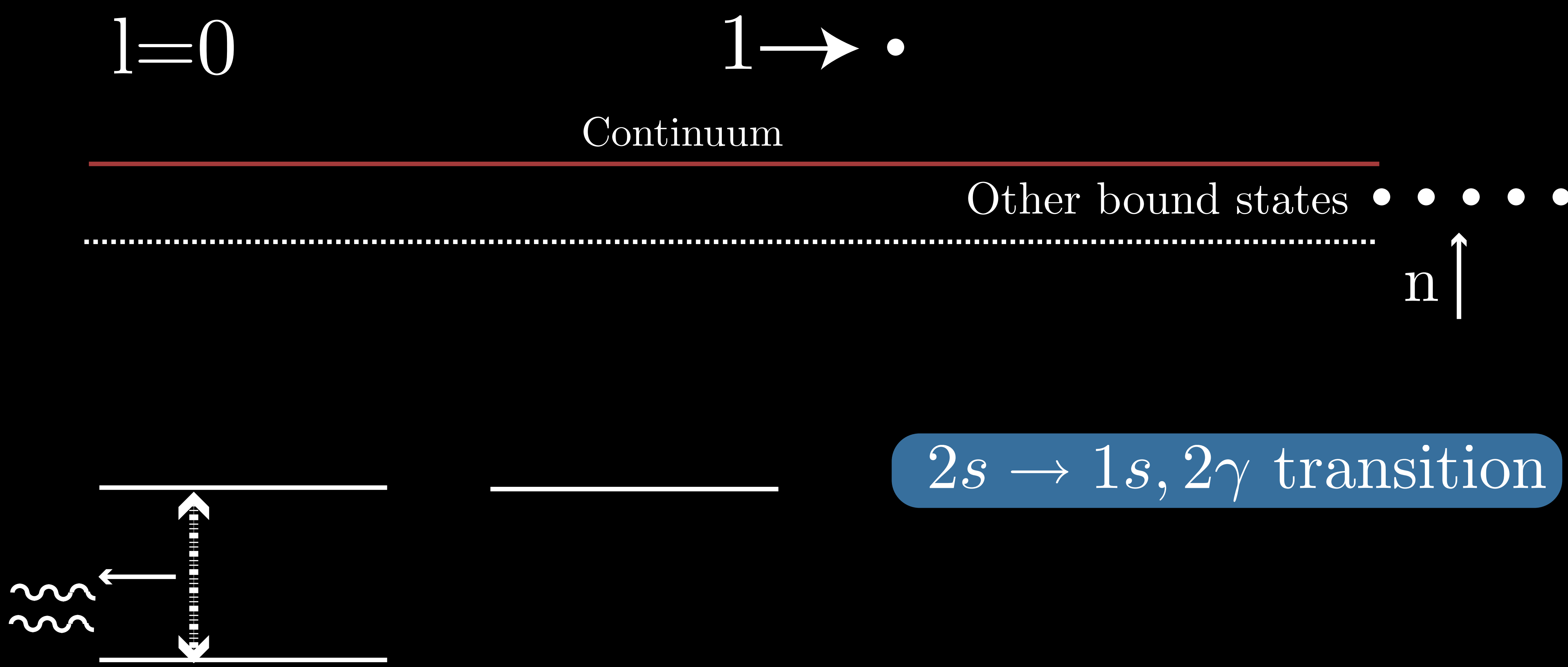

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RECSPARSE AND THE MULTI-LEVEL ATOM

- Free electron fraction evolved according to

$$\begin{aligned}\dot{x}_e &= -\dot{x}_{1s} \\ &= -\Lambda_{2s\to 1s}\left(x_{2s} - x_{1s}e^{-E_{2s\to 1s}/T_\gamma}\right) + \sum_{n,l>1s} A^{l\,0}_{n1} P^{l0}_{n1}\left\{g(T,n,l)\right\}\end{aligned}$$

2s-1s decay rate

RECSPARSE AND THE MULTI-LEVEL ATOM

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$$

Lyman series current to ground state

RECSPARSE AND THE MULTI-LEVEL ATOM

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Escape probability

RADIATION FIELD: BLACK BODY +

- Escape probability treated in Sobolev approx: depends on steady-state and incoherent scattering approximations

$$P_{n,n'}^{l,l'} = \frac{1-e^{-\tau_s}}{\tau_s}$$

$$\tau_s \propto \frac{n_{\mathrm{H}} x_n^l A_{nn'}^{ll'}}{H\left(z\right)} \quad n' > n$$

- RecSparse includes radiative feedback

- Ongoing work in field focuses on corrections to simple radiative transfer picture

- Ultimate goal is to combine all new atomic physics effect in one fast recombination code

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Resonant absorber density

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Einstein coefficient

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Cosmological expansion

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OTHER CORRECTIONS TO RECOMBINATION

- Deviations from steady-state approx (Chluba/Sunyaev 2008)
- Coherent scattering (Forbes and Hirata 2009, Switzer/Hirata 2007)
- Atomic recoil (Forbes and Hirata 2009, Dubrovich and Grachev 2008)
- Diffusion near resonance lines
- Line overlap (Ali-Haimoud, Grin, Hirata in progress)
- Feedback from hydrogen/helium (Chluba/Sunyaev 2007)
- Higher-n two-photon processes (Chluba/Sunyaev 2007, Hirata 2008) in hydrogen and Helium (Switzer/Hirata 2007)
- Deuterium
- Additional effects in Helium (Switzer/Hirata 2007)

STEADY-STATE FOR EXCITED LEVELS

- Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

Steady-state for excited levels

✳ Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

$$\vec{x} = \begin{pmatrix} \vec{x}_0 \\ \vec{x}_1 \\ \cdots \\ \vec{x}_l \\ \cdots \\ \vec{x}_{l_{\rm max}} \end{pmatrix}$$

21

STEADY-STATE FOR EXCITED LEVELS

* Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

$$\vec{x}_l = \begin{pmatrix} x_{l,l+1} \\ \cdots \\ x_{l,n_{\max}} \end{pmatrix} \longleftarrow \qquad \vec{x} = \begin{pmatrix} \vec{x}_0 \\ \vec{x}_1 \\ \cdots \\ \boxed{\vec{x}_l} \\ \cdots \\ \vec{x}_{l_{\max}} \end{pmatrix}$$

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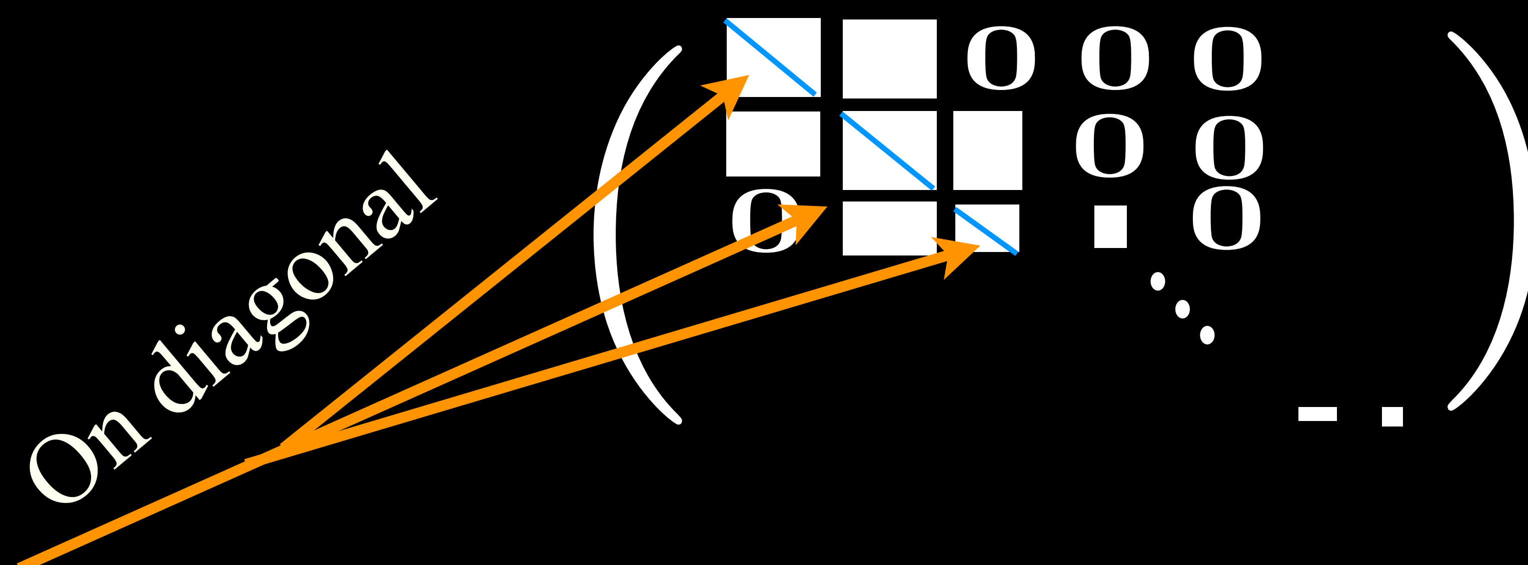

For state l, includes BB transitions out of l to all other l”, photo-ionization, 2γ transitions to ground state

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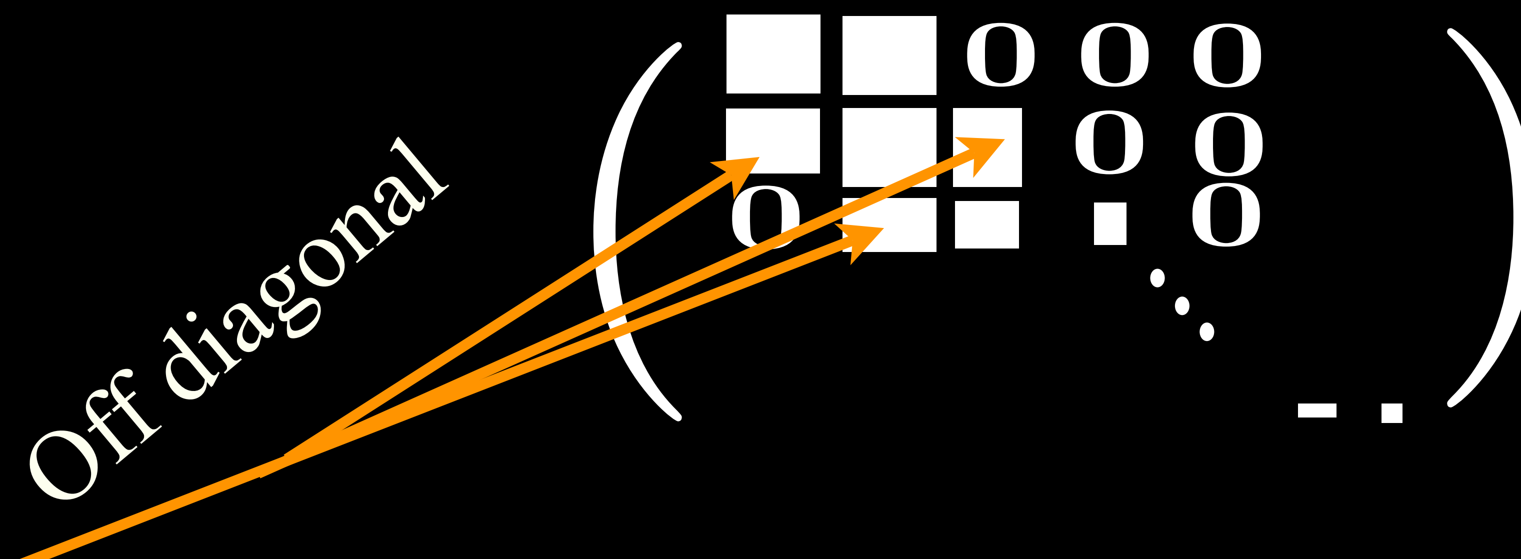

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Includes recombination to l,
1 and 2γ transitions from ground state

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$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

For n>1, $\mathbf{R}$, $\vec{s} \geq 1\ \mathrm{s}^{-1}$ e.g. Lyman-α

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Steady-state for excited levels

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$$\cancel{\frac{d\vec{x}}{dt}} = \mathbf{R}\vec{x} + \vec{s}$$

LHS $\ll$ RHS

$\vec{x} \simeq -\mathbf{R}^{-1}\vec{s}$

$t_{\rm rec}^{-1} \sim 10^{-12}\ {\rm s}^{-1}$

For n>1, $\mathbf{R}$, $\vec{s} \geq 1\ {\rm s}^{-1}$ e.g. Lyman-α

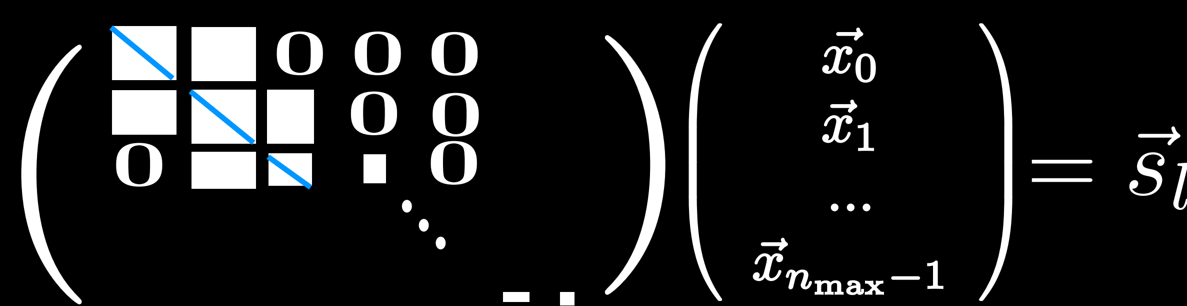
RAPID MATRIX INVERSION: SPARSITY TO THE RESCUE

- Matrix is $\sim n_{max}^2 \times n_{max}^2$

- Brute force would require $\mathrm{A}n_{max}^6 \sim 10^5$ s for $n_{\max} = 200$ for a single time step

- Dipole selection rules: $\Delta l = \pm 1$

  $\mathrm{M}_{l,\boldsymbol{l-1}}\vec{x}_{\boldsymbol{l-1}} + \mathbf{M}_{l,l}\vec{x}_l + \mathbf{M}_{l,\boldsymbol{l+1}}\vec{x}_{\boldsymbol{l+1}} = \vec{s}_l$



- Physics imposes sparseness on the problem. Solved in closed form to yield algebraic $\vec{x}_{l_{\max}}$, then $\vec{x}_l$ in terms of $\vec{x}_{l+1}$

22

Rapid matrix inversion: Sparsity to the rescue

- Einstein coefficients to states with $n > n_{\max}$ are set $A = 0$: more later!

- RecSparse generates rec. history with computation time ~ $n_{max}^{2.5}$: Huge improvement!

- Case of $n_{\max} = 100$ runs in less than a day, $n_{\max} = 200$ takes ~ 4 days.

FORBIDDEN TRANSITIONS AND RECOMBINATION

- Higher-n 2γ transitions in H important at 7-σ for Planck (TT/EE) data analysis (Hirata 2008, Kholupenko 2006)
- Some forbidden transitions are important in Helium recombination (Dubrovich 2005, Lewis 2006) and would bias cosmological parameter estimation.
- *Are other forbidden transitions in hydrogen important, particularly for Planck data analysis? How about electric quadrupole (E2) transitions?*

QUADRUPOLE TRANSITIONS AND RECOMBINATION

- Ground-state electric quadrupole (E2) lines are optically thick!

$$P_{n,n'}^{l,l'} = \frac{1 - e^{-\tau_s}}{\tau_s}$$

$$R \propto AP \propto A/\tau \text{ if } \tau \gg 1$$

$$\tau \propto A \rightarrow R \rightarrow A/A \rightarrow \text{const}$$

- Coupling to ground state will overwhelmingly dominate:

$$\frac{A_{n,2\rightarrow 1,0}^{\text{quad}}}{A_{n,2\rightarrow m,0}^{\text{quad}}} \propto \frac{\omega_{n1}^5}{\omega_{nm}^5} \geq 10^3 \text{ if } m \geq 2$$

QUADRUPOLE TRANSITIONS AND RECOMBINATION

- Lyman lines are optically thick, so $nd \rightarrow 1s$ immediately followed by $1s \rightarrow np$, so this can be treated as an effective $d \rightarrow p$ process with rate $A_{nd\rightarrow 1s} x_{nd}$.

