COSMOLOGICAL HYDROGEN RECOMBINATION: THE EFFECT OF HIGH-N STATES

Daniel Grin

in collaboration with Chris Hirata

Caltech

NPAC Talk
UW Madison, 1/18/09

OUTLINE

- Cosmological Recombination in a nutshell

- Breaking the naive model

- Why should you care? Effects on CMB, inferences about primordial physics

- Our tools

- Preliminary results!

SAHA EQUILIBRIUM IS INADEQUATE

$$p + e^- \leftrightarrow H^{(n)} + \gamma^{(nc)}$$

- Chemical equilibrium does reasonably well predicting "moment of recombination"

$$\frac{x_e^2}{1 - x_e} = \left(\frac{13.6}{T_{\mathrm{eV}}}\right)^{3/2} e^{35.9 - 13.6/T_{\mathrm{eV}}}$$

$$x_e = 0.5 \text{ when } T = T_{\mathrm{rec}} \simeq 0.3 \text{ eV} \qquad z_{\mathrm{rec}} \simeq 1300$$

- Further evolution falls prey to reaction freeze-out

$$\Gamma = 6 \times 10^{-22} \text{ eV } x_e(T) \left(13.6/T_{\mathrm{eV}}\right)^{-5/2} \ln\left(13.6/T_{\mathrm{eV}}\right)$$

$$H = 1.1 \times 10^{-26} \text{ eV } T_{\mathrm{eV}}^{3/2}$$

$$\Gamma < H \text{ when } T < T_{\mathrm{F}} \simeq 0.25 \text{ eV}$$

Bottlenecks and Escape Routes

- BOTTLENECKS
  - Ground state recombinations are ineffective
  $$\tau_{c \to 1s}^{-1} = 10^{-1} \text{ s}^{-1} \gg H \simeq 10^{-12} \text{ s}^{-1}$$
  - Resonance photons are re-captured, e.g. Lyman α
  $$\tau_{2p \to 1s}^{-1} = 10^{-2} \text{ s}^{-1} \gg H \simeq 10^{-12} \text{ s}^{-1}$$
- ESCAPE ROUTES (e.g. n=2)
  - Two-photon processes
  $$H^{2s} \to H^{1s} + \gamma + \gamma \qquad \mathbf{\Lambda_{2s \to 1s} = 8.22 \text{ s}^{-1}}$$
  - Redshifting off resonance $\quad R \sim \left(n_{\text{H}} \lambda_{\alpha}^3\right)^{-1} \left(\dfrac{\dot{a}}{a}\right)$

EQUILIBRIUM ASSUMPTIONS

- Radiative eq. between different n-states

$$\mathcal{N}_n = \mathcal{N}_2 e^{-(E_n - E_2)/T}$$

- Radiative/collisional eq. between different l

$$\mathcal{N}_{nl} = \mathcal{N}_n \frac{(2l + 1)}{n^2}$$

- Matter in eq. with radiation due to Thompson scattering

$$T_m = T_\gamma \text{ since } \frac{\sigma_{\mathrm{T}} a T_\gamma^4 c}{m_e c^2} < H(T)$$

THE PEEBLES PUNCHLINE

- Only n=2 bottlenecks are treated

$$\Gamma_{\text{net,H}} = \Lambda_{2s \to 1s} \left[n_{2s} - n_{1s} e^{-(B_1 - B_2)/kT} \right] + \frac{8\pi}{\lambda_\alpha^3} \frac{\dot{a}}{a} \times \left(f_\alpha - e^{-h\nu_\alpha/kT} \right)$$

- Net Rate is suppressed by bottleneck vs. escape factor

$$-\frac{dx_e}{dt} = \sum_{n,l} \alpha_{nl}(T) \left\{ n x_e^2 + (2l+1) e^{-(B_1 - B_n)/kT} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} \right\} \mathcal{C}$$

$$\mathcal{C} = \frac{\frac{8\pi}{\lambda_\alpha^3 n_{1s}} \frac{\dot{a}}{a} + \Lambda_{2s \to 1s}}{\frac{8\pi}{\lambda_\alpha^3 n_{1s}} \frac{\dot{a}}{a} + (\Lambda_{2s \to 1s} + \beta_c)}$$

THE PEEBLES PUNCHLINE

- Net Rate is suppressed by bottleneck vs. escape factor

$$\mathcal{C} = \frac{\frac{8\pi}{\lambda_\alpha^3 n_{1s}} \frac{\dot{a}}{a} + \Lambda_{2s \to 1s}}{\frac{8\pi}{\lambda_\alpha^3 n_{1s}} \frac{\dot{a}}{a} + (\Lambda_{2s \to 1s} + \beta_c)}$$

THE PEEBLES PUNCHLINE

- Net Rate is suppressed by bottleneck vs. escape factor

$$\mathcal{C} = \frac{\frac{8\pi}{\lambda_\alpha^3 \, n_{1s}} \frac{\dot{a}}{a} + \Lambda_{2s \rightarrow 1s}}{\frac{8\pi}{\lambda_\alpha^3 \, n_{1s}} \frac{\dot{a}}{a} + (\Lambda_{2s \rightarrow 1s} + \beta_c)}$$

Redshifting term

THE PEEBLES PUNCHLINE

- Net Rate is suppressed by bottleneck vs. escape factor

$$\mathcal{C} = \frac{\frac{8\pi}{\lambda_\alpha^3 n_{1s}}\frac{\dot{a}}{a} + \Lambda_{2s \to 1s}}{\frac{8\pi}{\lambda_\alpha^3 n_{1s}}\frac{\dot{a}}{a} + (\Lambda_{2s \to 1s} + \beta_c)}$$

2γ term

THE PEEBLES PUNCHLINE

- Net Rate is suppressed by bottleneck vs. escape factor

$$\mathcal{C} = \frac{\frac{8\pi}{\lambda_\alpha^3 n_{1s}}\frac{\dot{a}}{a} + \Lambda_{2s\to1s}}{\frac{8\pi}{\lambda_\alpha^3 n_{1s}}\frac{\dot{a}}{a} + (\Lambda_{2s\to1s} + \beta_c)}$$

Ionization Term

THE PEEBLES PUNCHLINE

- Net Rate is suppressed by bottleneck vs. escape factor

$$\mathcal{C} = \frac{\frac{8\pi}{\lambda_\alpha^3 n_{1s}} \frac{\dot{a}}{a} + \Lambda_{2s \to 1s}}{\frac{8\pi}{\lambda_\alpha^3 n_{1s}} \frac{\dot{a}}{a} + (\Lambda_{2s \to 1s} + \beta_c)}$$

$$\frac{\text{redshift term}}{2\gamma \text{ term}} \simeq 0.02 \frac{\Omega_m^{1/2}}{(1 - x_e[z])\left(\frac{1+z}{1100}\right)^{3/2}}$$

2γ process dominates until late times $(z \lesssim 850)$

PEEBLES MODEL ASSUMPTIONS/RESULTS

$\Omega_m h^2 \uparrow$
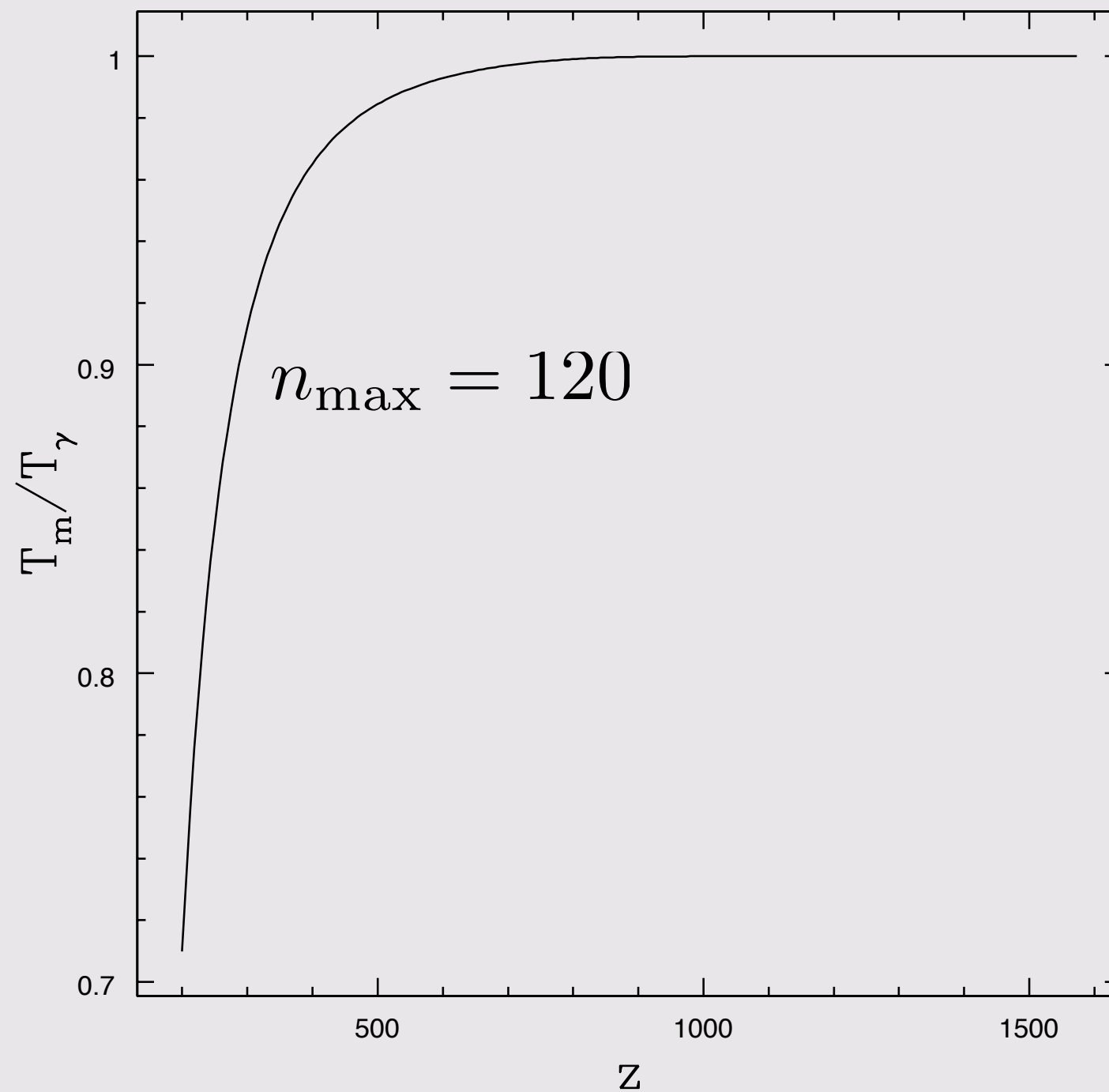
$\Omega_b h^2 \uparrow$
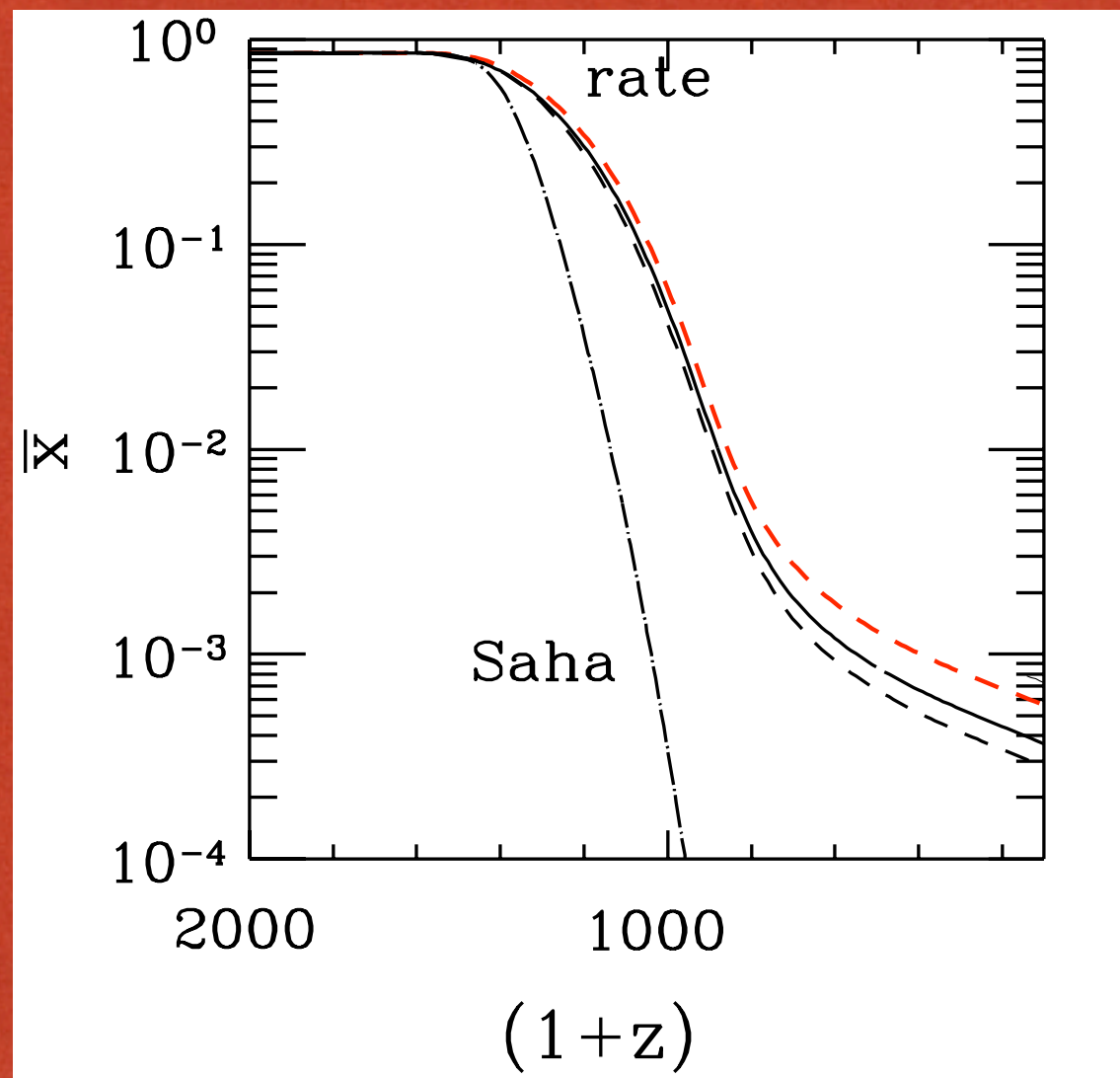
- State of the Art for 30 years!