- Same sparsity pattern of rate matrix, similar to l-changing collisions

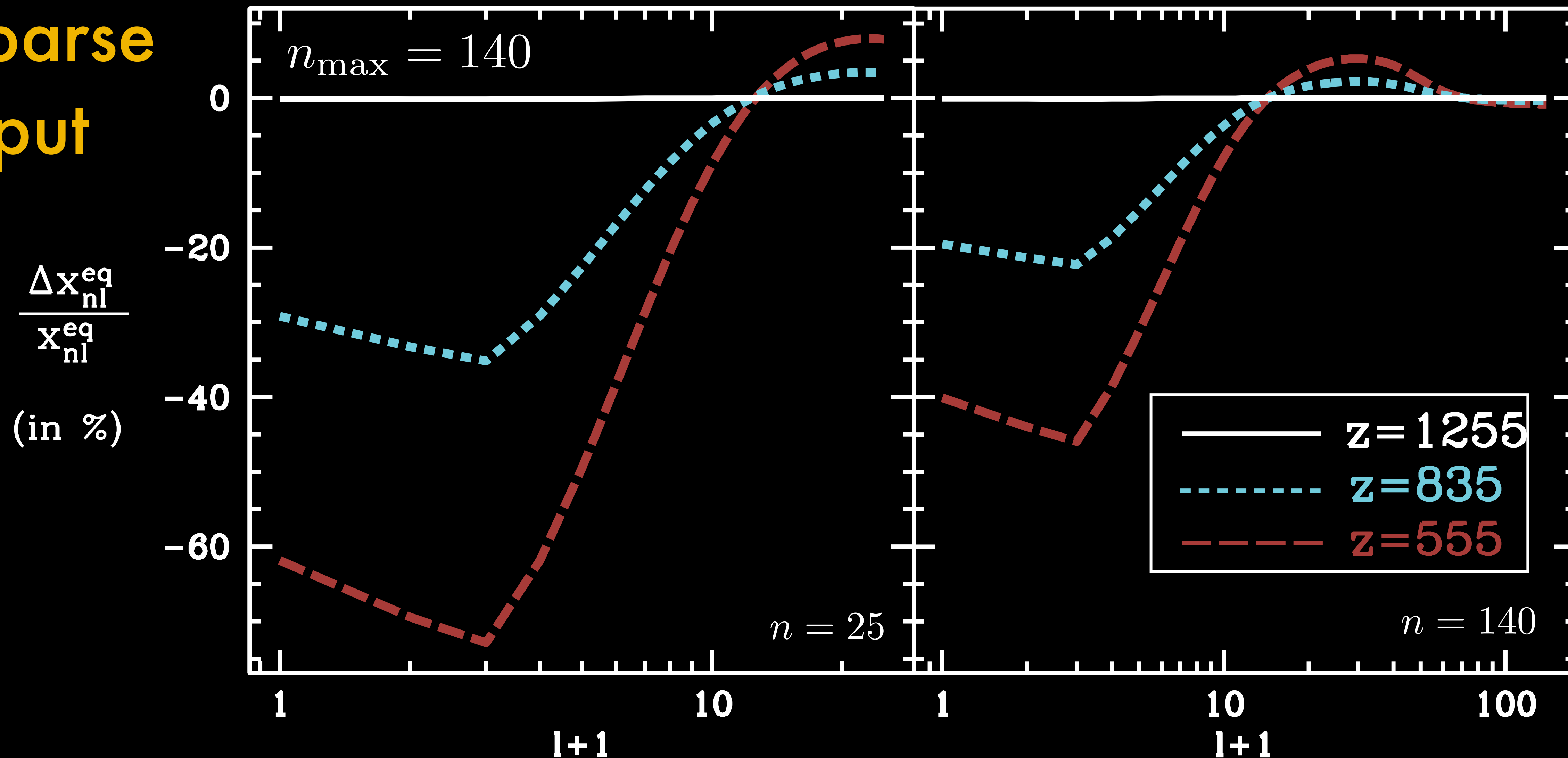
- Detailed balance yields net rate

$$R^{\mathrm{quad}}_{nd\rightarrow np} = A_{nd\rightarrow 1s}\left(x_{nd} - \frac{5}{3}x_{np}\right)$$

RESULTS: STATE OF THE GAS

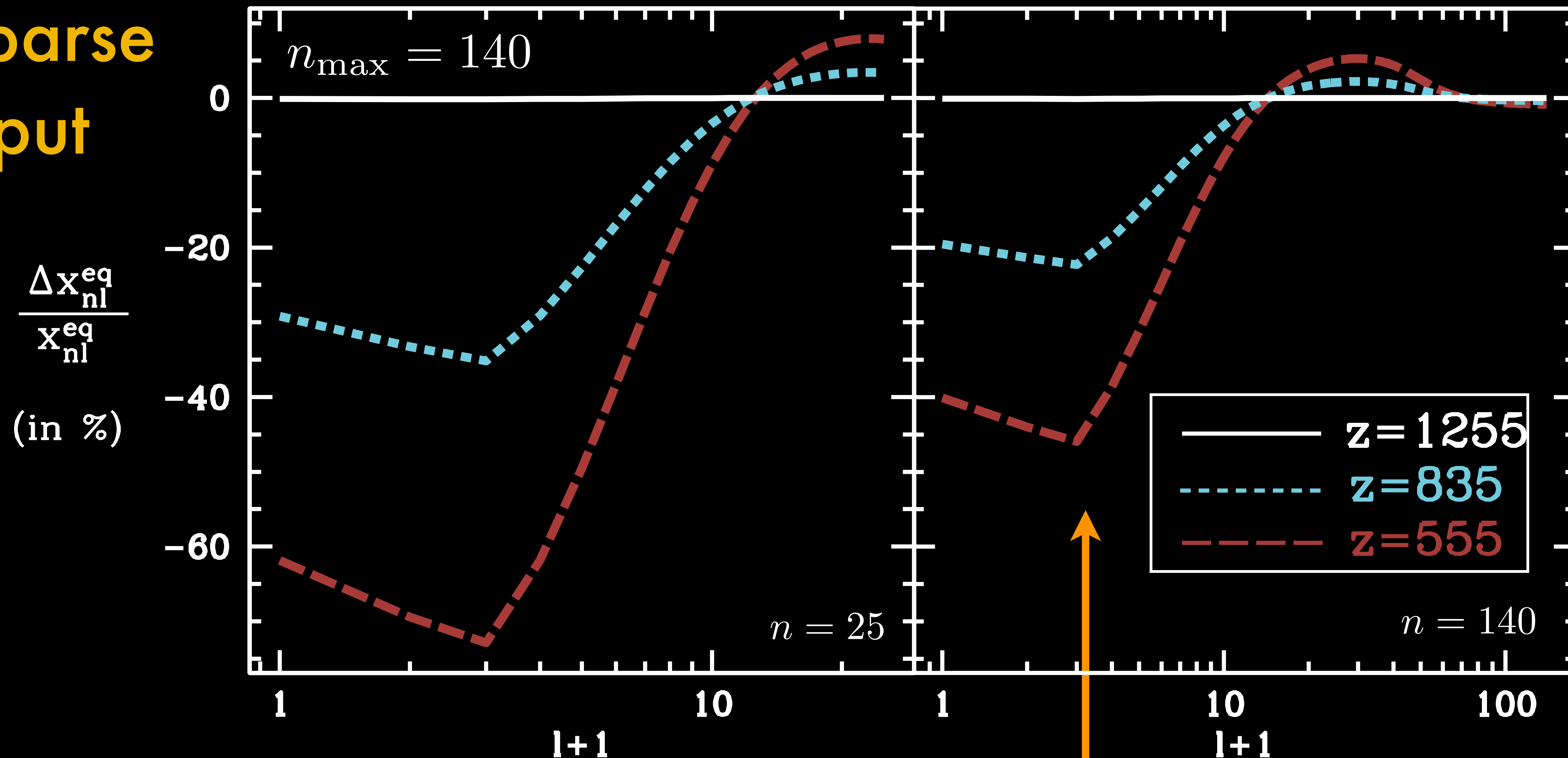
DEVIATIONS FROM BOLTZMANN EQ: l-SUBSTATES

RecSparse output



DEVIATIONS FROM BOLTZMANN EQ: l-SUBSTATES

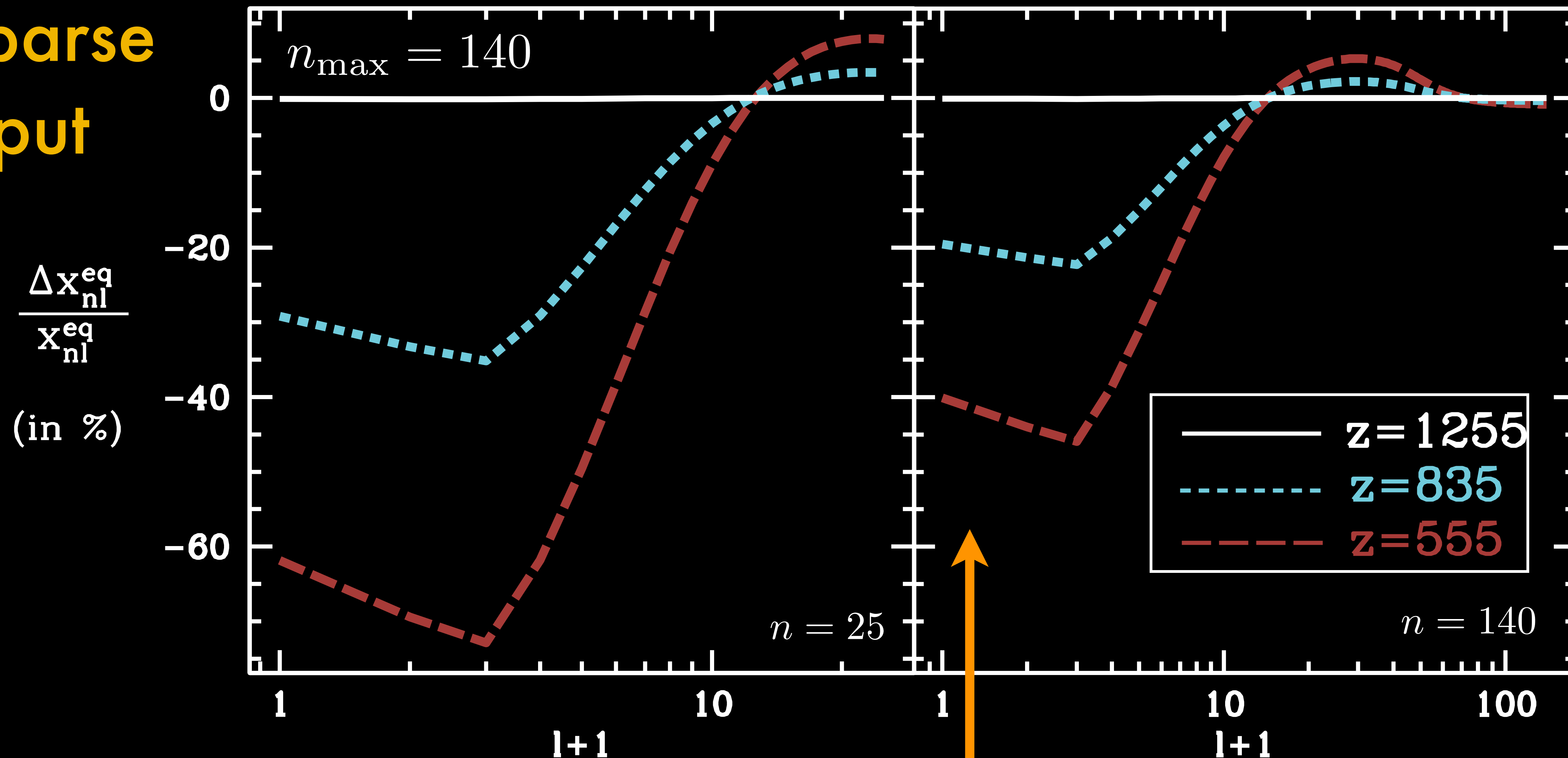
RecSparse output



Lower l states can easily cascade down, and are relatively under-populated

DEVIATIONS FROM BOLTZMANN EQ: l-SUBSTATES

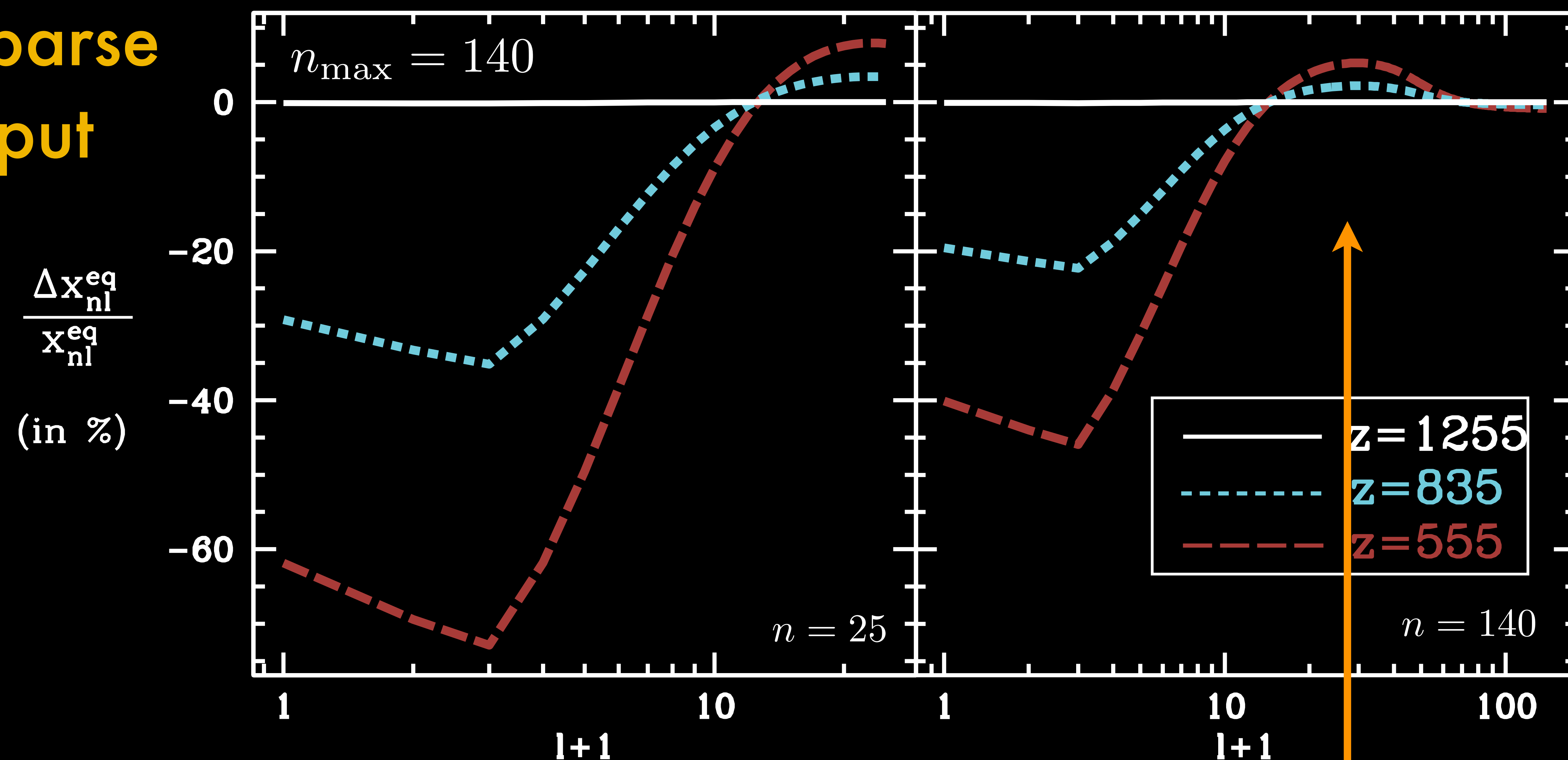
RecSparse output



l=0 can't cascade down, so s states are not as under-populated

DEVIATIONS FROM BOLTZMANN EQ: l-SUBSTATES

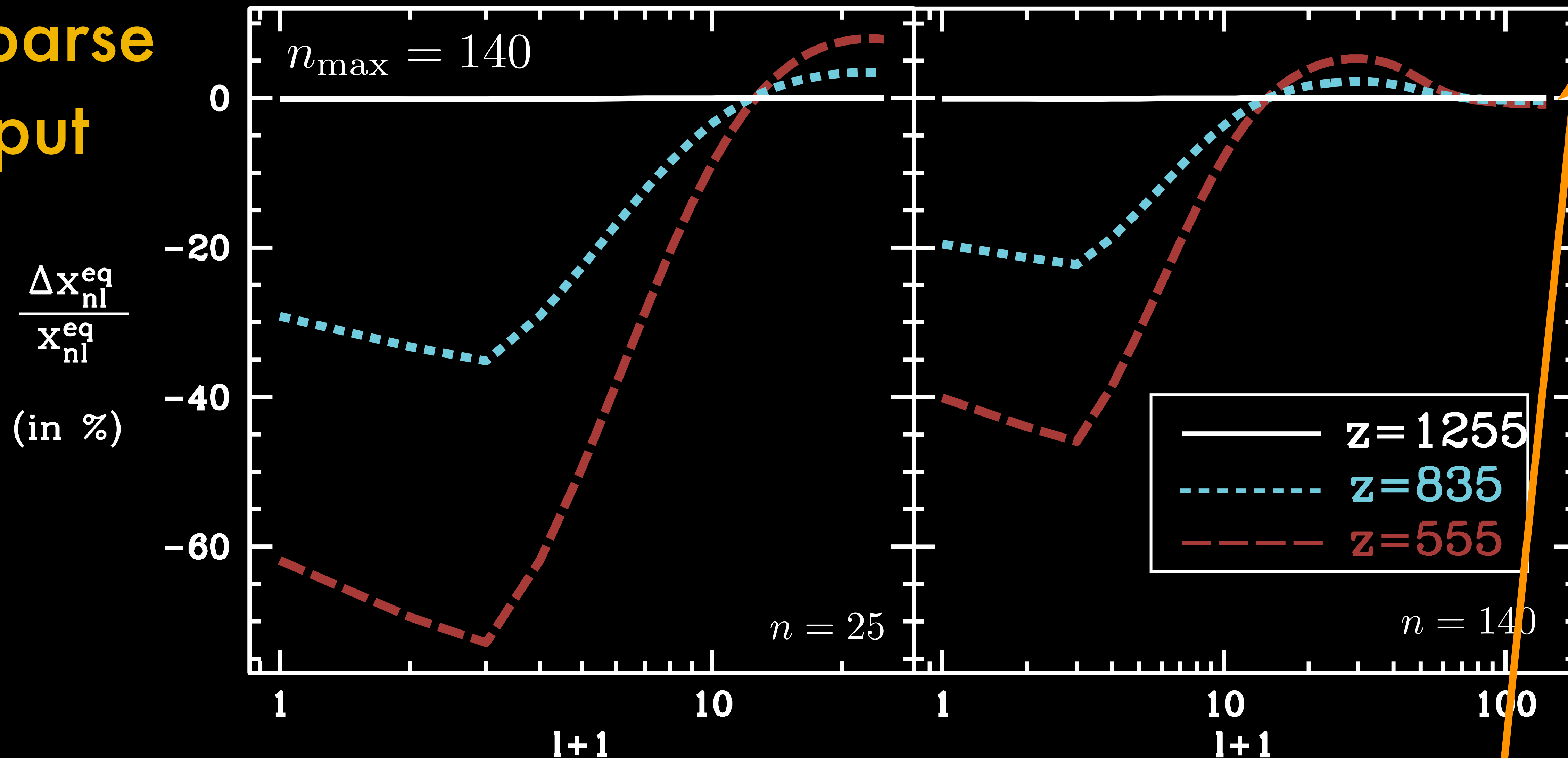
RecSparse output



Higher l are bottlenecked by $\Delta l = \pm 1$ (over-pop)