BREAKING THE NAIVE MODEL

- Radiation field is cool: ~~Boltzmann eq. of higher n~~

- Treated by Seager et al. (2000) $n_{\mathrm{max}} = 300$ RecFAST!!!

- ~~Equilibrium~~ between *l states*

- Treated by Chluba et al. (2005) for $n_{\mathrm{max}} = 100$

- Radiation and matter field fall out of eq.

$$\dot{T}_M + 2HT_m = \frac{8x_e\sigma_{\mathrm{T}}aT_\gamma^4}{3m_ec\left(1 + f_{\mathrm{He}} + x_e\right)}\left(T_M - T_\gamma\right)$$

- Higher-order 2γ transitions, (Hirata, Ali-Haimoud, in progress)

DECOUPLING OF MATTER AND RADIATION

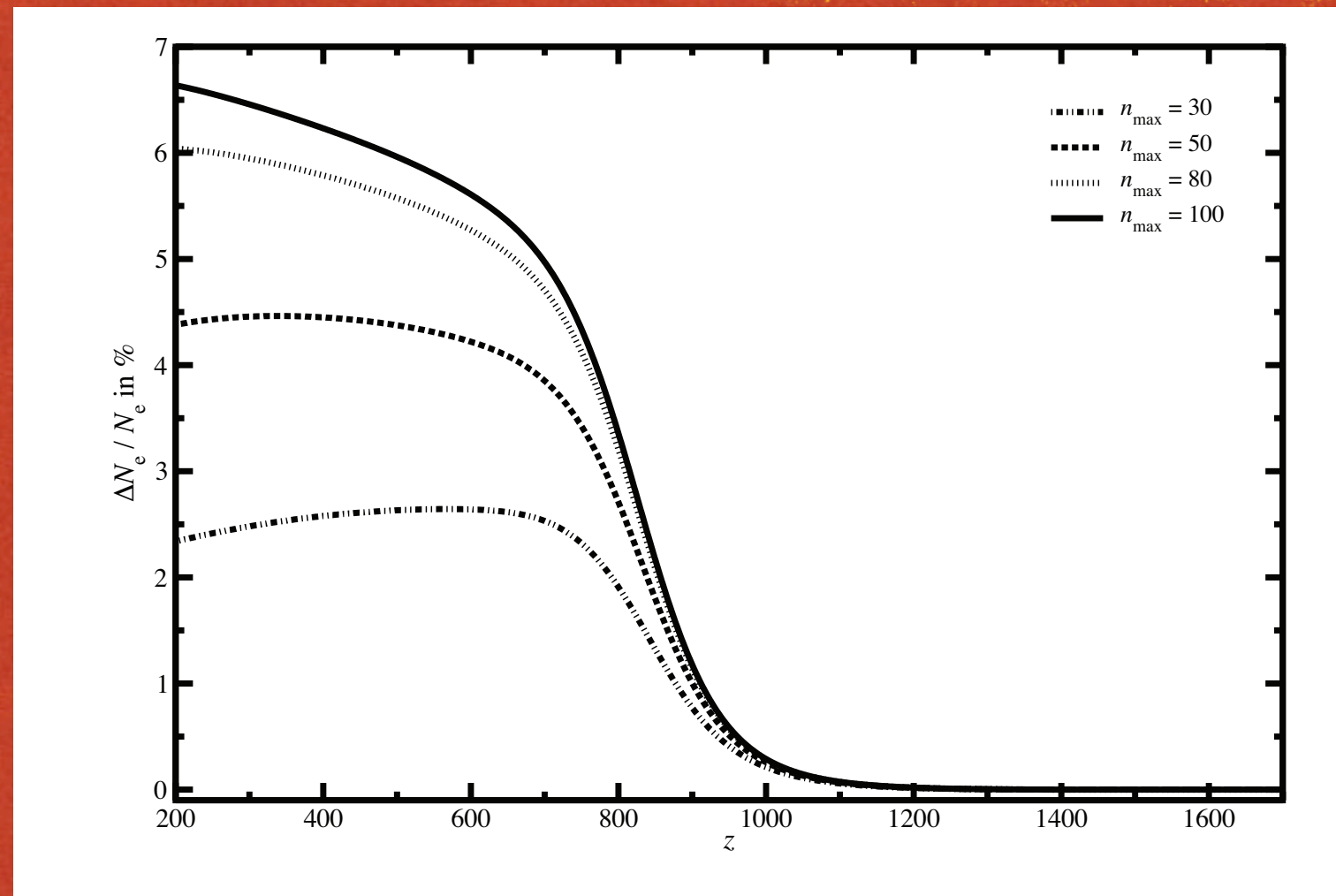$$n_{\mathrm{max}} = 120$$

BREAKING THE NAIVE MODEL

- Radiation field is cool: ~~Boltzmann eq. of higher n~~

- Treated by Seager et al. (2000) $n_{\max} = 300$ RecFAST!!!

- ~~Equilibrium~~ between *l states*

- Treated by Chluba et al. (2005) for $n_{\max} = 100$

- Beyond this, testing convergence with $n_{\max}$ is hard!

  $$t_{\text{compute}} \sim \mathcal{O}\left(\text{weeks}\right)$$

How to proceed if we want 0.1% accuracy in $x_e(z)$?

THE EFFECT OF RESOLVING l- SUBSTATES

Putting free-electrons in 'bottlenecked' l-substates slows down the decay to 1s: Recombination is slower

BREAKING THE NAIVE MODEL

- Radiation field is cool: ~~Boltzmann eq. of higher n~~

- Treated by Seager et al. (2000) $n_{\max} = 300$ RecFAST!!!

- ~~Eq.~~ between *l states*: dipole selection bottleneck: $\Delta l = \pm 1$

- Treated by Chluba et al. (2005) for $n_{\max} = 100$

- Beyond this, testing convergence with $n_{\max}$ is hard!
$$t_{\mathrm{compute}} \sim \mathcal{O}\,(\mathrm{weeks})$$

WHY PROCEED?

WHO CARES?
I. SMEARING AND MOVING THE SURFACE OF LAST SCATTERING (SLSS)

- Photons kin. decouple when Thompson scattering freezes out

$$\gamma + e^- \Leftrightarrow \gamma + e^-$$

$$\Gamma = n_e \sigma_{\mathrm{T}} c = 2.2 \times 10^{-19} \ \mathrm{s}^{-1} \frac{x_e \Omega_b h^2}{a^3} =$$

$$H = H_0 \Omega_m^{1/2} a^{-3/2} \left[1 + \frac{a_{\mathrm{eq}}}{a} \right]^{1/2}$$