DEVIATIONS FROM BOLTZMANN EQ: l-SUBSTATES

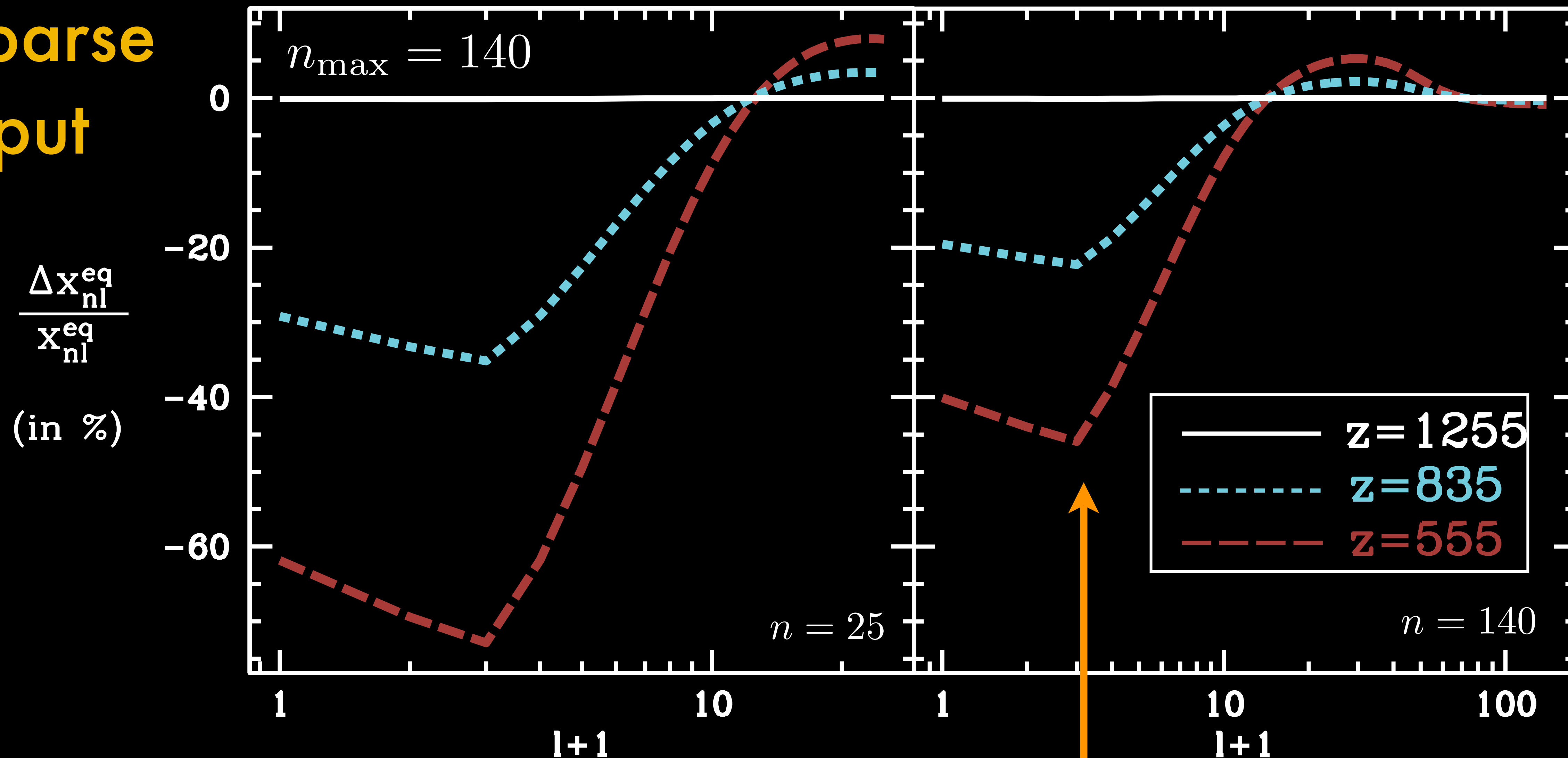
RecSparse output



Highest l states recombine inefficiently, and are under-populated

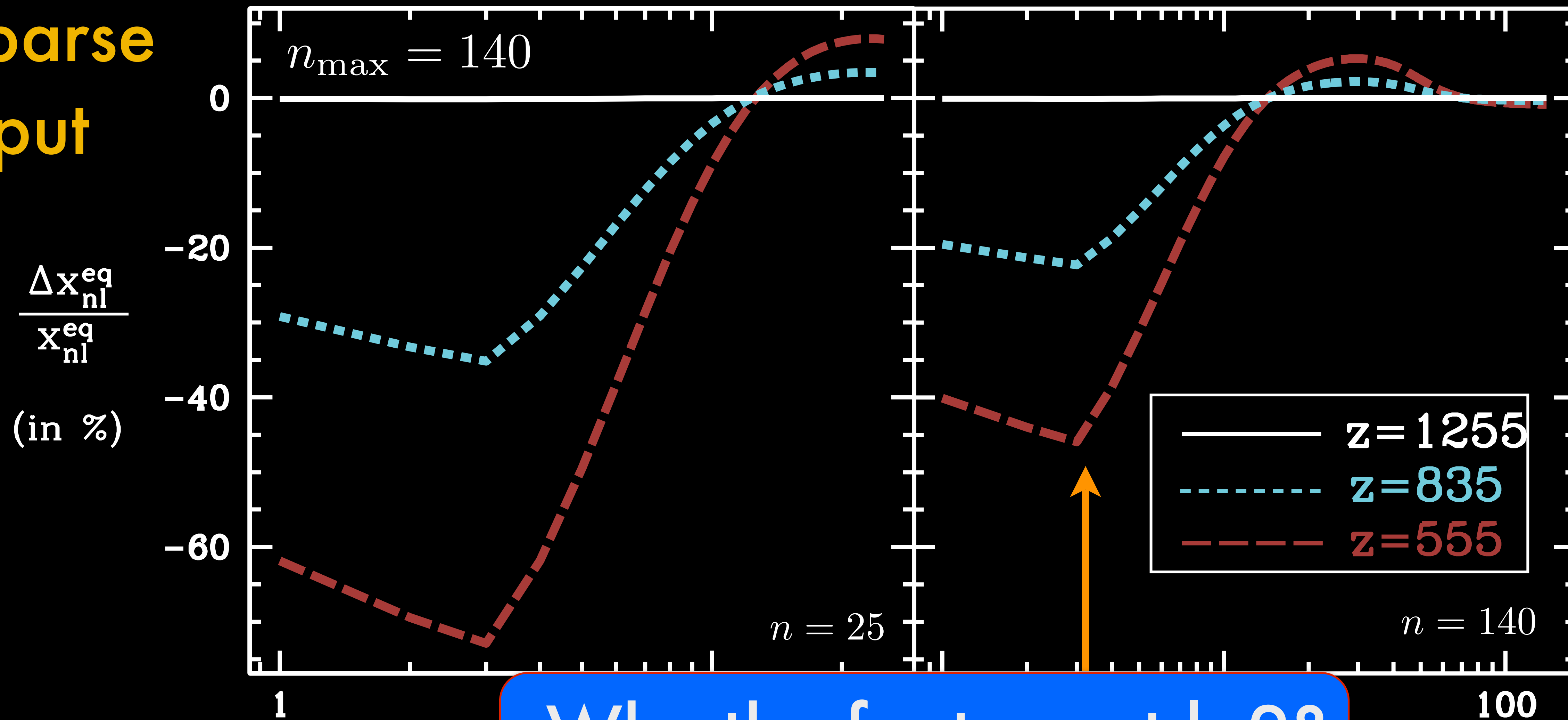
DEVIATIONS FROM BOLTZMANN EQ: l-SUBSTATES

RecSparse output



l-substates are highly out of Boltzmann eqb'm at late times

RecSparse output



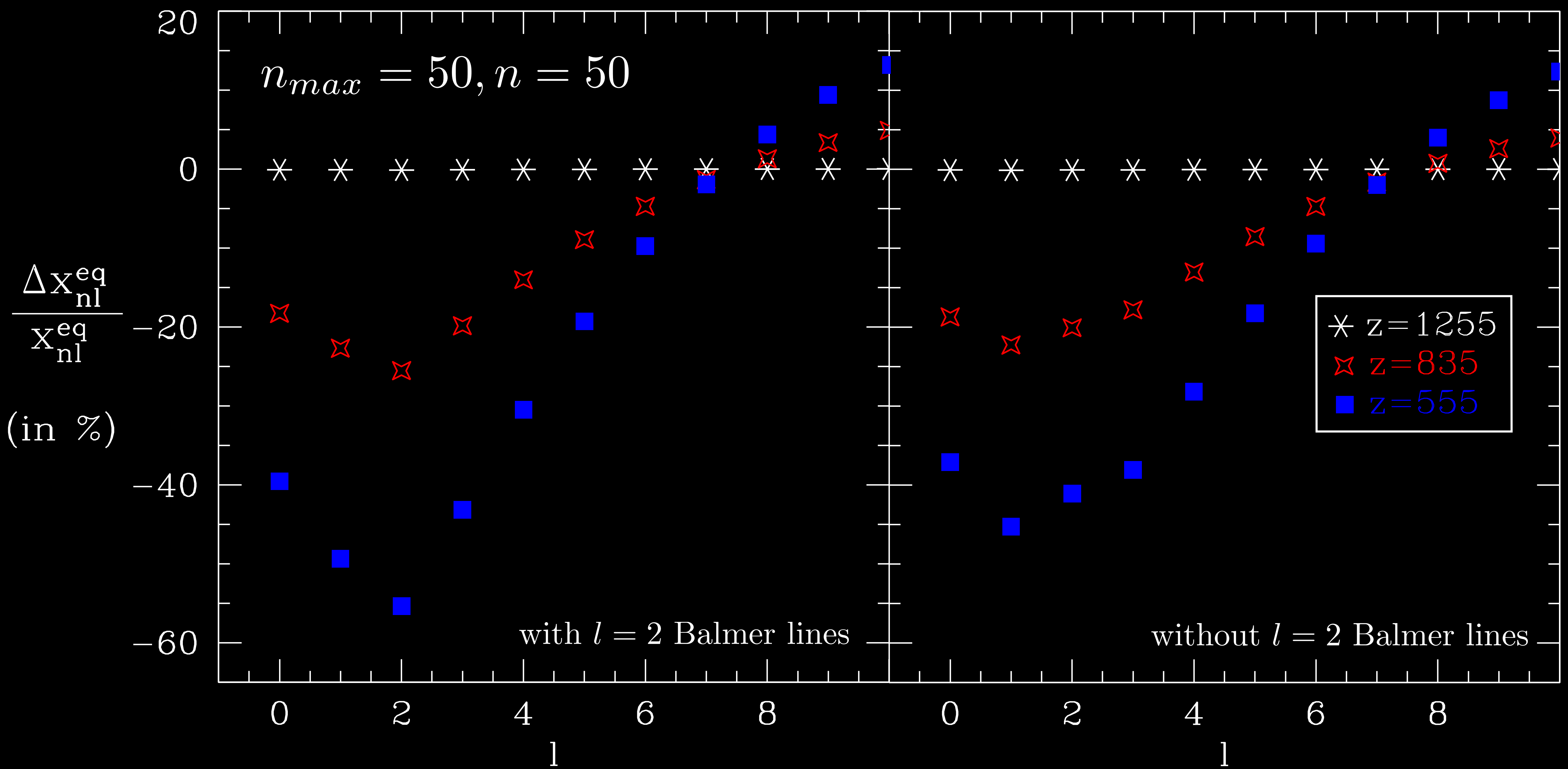
Why the feature at l=2?

WHAT IS THE ORIGIN OF THE l=2 DIP?

$$A_{\mathrm{nd}\to 2\mathrm{p}} > A_{\mathrm{np}\to 2\mathrm{s}} > A_{\mathrm{ns}\to 2\mathrm{p}}$$

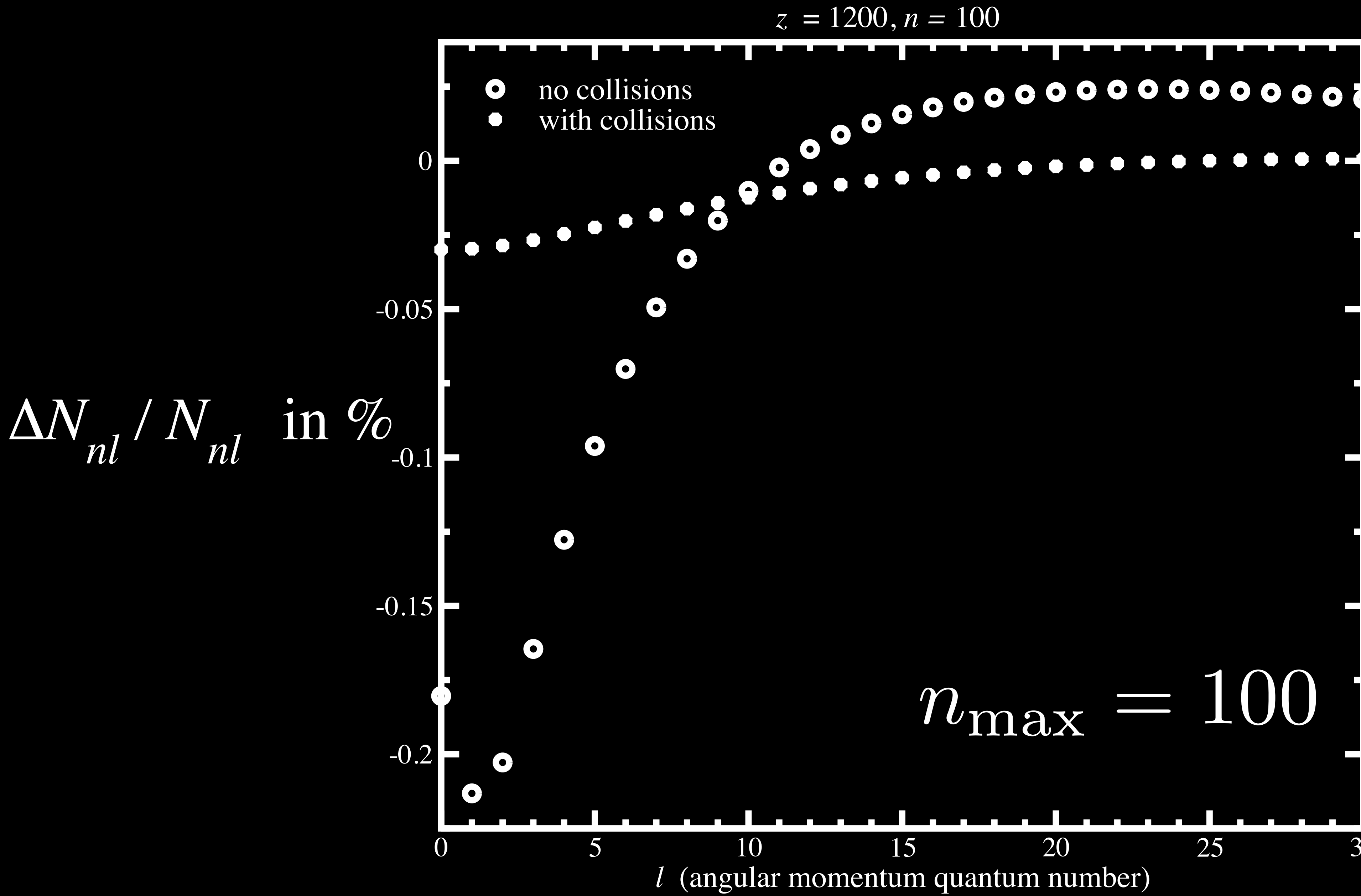
- l=2 depopulates more rapidly than l=1 for higher (n>2) excited states
- We can test if this explains the dip at l=2 by running the code with these Balmer transitions the blip should move to l=1