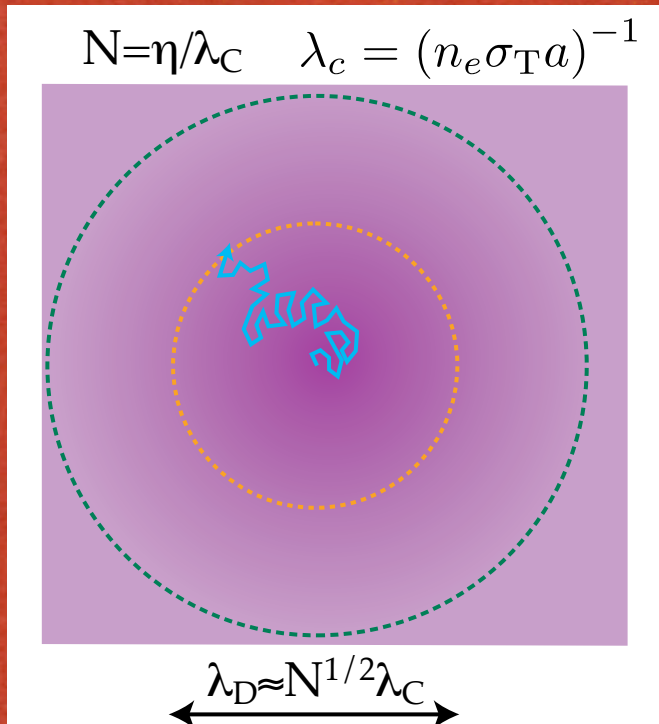- $z_{\mathrm{dec}} \simeq 1100$:Decoupling occurs during recombination

$$C_l \rightarrow C_l e^{-2\tau} \quad \text{if } l > \frac{\eta_0}{\eta_{\mathrm{rec}}}.$$

$$\tau = \int_0^{\eta_{\mathrm{dec}}} d\eta \, n_e [\eta] \, \sigma_{\mathrm{T}} a (\eta)$$

- From Wayne Hu's website

$$l_{\mathrm{damp}} \sim 1000$$
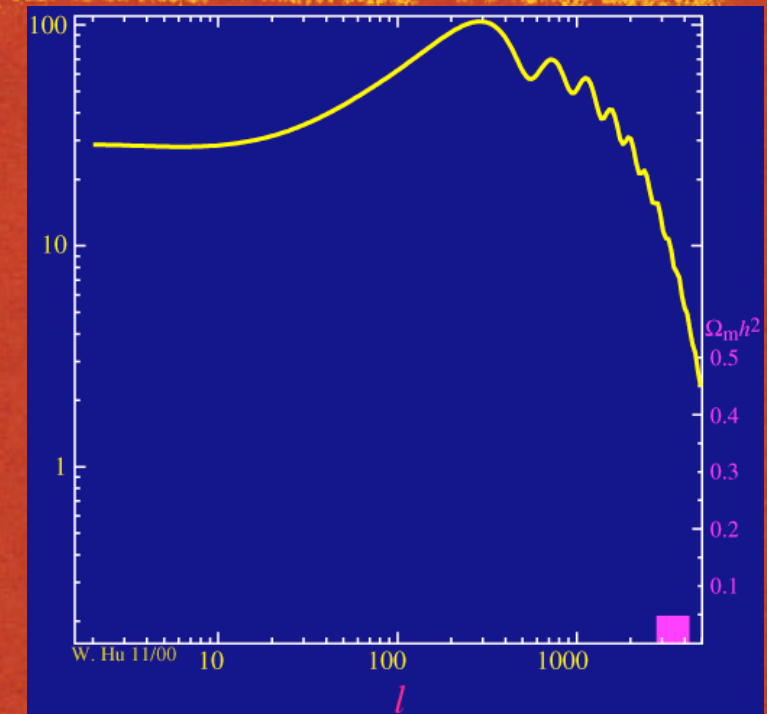
$N=\eta/\lambda_{\mathrm{C}}$ $\lambda_c = (n_e \sigma_{\mathrm{T}} a)^{-1}$

$\lambda_{\mathrm{D}} \approx N^{1/2} \lambda_{\mathrm{C}}$

- Inhomogeneities are damped for $\lambda < \lambda_D$

$$k_D^{-2}(\eta) \simeq \int_0^\eta \frac{d\eta'}{6(1+R)n_e[\eta']\sigma_{\mathrm{T}}a[\eta']} \left[\frac{R^2}{1+R} + \frac{8}{9} \right]$$
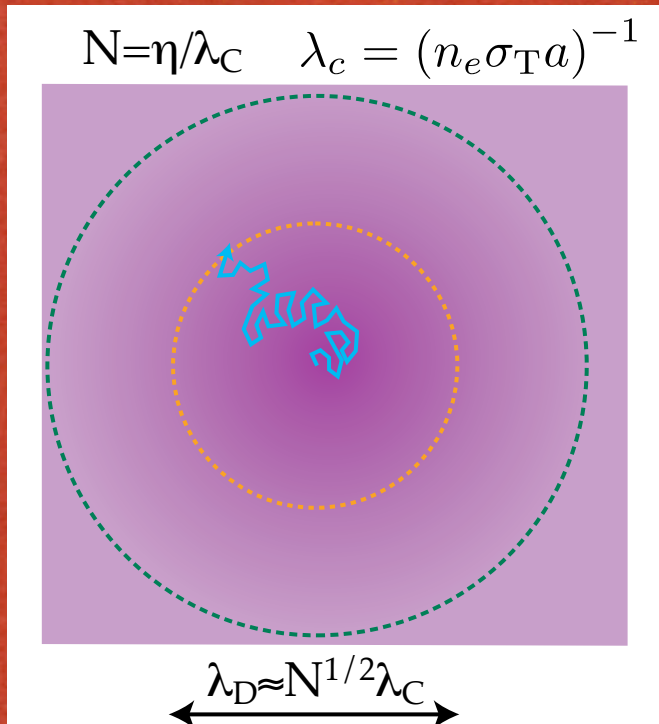
$$R = \frac{3\rho_b^0}{4\rho^\gamma}$$

$$|\Theta_l(\eta_0)| \simeq \int_0^{\eta_0} d\eta \; \dot{\tau} e^{-\tau(\eta)} e^{ik \int d\eta c_s} e^{-k^2/k_D^2(\eta)} \tilde{\delta}(k) j_l(k(\eta - \eta_0)) dk$$

- From Wayne Hu's website

N=η/λ_C $\lambda_c = (n_e \sigma_T a)^{-1}$

$\lambda_D \approx N^{1/2} \lambda_C$

W. Hu 11/00

$\Omega_m h^2$

l

$$l_{\mathrm{damp}} \sim 1000$$

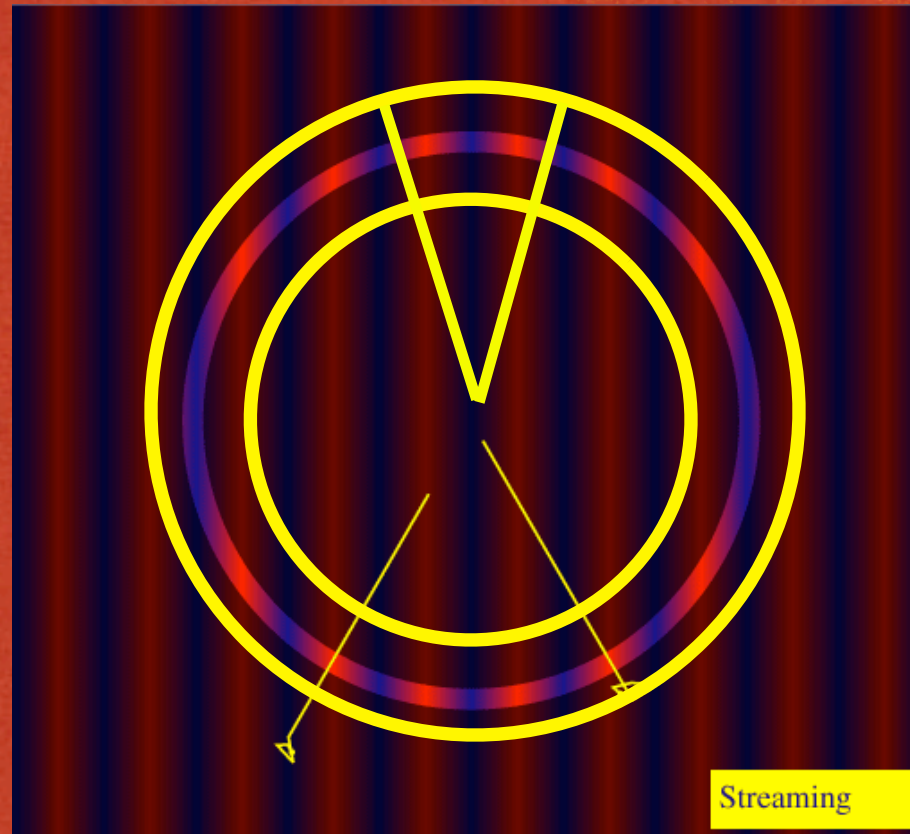- Inhomogeneities are damped for $\lambda < \lambda_D$

$$k_D^{-2}(\eta) \simeq \int_0^\eta \frac{d\eta'}{6(1+R)n_e[\eta']\sigma_T a[\eta']} \left[\frac{R^2}{1+R} + \frac{8}{9} \right]$$

$$R = \frac{3\rho_b^0}{4\rho^\gamma}$$

$$|\Theta_l(\eta_0)| \simeq \int_0^{\eta_0} d\eta \, \dot{\tau} e^{-\tau(\eta)} e^{ik \int d\eta c_s} e^{-k^2/k_D^2(\eta)} \tilde{\delta}(k) j_l(k(\eta - \eta_0)) dk$$

Streaming

- Additional damping of form

$$|\Theta_l\left(\eta_0, k\right)| \rightarrow |\Theta_l\left(\eta_0, k\right)| e^{-\sigma^2 \eta_{\mathrm{rec}}^2 k^2}$$