L-SUBSTATE POPULATIONS, BALMER LINES OFF



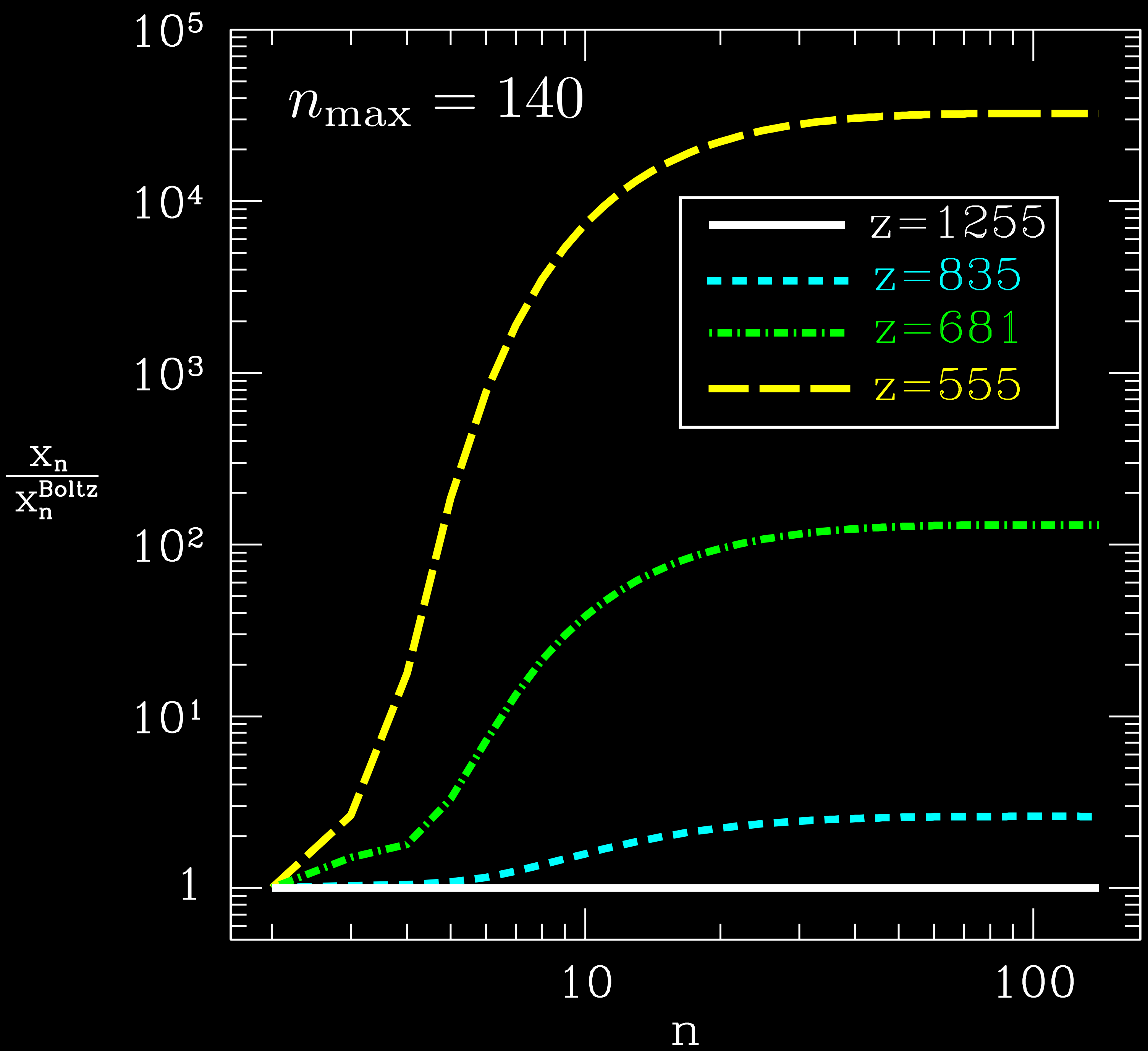
Dip moves as expected when Balmer lines are off!

ATOMIC COLLISIONS



- ✳ l-changing collisions bring l-substates closer to statistical equilibrium (SE) (Chluba, Rubino Martin, Sunyaev 2006)
- ✳ Theoretical collision rates unknown to factors of 2!

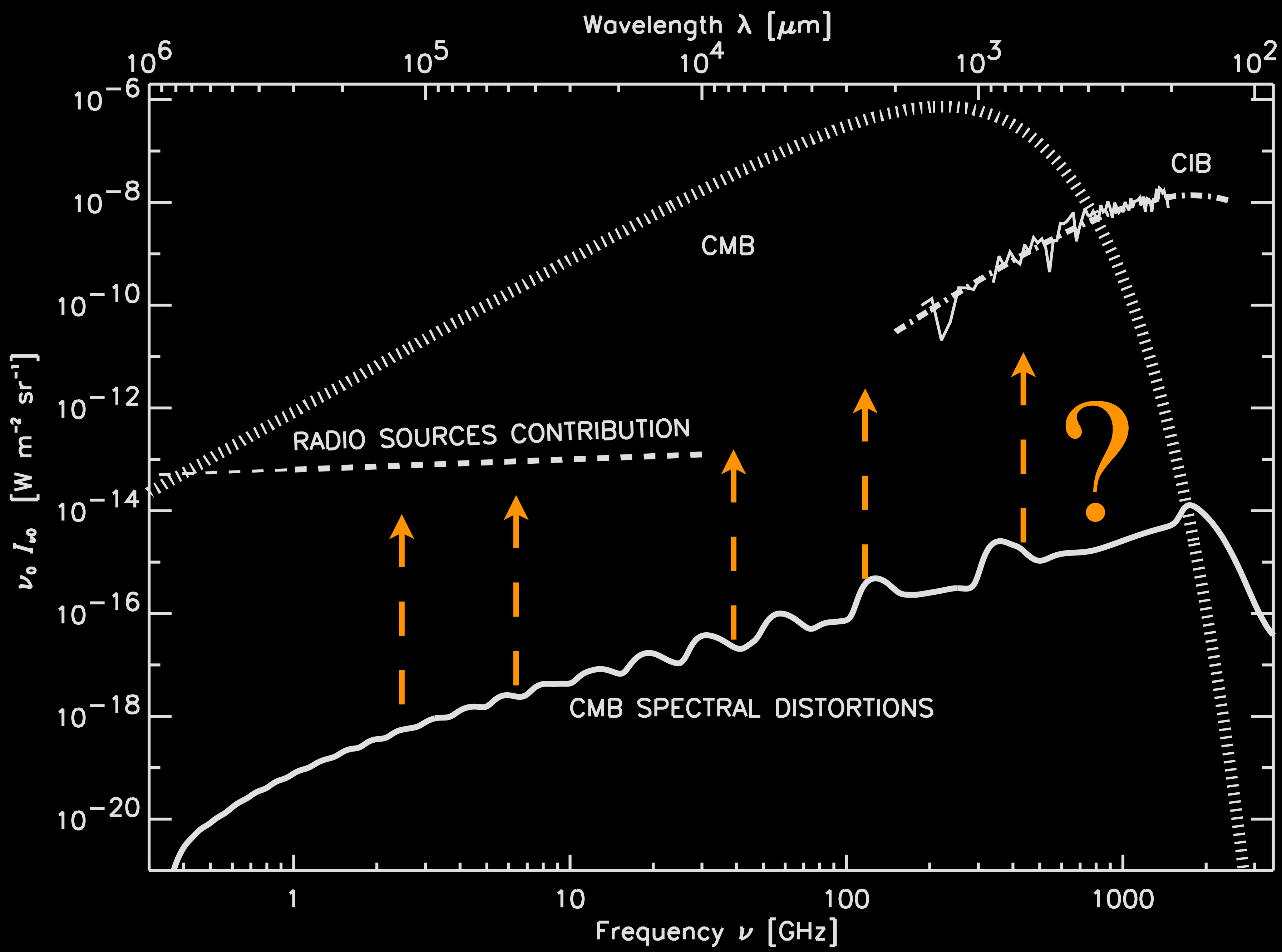
DEVIATIONS FROM BOLTZMANN EQUILIBRIUM: DIFFERENT n-SHELLS



$$\alpha_n n_e > \sum_{n'l}^{n'<n} A_{nn'}^{ll\pm1}$$

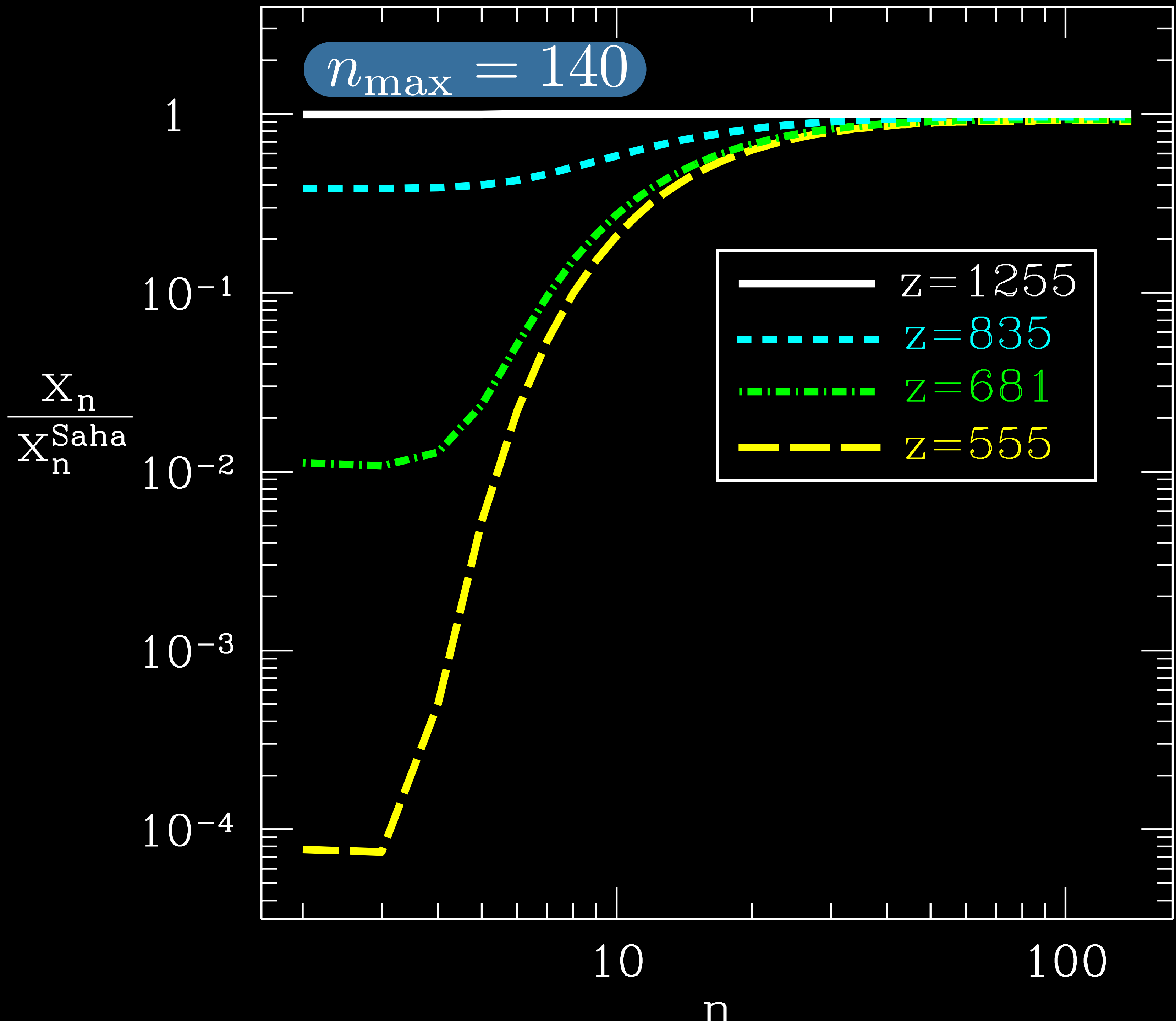
- No inversion relative to n=2 (just over-population)
- Population inversion seen between some excited states: Does radiation stay coherent? Does recombination mase?

DEVIATIONS FROM BOLTZMANN EQUILIBRIUM: DIFFERENT n-SHELLS



Masing could make spectral distortions detectable!

DEVIATIONS FROM SAHA EQUILIBRIUM

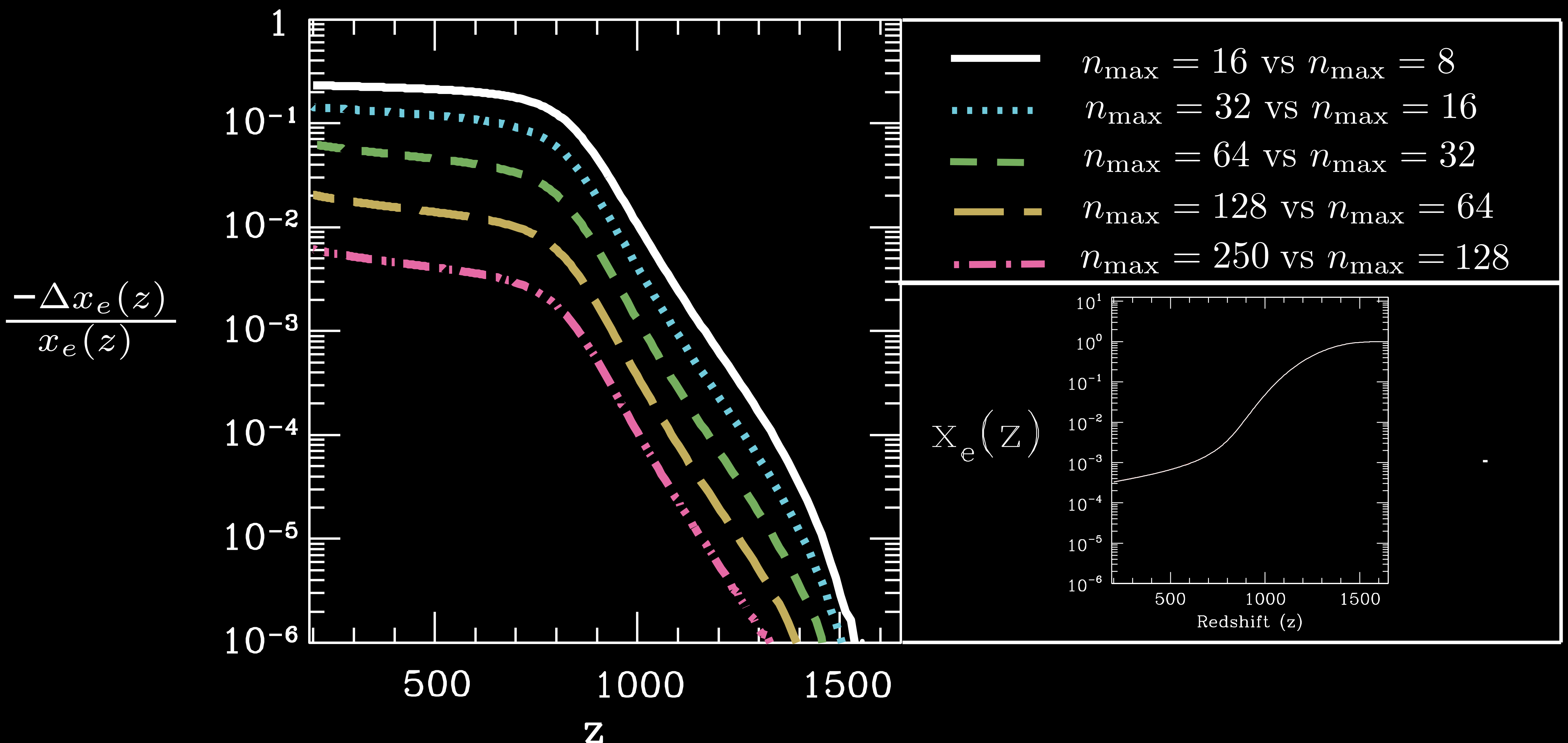


HUGE DEVIATIONS FROM SAHA EQ!