Who Cares?
IV. CMB Polarization
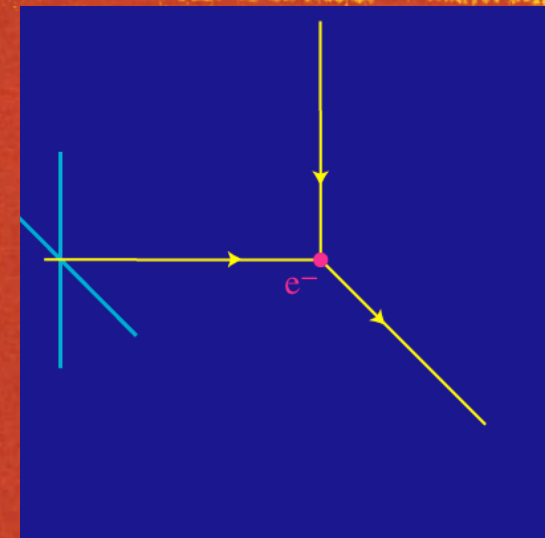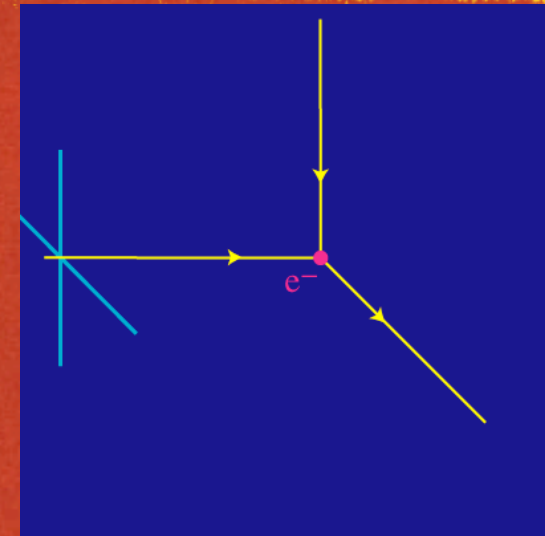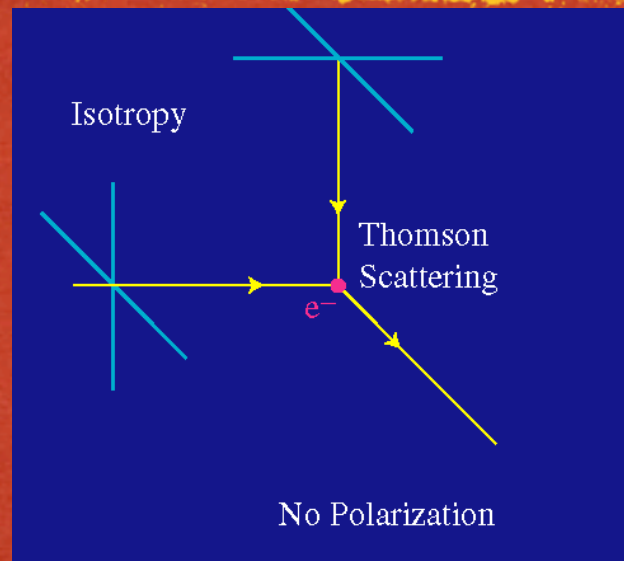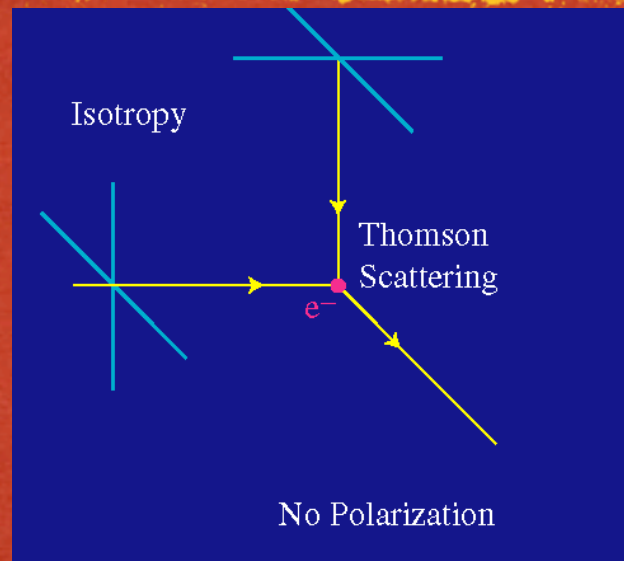
- Need to scatter quadrapole to polarize CMB

$$\Theta_l^P(k) = \int d\eta \dot{\tau} e^{-\tau(\eta)} \Theta_{T,2}(k,\eta) \frac{l^2}{(k\eta)^2} j_l(k\eta)$$

- Need time to develop a quadrapole

$$\Theta_l(k\eta) \sim \frac{k\eta}{2\tau} \Theta_l(k\eta) \ll \Theta_l(\eta) \text{ if } l \geq 2, \text{ in tight coupling regime}$$

WHO CARES?
IV. CMB POLARIZATION

- Need to scatter quadrapole to polarize CMB

$$\Theta_l^P\left(k\right) = \int d\eta\,\dot{\tau}e^{-\tau(\eta)}\Theta_{T,2}\left(k,\eta\right)\frac{l^2}{\left(k\eta\right)^2}j_l\left(k\eta\right)$$

- Need time to develop a quadrapole

$$\Theta_l\left(k\eta\right) \sim \frac{k\eta}{2\tau}\Theta_l\left(k\eta\right) \ll \Theta_l\left(\eta\right) \text{ if } l \geq 2, \text{ in tight coupling regime}$$

WHO CARES?
IV. CMB POLARIZATION

- Need to scatter quadrapole to polarize CMB

$$\Theta_l^P(k) = \int d\eta\, \dot{\tau}\, e^{-\tau(\eta)} \Theta_{T,2}(k,\eta) \frac{l^2}{(k\eta)^2} j_l(k\eta)$$
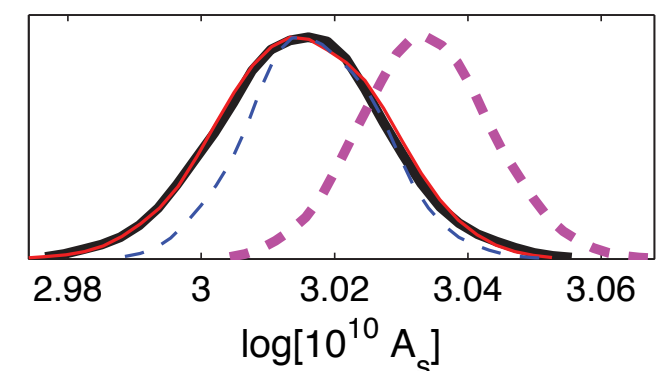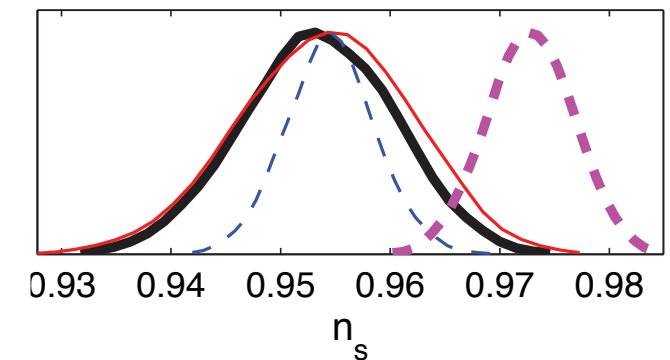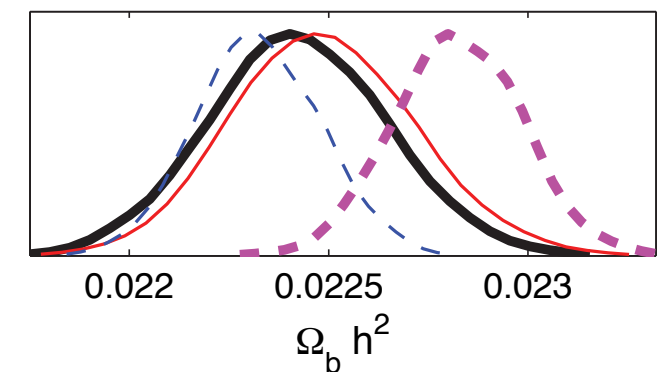
- Need time to develop a quadrapole