- Effect of states with $n > n_{max}$ could be approximated using asymptotic Einstein coeffs. and Saha eq, but Saha is elusive at high n/late times.
- At z=200, n_{max}~1000 needed, unless collisions included

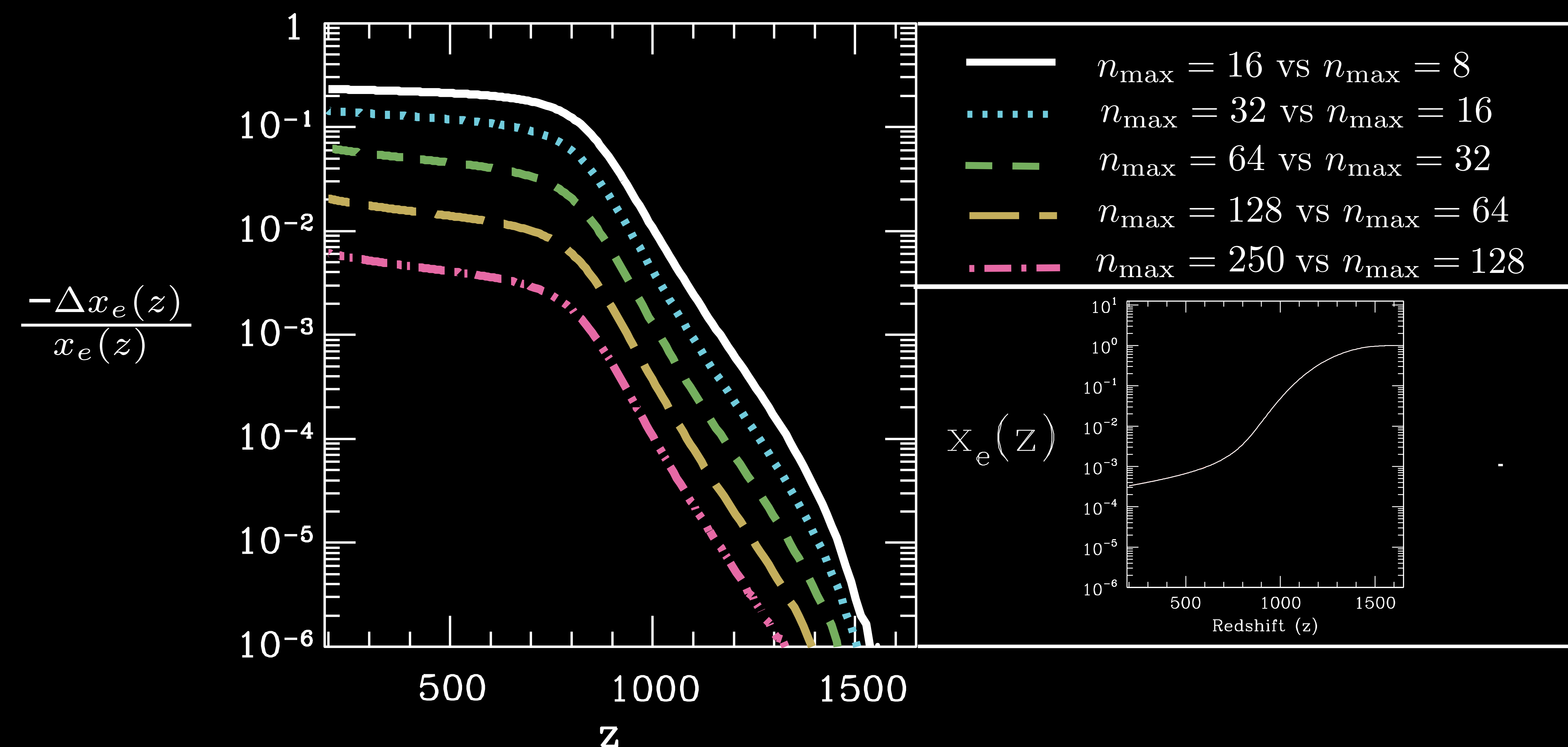
RESULTS: RECOMBINATION HISTORIES

RESULTS: RECOMBINATION HISTORIES INCLUDING HIGH-n



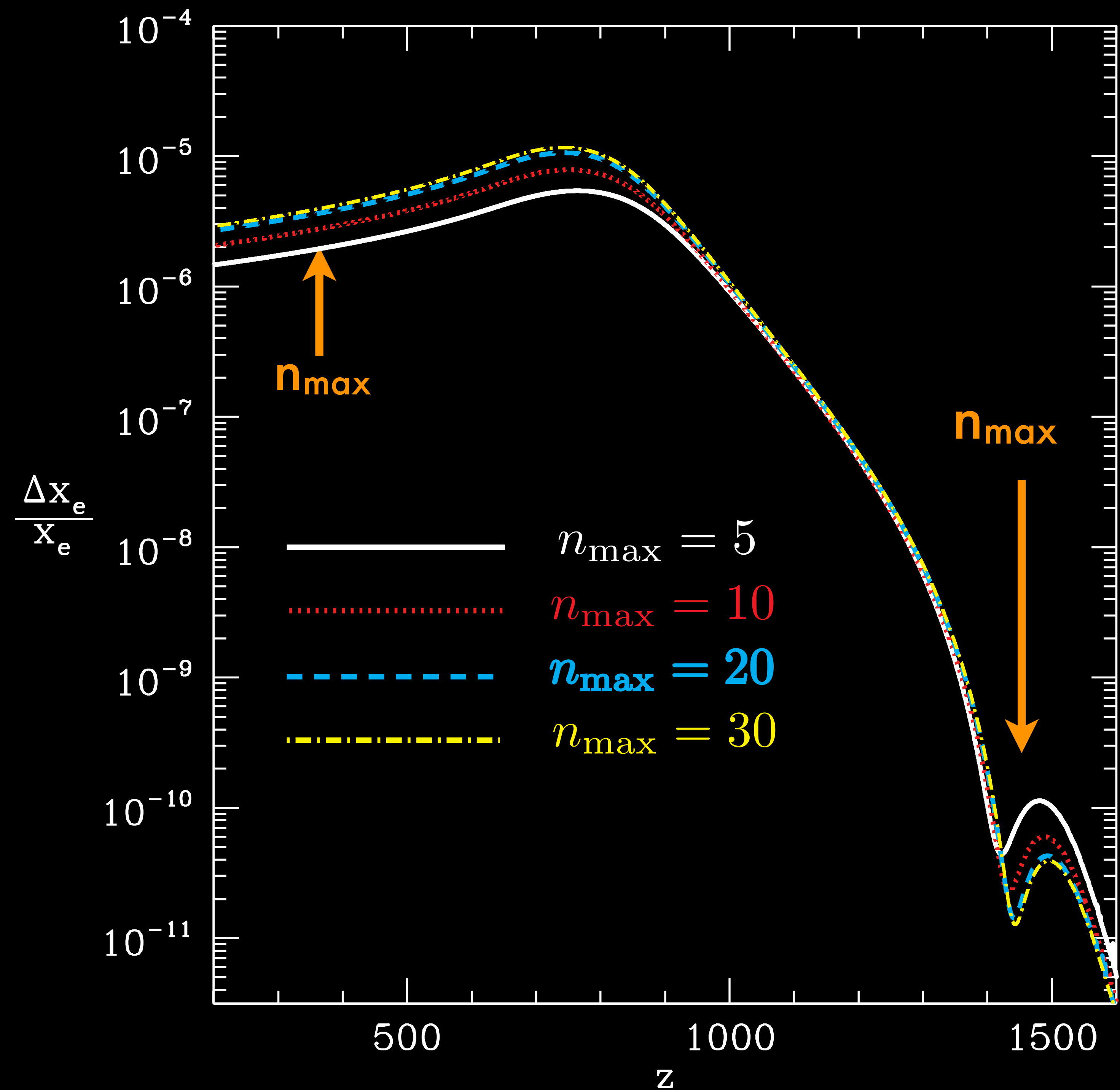
- $x_e(z)$ falls with increasing $n_{\max} = 10 \rightarrow 250$, as expected.
- Rec Rate>downward BB Rate> Ionization, upward BB rate
- For $n_{max} = 100$, code computes in only 2 hours

RESULTS: RECOMBINATION HISTORIES INCLUDING HIGH-n



- Relative convergence is not the same thing as absolute convergence: Want to see Saha asymptote and impose well-motivated cutoff! Collisions could help
- These are lower limits to the actual error
- n_{max}=300 just completed

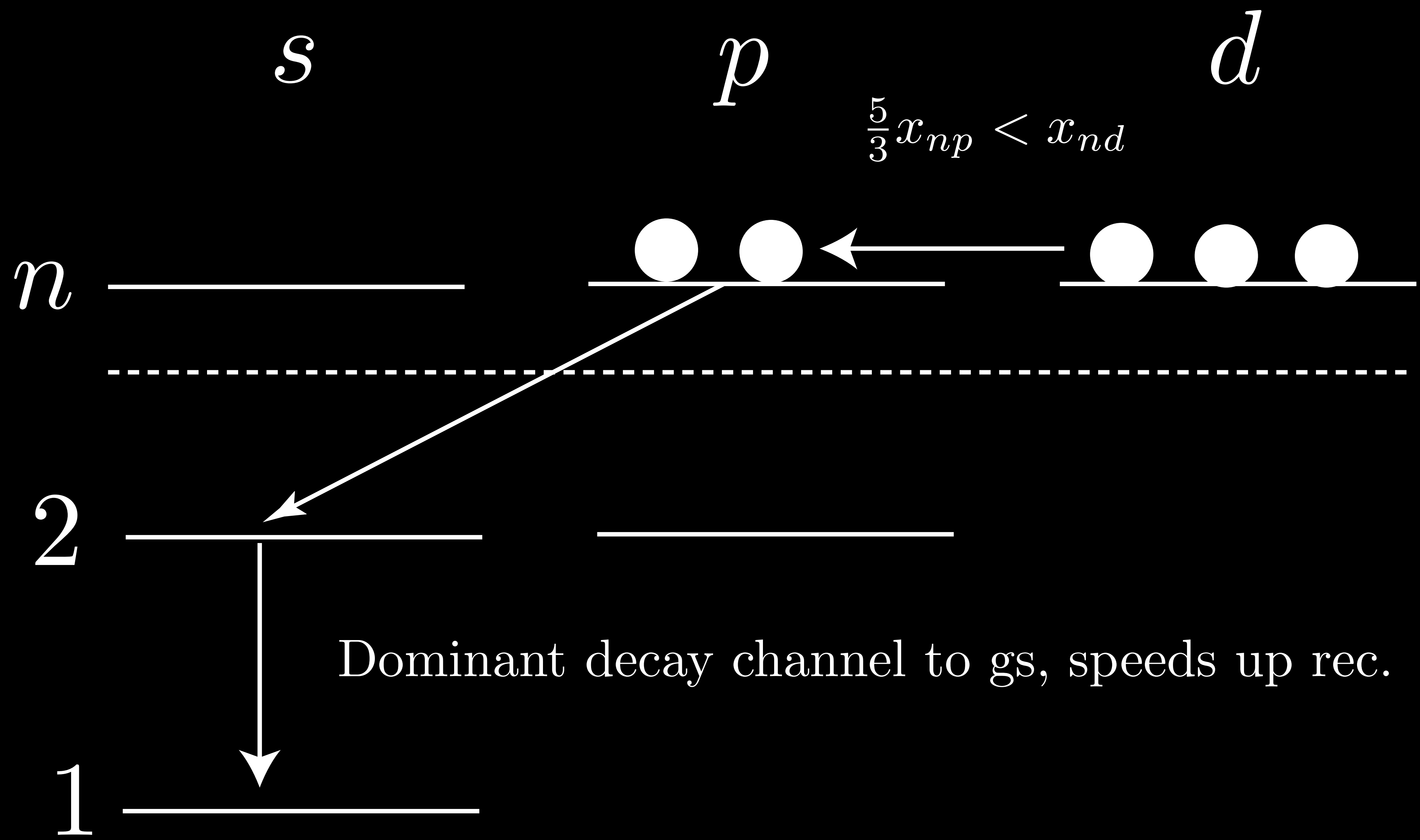
RESULTS: RECOMBINATION WITH HYDROGEN



$$\Delta x_e \equiv x_e\big|_{\text{no } E2 \text{ transitions}} - x_e\big|_{\text{with } E2 \text{ transitions}}$$

Negligible for Planck!

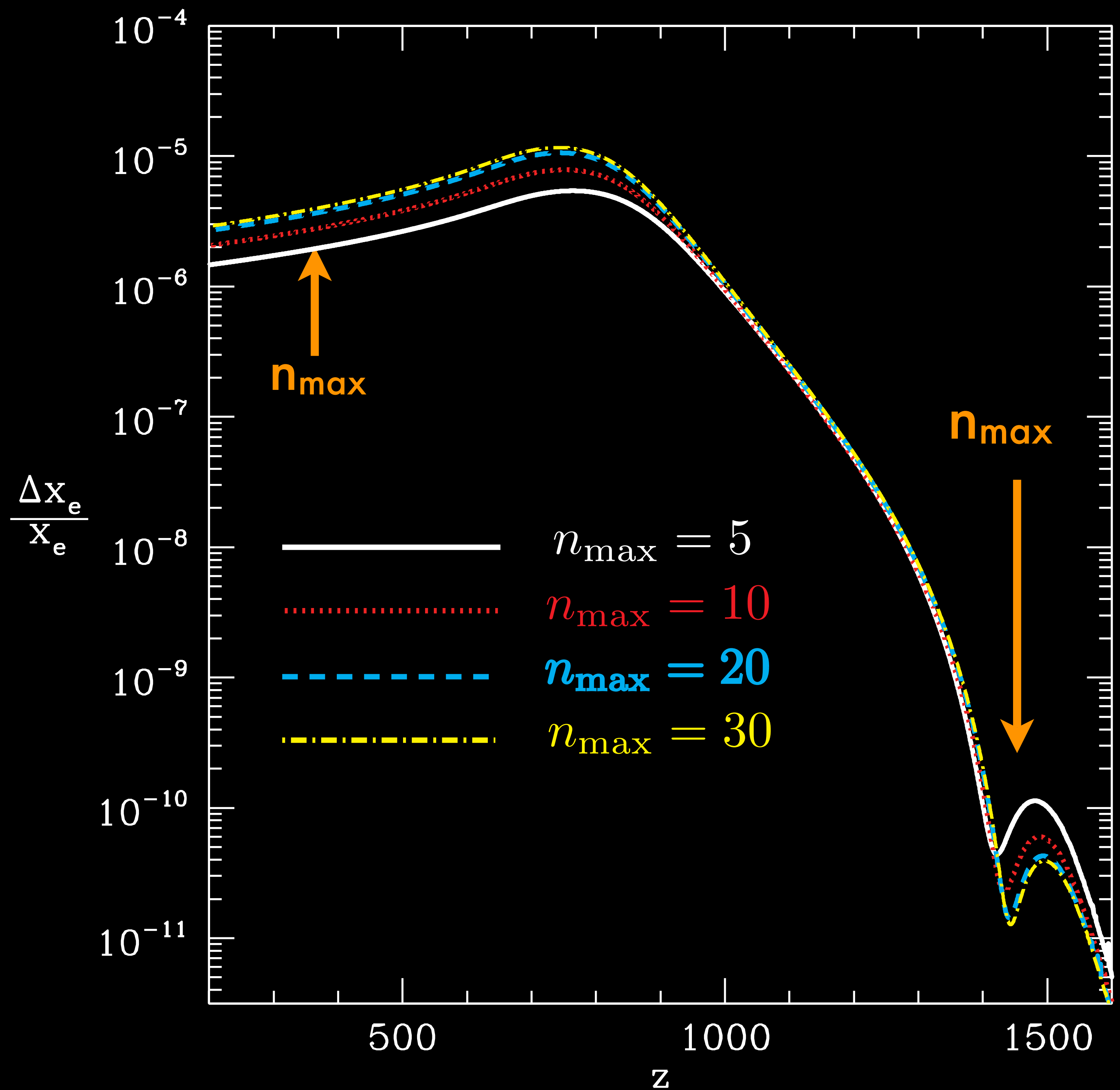
RESULTS: RECOMBINATION WITH HYDROGEN



$$R^{\text{quad}}_{nd\to np} = A_{nd\to 1s}\left(x_{nd} - \frac{5}{3}x_{np}\right)$$

$n < 5$, early times

RESULTS: RECOMBINATION WITH HYDROGEN



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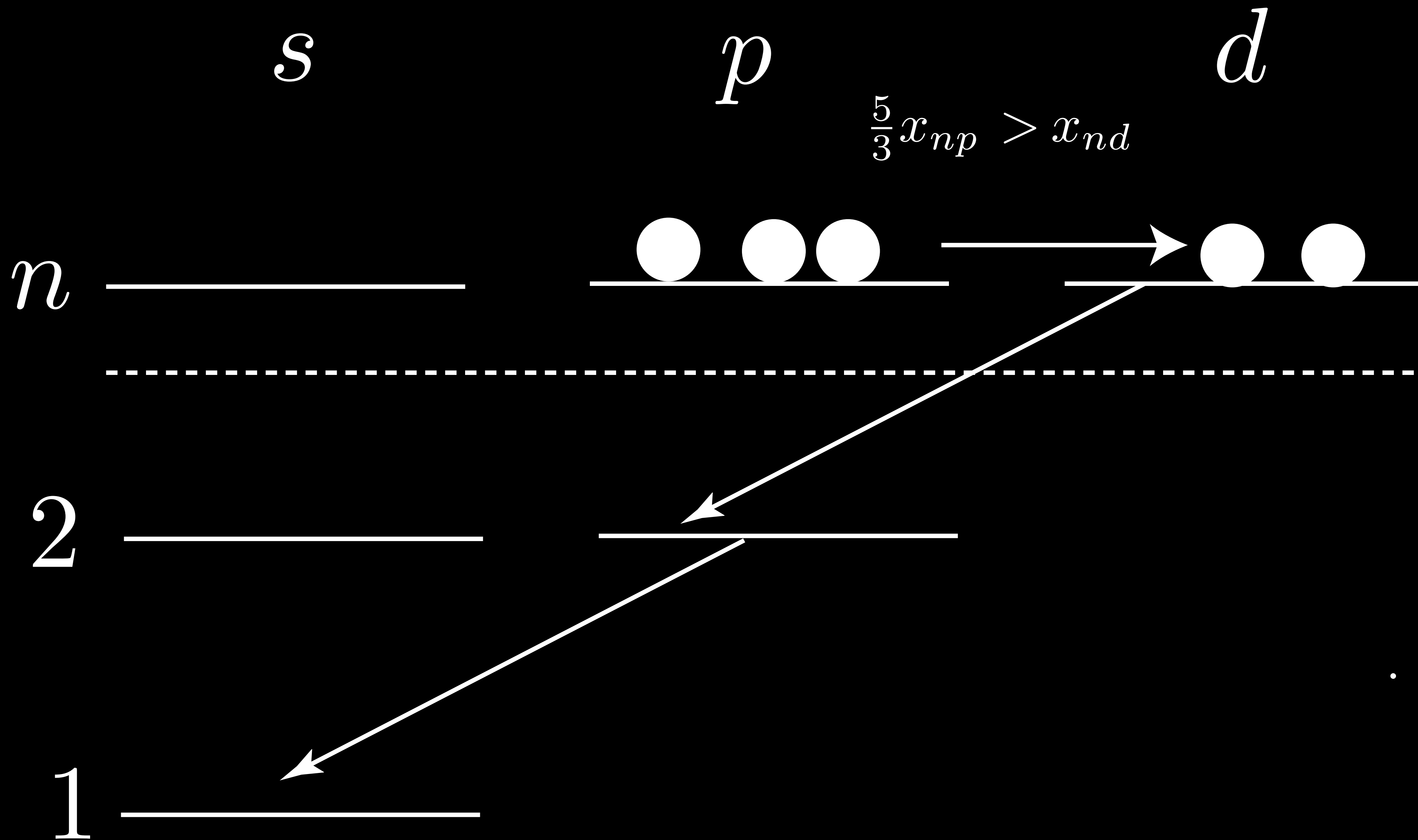
Negligible for Planck!

RESULTS: RECOMBINATION WITH HYDROGEN

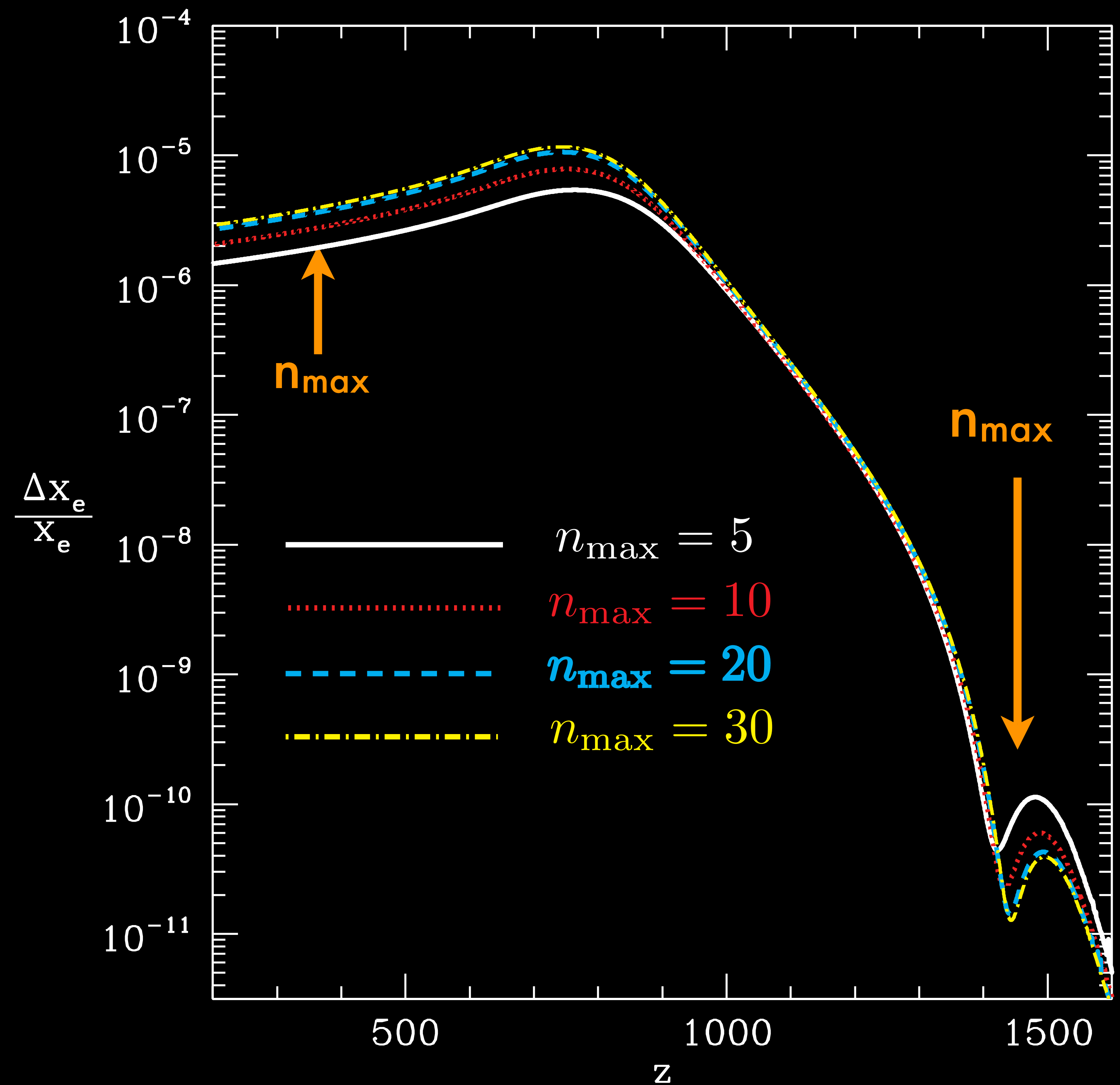


Sub-Dominant decay channel to gs, slows rec down rel. to $n < 5$

$$R^{\mathrm{quad}}_{nd\to np} = A_{nd\to 1s}\left(x_{nd} - \frac{5}{3}x_{np}\right)$$

$n \geq 5$, early times

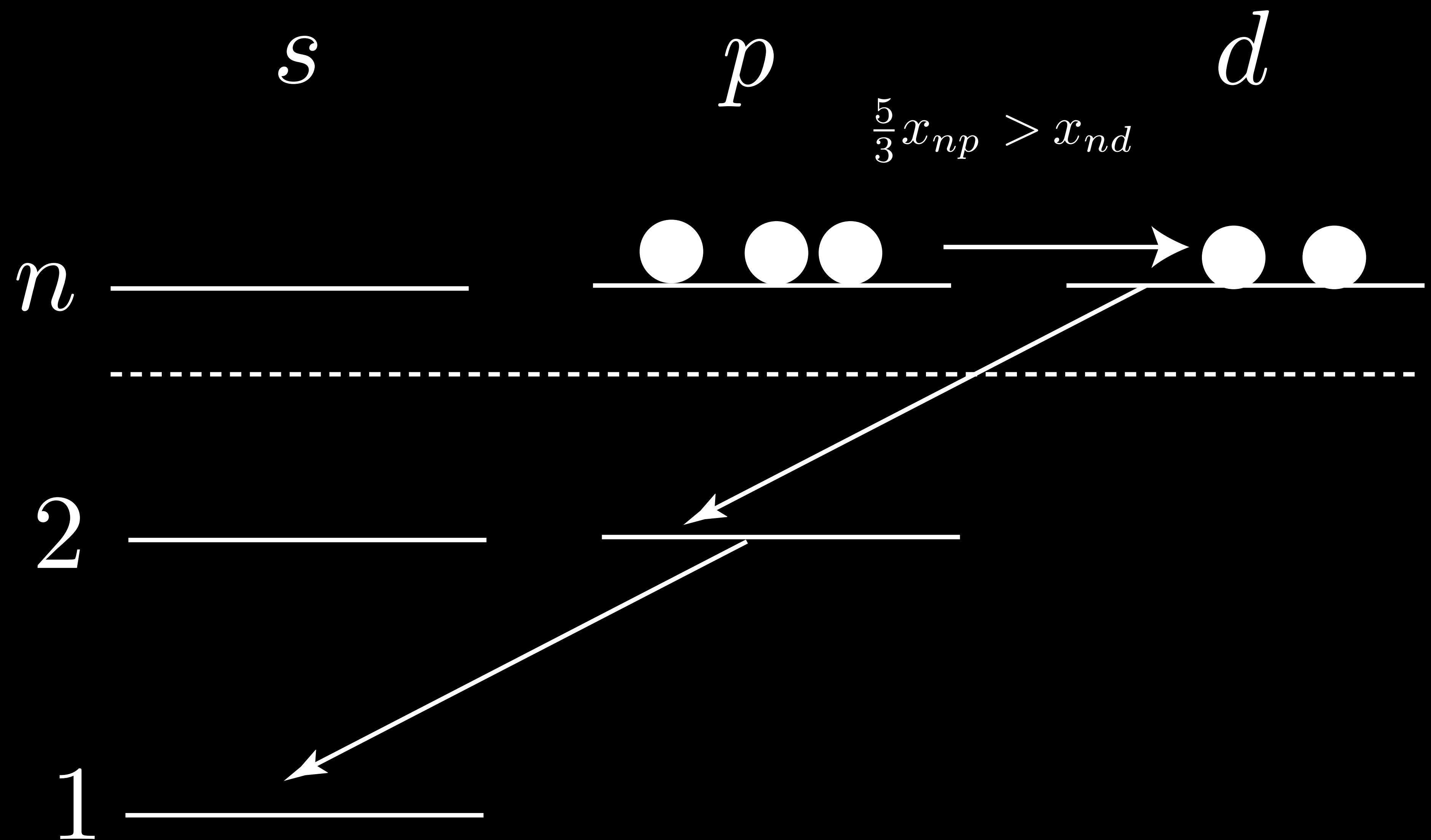
RESULTS: RECOMBINATION WITH HYDROGEN



$$\Delta x_e \equiv x_e\big|_{\text{no } E2 \text{ transitions}} - x_e\big|_{\text{with } E2 \text{ transitions}}$$

Negligible for Planck!

RESULTS: RECOMBINATION WITH HYDROGEN

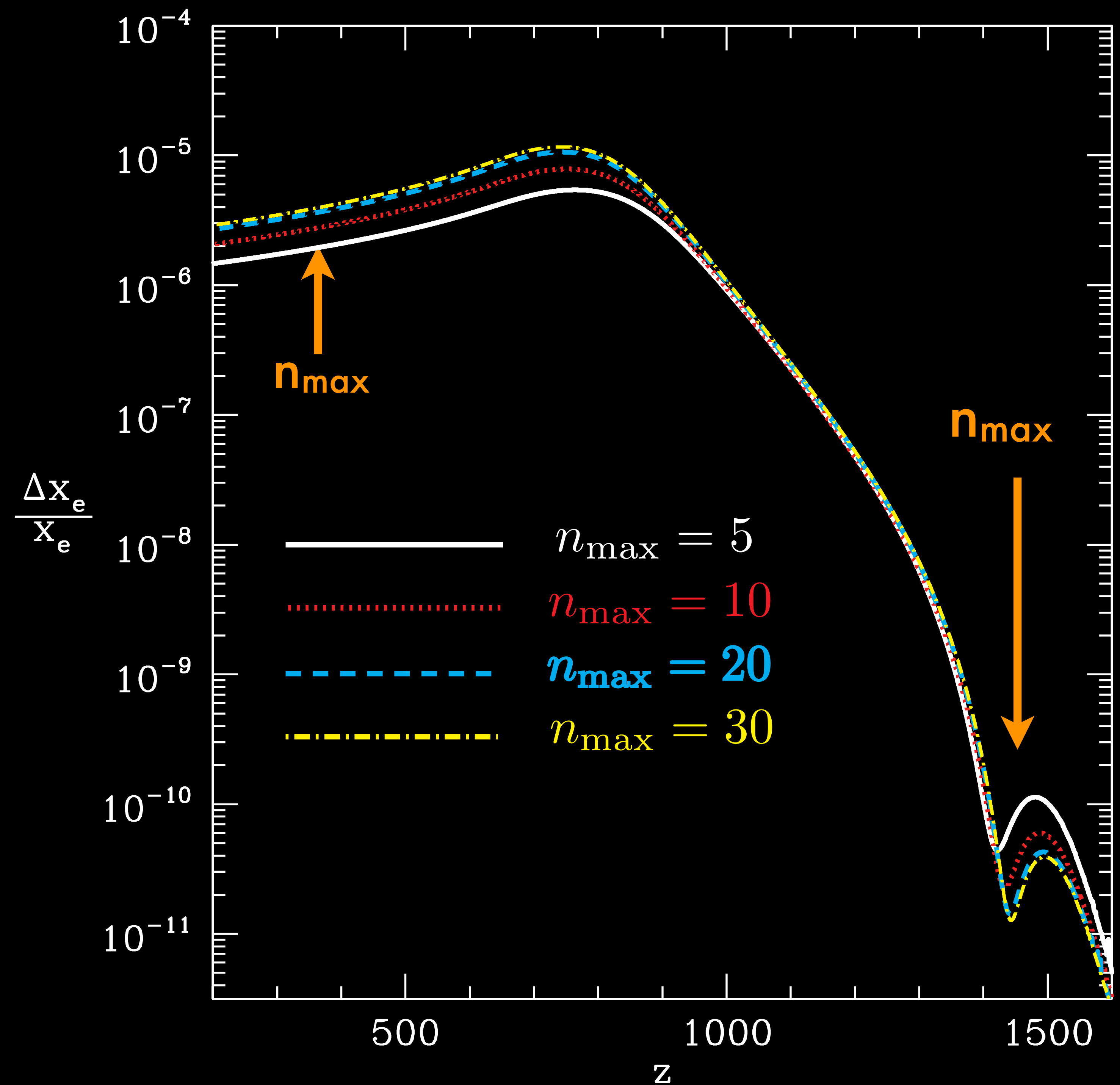


Dominant decay channel to gs, speeds up rec

$$R^{\rm quad}_{nd\rightarrow np} = A_{nd\rightarrow 1s}\left(x_{nd} - \frac{5}{3}x_{np}\right)$$

All n, late times

RESULTS: RECOMBINATION WITH HYDROGEN

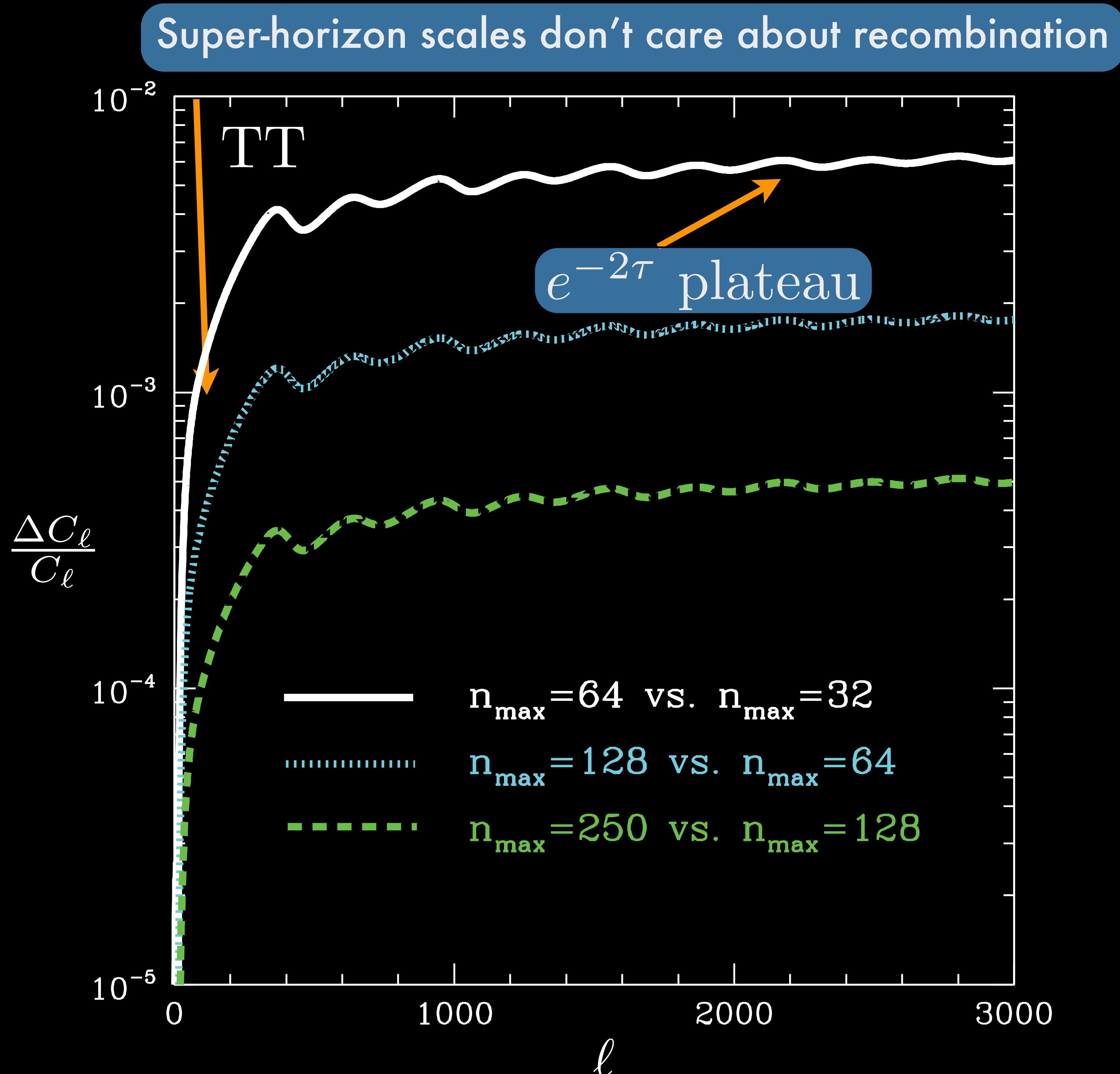


$$\Delta x_e \equiv x_e\big|_{\text{no } E2 \text{ transitions}} - x_e\big|_{\text{with } E2 \text{ transitions}}$$

Negligible for Planck!

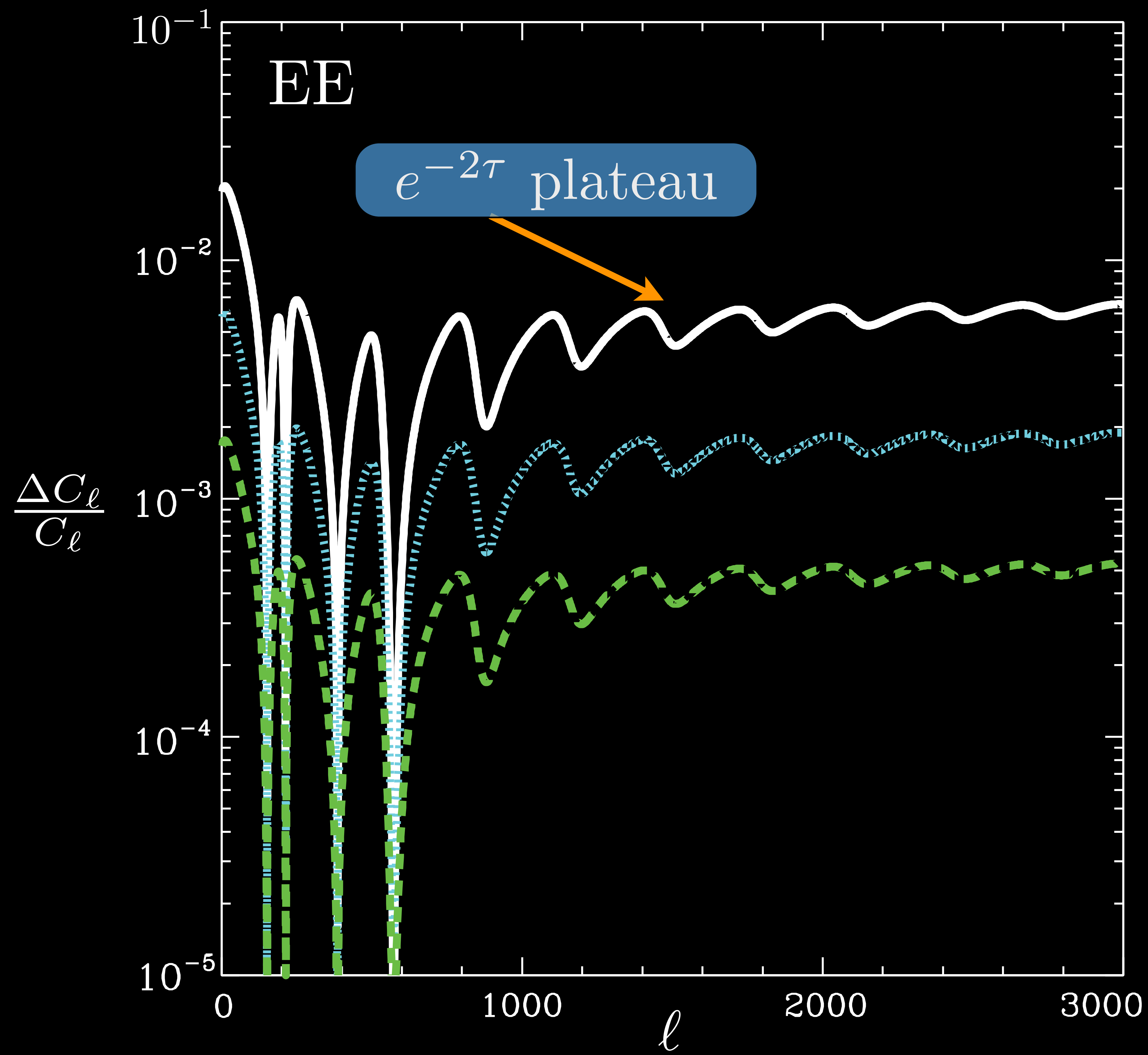
RESULTS: CMB ANISOTROPIES

RESULTS: TT $C_l s$ WITH HIGH-N STATES



38

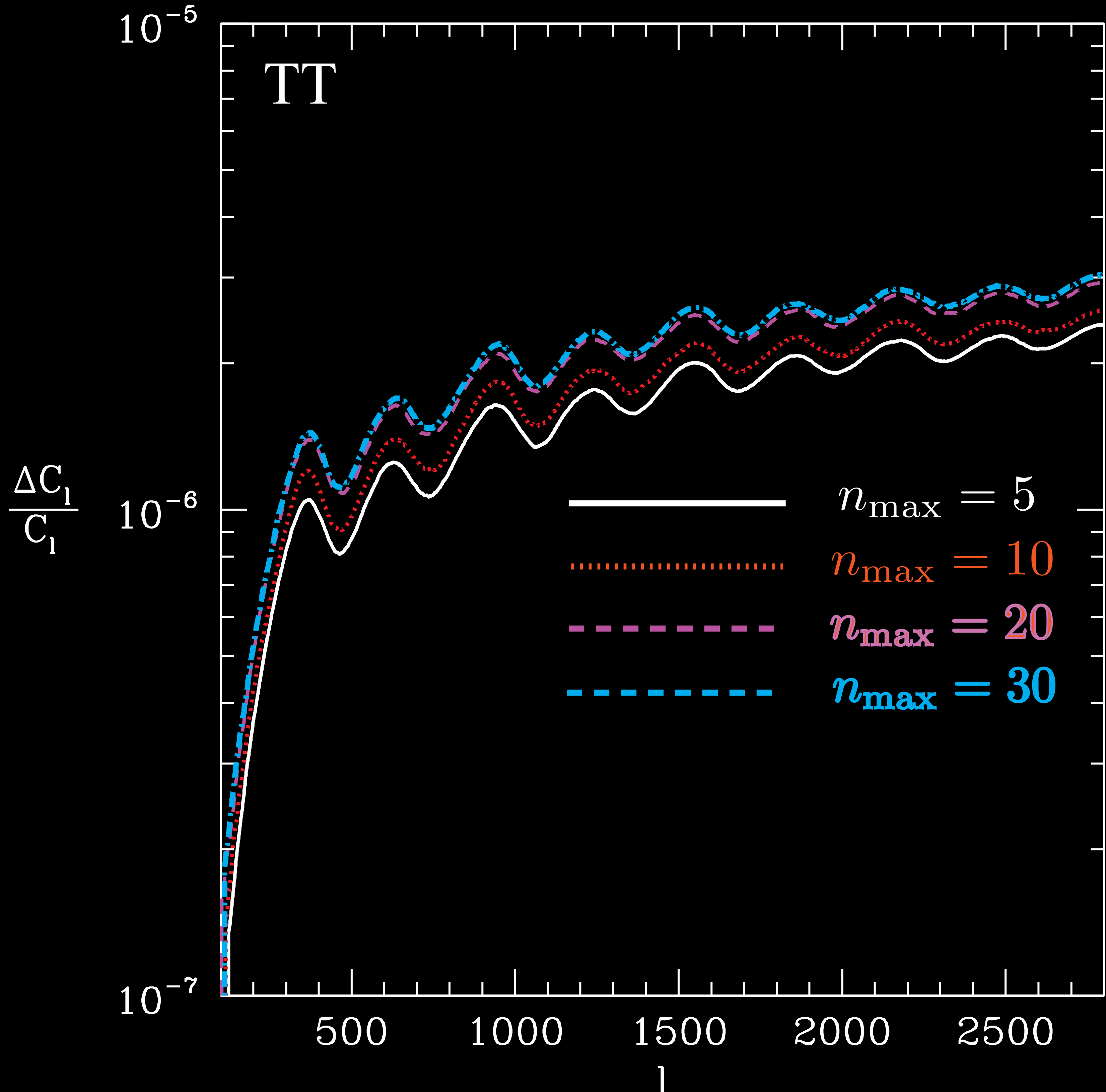
RESULTS: EE $C_l s$ WITH HIGH-N STATES



39

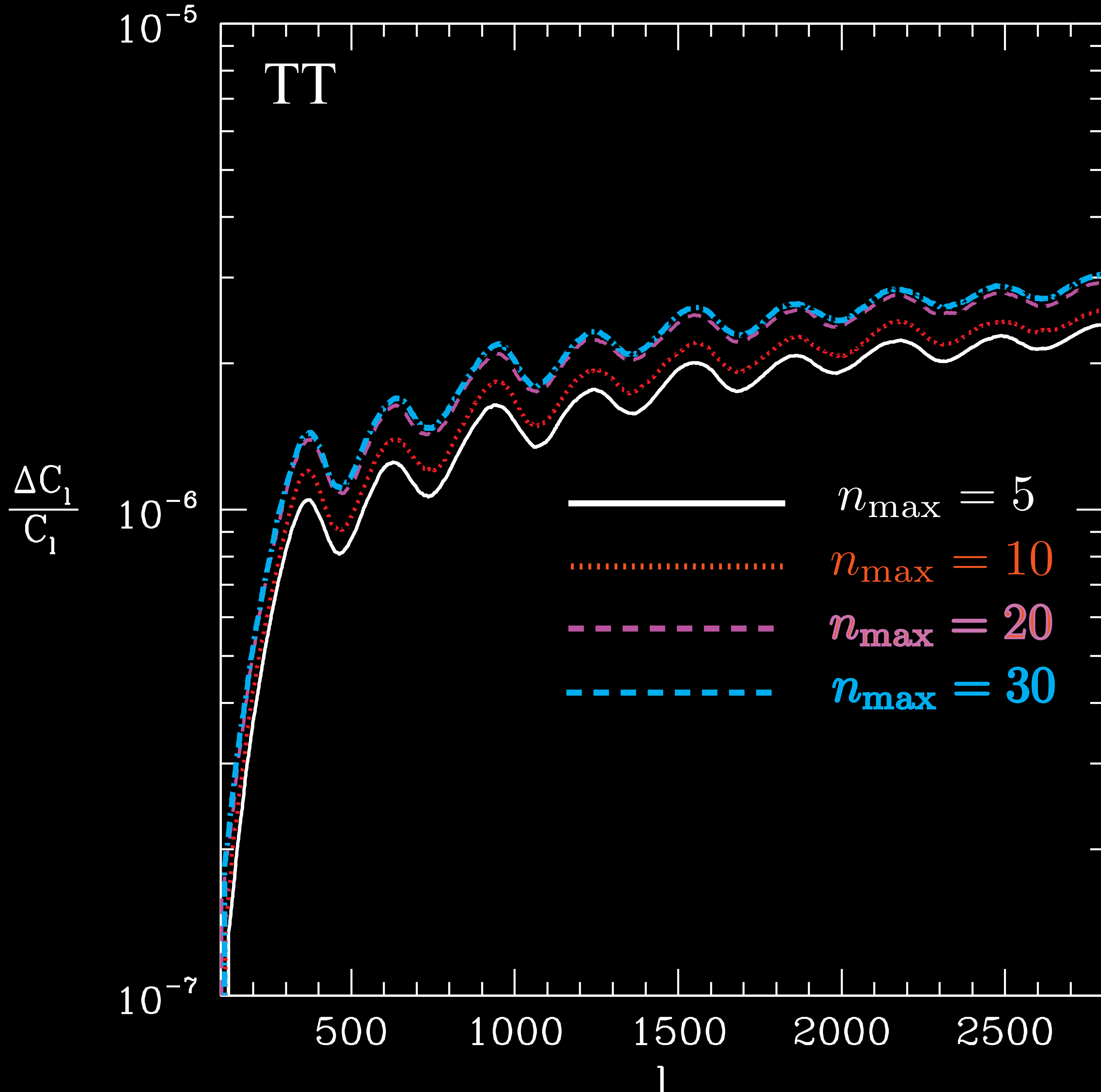
RESULTS: TEMPERATURE (TT) $C_l s$ WITH HYDROGEN QUADRUPOLES,

Bulk of integral from late times, higher n_{max} $\rightarrow$ lower x_e $\rightarrow$ lower τ $\rightarrow$ higher $e^{-2\tau}$ $\rightarrow$ higher C_l



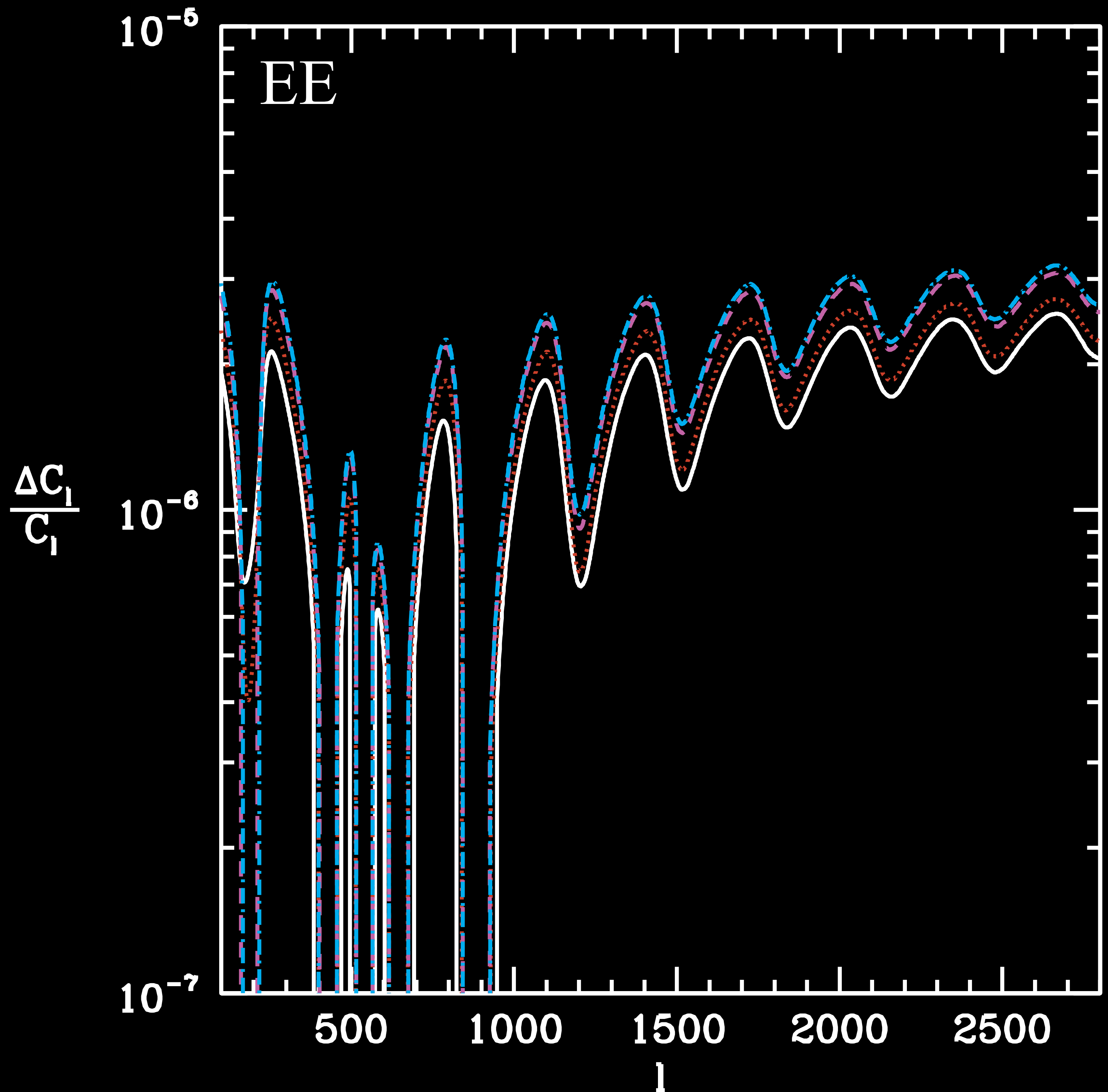
RESULTS: TEMPERATURE (TT) $C_l s$ WITH HYDROGEN QUADRUPOLES,

Bulk of integral from late times, higher $\mathrm{n_{max}}$ $\longrightarrow$ lower x_e $\longrightarrow$ lower τ $\longrightarrow$ higher $e^{-2\tau}$ $\longrightarrow$ higher C_l



Overall effect is negligible for CMB experiments!

RESULTS: POLARIZATION (EE) C_ls WITH HYDROGEN QUADRUPOLES

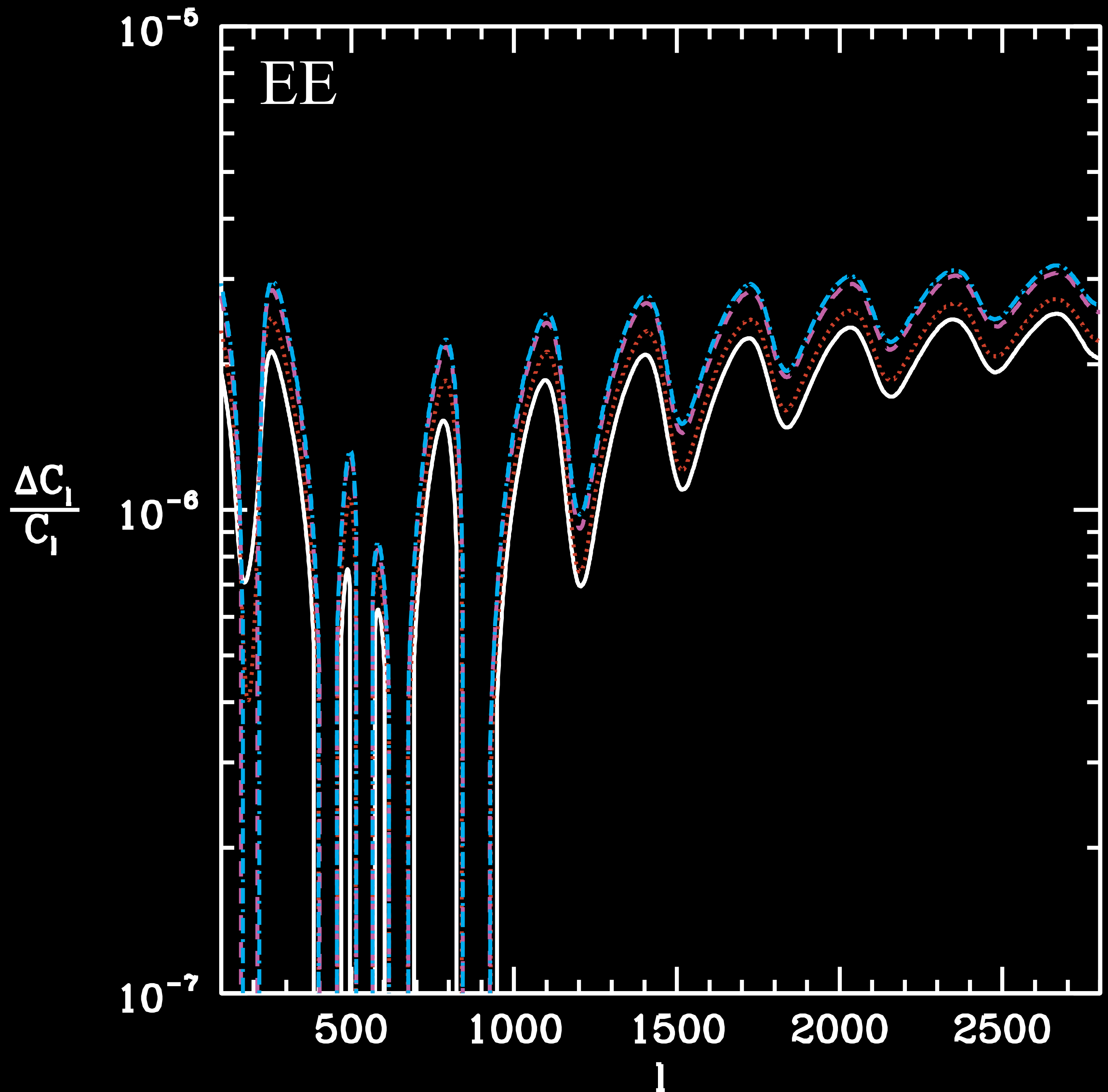


$$\Delta C_l \equiv C_l|_{\text{with } E2 \text{ transitions}} - x_e|_{\text{no } E2 \text{ transitions}}.$$

Bulk of integral from late times, higher $n_{\max} \rightarrow$ lower x_e $\rightarrow$ lower $\tau \rightarrow$ higher $e^{-2\tau} \rightarrow$ higher C_l

41

RESULTS: POLARIZATION (EE) C_ls WITH HYDROGEN QUADRUPOLES

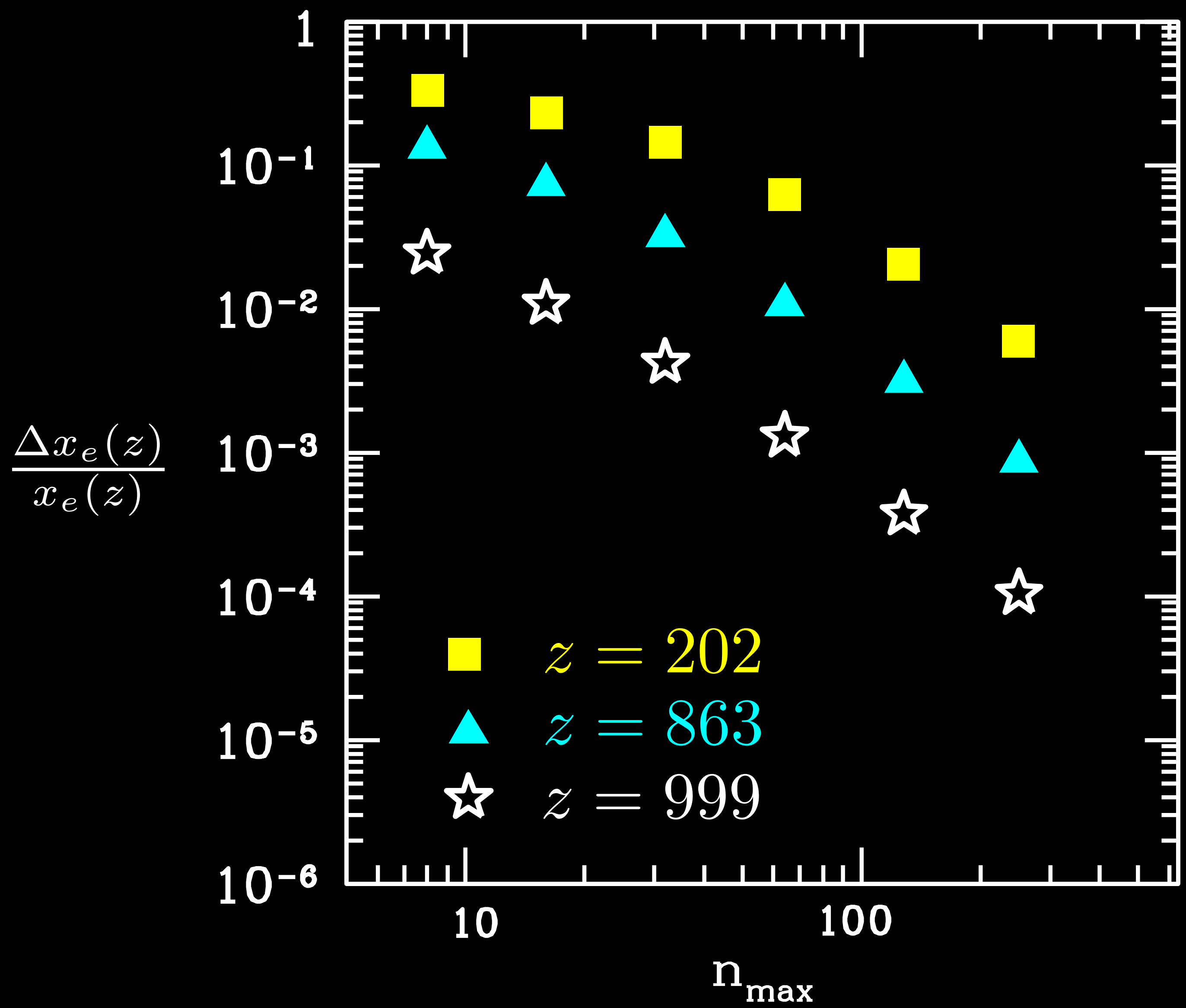


$$\Delta C_l \equiv C_l|_{\text{with } E2 \text{ transitions}} - x_e|_{\text{no } E2 \text{ transitions}} .$$

Overall effect is negligible for upcoming CMB experiments!

Bulk of integral from late times, higher n_{max} $\rightarrow$ lower x_e $\rightarrow$ lower τ $\rightarrow$ higher $e^{-2\tau}$ $\rightarrow$ higher C_l

CONVERGENCE



- ✳ Relative error well described by power law at high n_{max}

$$\Delta x_e / x_e \propto n_{\mathrm{max}}^{-1.9}$$

- ✳ Can extrapolate to absolute error

THE UPSHOT FOR COSMOLOGY

- Can explore effect on overall Planck likelihood analysis

$$Z^2 = \sum_{ll',X,Y} F_{ll'} \Delta C_l^{\mathrm{X}} \Delta C_l^{\mathrm{Y}}$$

$$Z = 1.8 \text{ if } n_{\max} = 64,$$
$$Z = 0.50 \text{ if } n_{\max} = 128,$$
$$Z = 0.14 \text{ if } n_{\max} = 250.$$

CONCLUSIONS

- RecSparse: a new tool for MLA recombination calculations (*arXiv:0911.1359*)

- Highly excited levels (n~64 and higher) are relevant for Planck CMB data analysis

- E2 transitions in H are not relevant for Planck CMB data analysis

FUTURE WORK

- Include line-overlap
- Develop cutoff method for excluded levels
- Generalize RecSparse to calc. rec. line. spectra
- Compute and include collisional rates
- Monte-Carlo analyses
- Cosmological masers

Bound-free rates

- Using continuum wave functions, bound-free rates are obtained (Burgess 1957)
- Bound-free matrix elements satisfy a convenient recursion relation:
- Matrix elements compared with Burgess 1965 (5 digits) and with WKB approximation (5%):
- At each temperature, thermal recombination/ionization rates obtained using 11-point Newton-Cotes formula, agreement with Burgess to 4 published digits

BB Rate coefficients: verification

- WKB estimate of matrix elements $$\rho(n'l', nl) = a_0 n^2 \int_{-\pi}^{\pi} d\tau e^{i\Omega\tau} \left(1 + cos\eta\right)$$

$$\Omega = \omega_n - \omega_{n'}$$
$$r = r_{\max}\left(1 + \cos\eta\right)/2$$
$$\tau = \eta + \sin\eta$$

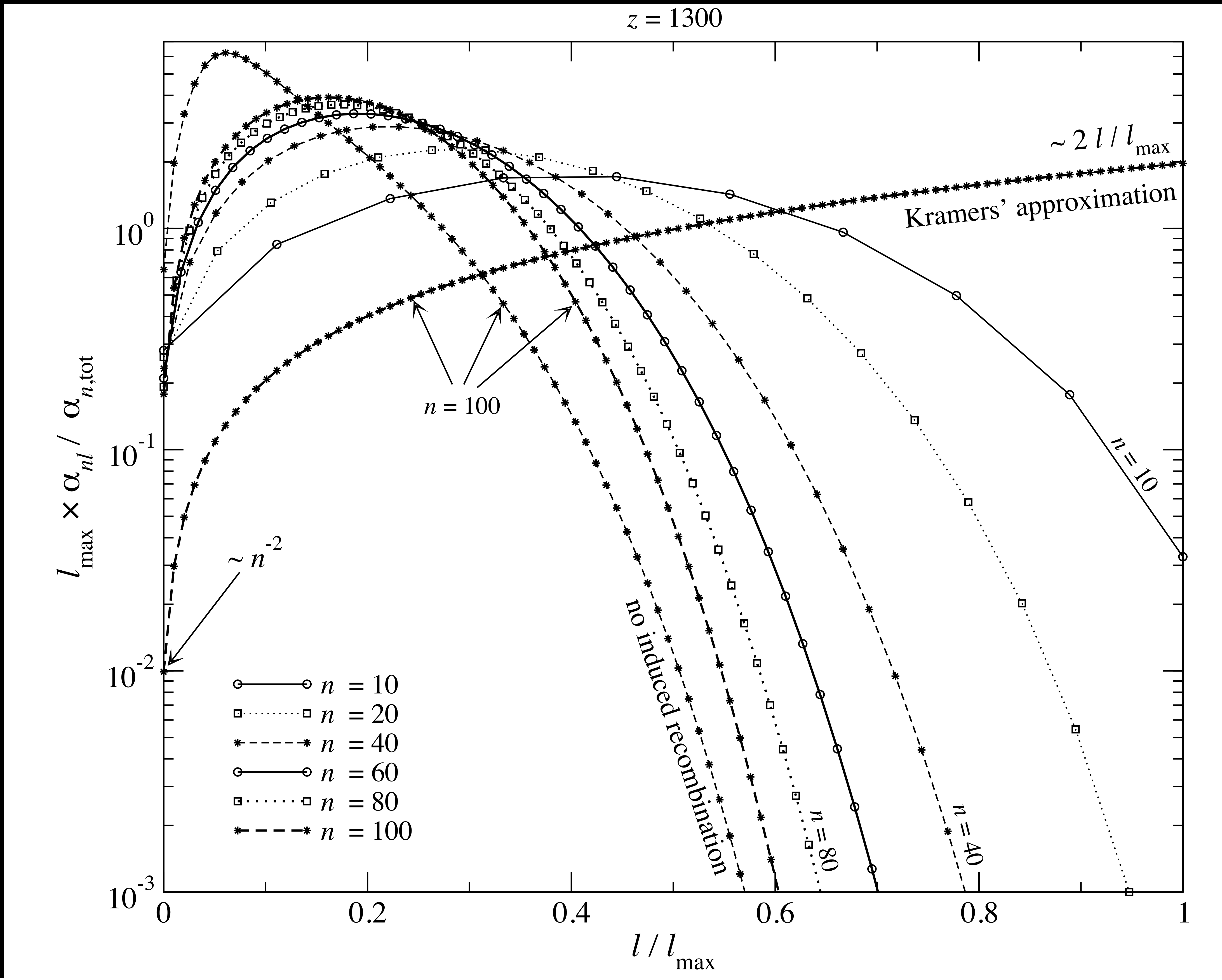
Fourier transform of classical orbit!
Application of correspondence principle!

$$\rho^{\text{dipole}}(n, l, n', l') = \frac{n_c^2}{s}\left\{J_{s-1}(s\epsilon) - \frac{1 \mp \sqrt{1-\epsilon^2}}{\epsilon} J_s(s\epsilon)\right\}$$
$$\epsilon = \left(1 - \frac{l(l+1)}{n^2}\right)^{1/2}$$
$$s = n - n'$$

- Radial matrix elements checked against WKB (10%), published rates of Brocklehurst (1971), Green, Rush, and Chandler (1967) (agreement to their published 4 digits)

DEVIATIONS FROM BOLTZMANN EQ: L-SUBSTATES

Chluba/Rubino-Martin/Sunyaev 2006



48

Quadrupole rates: basic formalism

$$A^{\text{quad}}_{n_a,l_a\to n_b,l_b} = \frac{\alpha}{15}\frac{1}{2l_a+1}\frac{\omega_{ab}^5}{c^4}\left\langle l_a||C^{(2)}||l_b\right\rangle^2 \left({}^2R^{n_a l_a}_{n_b l_b}\right)^2$$

- Reduced matrix element evaluated using Wigner 3J symbols:

$$\left\langle l_a||C^{(2)}||l_b\right\rangle = (-1)^{l_a}\sqrt{(2l_a+1)(2l_b+1)}\begin{pmatrix} l_a & 2 & l_b \\ 0 & 0 & 0 \end{pmatrix}$$

- Radial matrix element evaluated using operator methods

$${}^2R^{n_a l_a}_{n_b l_b} \equiv \int_0^\infty r^4 R_{n_a l_a}(r) R_{n_b l_b}(r)\,dr$$

Quadrapole rates: Operator algebra

- Radial Schrödinger equation can be factored to yield:

$$^{-}\Omega_{nl} = \frac{1}{l A_{nl}}\left[1 - l\left(\frac{d}{dr} + \frac{l+1}{r}\right)\right] \qquad ^{+}\Omega_{nl} = \frac{1}{l A_{nl}}\left[1 + l\left(\frac{d}{dr} - \frac{l-1}{r}\right)\right]$$

$$^{-}\Omega_{nl} R_{nl}(r) = R_{n\ l-1}(r)$$
$$^{+}\Omega_{n\ l-1} R_{nl}(r) = R_{nl}(r)$$

$$A_{nl} = \frac{\sqrt{n^2 - l^2}}{nl}$$

- This algebra can be applied to radial matrix elements:

Quadrapole rates: Operator algebra

- Radial Schrödinger equation can be factored to yield:

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$$ {}^{-}\Omega_{nl}R_{nl}(r) = R_{n\ l-1}(r) $$
$$ {}^{+}\Omega_{n\ l-1}R_{nl}(r) = R_{nl}(r) $$

$$ A_{nl} = \frac{\sqrt{n^2 - l^2}}{nl} $$

- This algebra can be applied to radial matrix elements:

$$ {}^{2}R^{n\ l-1}_{n'\ l-1} = \frac{1}{A_{nl}}\left\{A_{n'l}{}^{2}R^{nl}_{n'l} + 2^{(1)}R^{nl}_{n'\ l-1}\right\} \qquad {}^{(2)}R^{n\ n'-1}_{n'\ n'-1} = \frac{2nn'}{\sqrt{n^2 - n'^2}}{}^{(1)}R^{nn'}_{n\ n'-1} $$

Diagonal!

Quadrapole rates: Operator algebra

- Radial Schrödinger equation can be factored to yield:

$$ {}^{-}\Omega_{nl} = \frac{1}{lA_{nl}}\left[1 - l\left(\frac{d}{dr} + \frac{l+1}{r}\right)\right] \qquad {}^{+}\Omega_{nl} = \frac{1}{lA_{nl}}\left[1 + l\left(\frac{d}{dr} - \frac{l-1}{r}\right)\right] $$

$$ {}^{-}\Omega_{nl} R_{nl}(r) = R_{n\ l-1}(r) $$
$$ {}^{+}\Omega_{n\ l-1} R_{nl}(r) = R_{nl}(r) \qquad A_{nl} = \frac{\sqrt{n^2 - l^2}}{nl} $$

- This algebra can be applied to radial matrix elements:

$$ l(2l+3)A_{n'l}\,{}^{(2)}R^{n\ l+1}_{n'\ l-1} = (2l+1)(l+2)A_{n\ l+2}\,{}^{(2)}R^{n\ l+2}_{n'l} + 2(l+1)A_{n'\ l+1}\,{}^{(2)}R^{n\ l+1}_{n'\ l+1} + 2(2l+1)(3l+5)\,{}^{(1)}R^{n\ l+1}_{n'l} \quad (1 \le l \le n'-1) $$

$$ {}^{(2)}R^{n\ n'-1}_{n'\ n'+1} = 0 $$

$$ {}^{(2)}R^{n\ n'+1}_{n'\ n'-1} = (-1)^{n-n'} 2^{2n'+4}\left[\frac{(n+n'+1)!}{(n-n'-2)!(2n'-1)!}\right]^{1/2} n'\,(nn')^{n'+3}\,\frac{(n-n')^{n-n'-3}}{(n+n')^{n+n'+3}} $$

Off-diagonal!