$$\Theta_l(k\eta) \sim \frac{k\eta}{2\tau} \Theta_l(k\eta) \ll \Theta_l(\eta) \text{ if } l \geq 2, \text{ in tight coupling regime}$$

WHO CARES?
V. PARAMETER DEGENERACIES

Planck will be CV limited (T and E) to $l \sim 2500$

0.1% accuracy required in $x_e(z)$

Planck uncertainty forecasts using MCMC

THE MULTI-LEVEL ATOM (MLA)

- Bound-free rate equation

$$\dot{x}_{nl}^{bf} = \int dE_{\mathrm{e}} P_M(T_m, E_{\mathrm{e}}) n_H x_e x_p \left[1 + f(E_e - E_n)\right] \alpha_{nl}(E_{\mathrm{e}})$$
$$- \int dE_{\mathrm{e}} \; g(E_{\mathrm{E}} - E_n) \; x_{nl} \; f(E_{\mathrm{e}} - E_{nl}) \alpha_{nl}(E_{\mathrm{E}})/g_{nl}$$

- Bound-bound rate equation

$$\dot{x}_{nl}^{bb} = \sum_{n',l'=l\pm 1} \left(A_{nn'}^{ll'} (1 + f_{nn'}) x_{n',l'} - \frac{g_{n'l'}}{g_{nl}} f_{nn'} x_{nl} \right) P_{nn'}^{ll'}$$

The multi-level atom (MLA)

- Bound-free rate equation

$$\dot{x}_{nl}^{bf} = \int dE_{\text{e}} P_M(T_m, E_{\text{e}}) n_H x_e x_p \left[1 + f(E_e - E_n)\right] \alpha_{nl}(E_{\text{e}})$$
$$- \int dE_{\text{e}} \, g(E_{\text{E}} - E_n) \, x_{nl} \, f(E_{\text{e}} - E_{nl}) \alpha_{nl}(E_{\text{E}})/g_{nl}$$

$$\boxed{\Omega_m, \Omega_b, h}$$

- Bound-bound rate equation

$$\dot{x}_{nl}^{bb} = \sum_{n',l'=l\pm1} \left(A_{nn'}^{ll'}(1 + f_{nn'}) x_{n',l'} - \frac{g_{n'l'}}{g_{nl}} f_{nn'} x_{nl}\right) P_{nn'}^{ll'}$$

THE MULTI-LEVEL ATOM (MLA)

- Bound-free rate equation

$$\dot{x}_{nl}^{bf} = \int dE_{\mathrm{e}} P_M(T_m, E_{\mathrm{e}}) n_H x_e x_p \left[1 + f(E_e - E_n)\right] \alpha_{nl}(E_{\mathrm{e}})$$
$$- \int dE_{\mathrm{e}} \; g(E_{\mathrm{E}} - E_n) \; x_{nl} \; f(E_{\mathrm{e}} - E_{nl}) \alpha_{nl}(E_{\mathrm{E}})/g_{nl}$$

- Bound-bound rate equation

$$\dot{x}_{nl}^{bb} = \sum_{n',l'=l\pm1} \left(A_{nn'}^{ll'}(1 + f_{nn'})x_{n',l'} - \frac{g_{n'l'}}{g_{nl}}f_{nn'}x_{nl}\right)P_{nn'}^{ll'}$$

- Phase-space density blueward of line

- Escape probability of γ in line

THE MULTI-LEVEL ATOM (MLA)

Stimulated emission/absorption

- Bound-free rate equation

$$\dot{x}_{nl}^{bf} = \int dE_{\text{e}} P_M(T_m, E_{\text{e}}) n_H x_e x_p \left[1 + f(E_e - E_n)\right] \alpha_{nl}(E_{\text{e}})$$
$$- \int dE_{\text{e}}\ g(E_{\text{E}} - E_n)\ x_{nl}\ f(E_{\text{e}} - E_{nl}) \alpha_{nl}(E_{\text{E}})/g_{nl}$$

- Bound-bound rate equation

$$\dot{x}_{nl}^{bb} = \sum_{n',l'=l\pm 1} \left(A_{nn'}^{ll'}(1 + f_{nn'}) x_{n',l'} - \frac{g_{n'l'}}{g_{nl}} f_{nn'} x_{nl}\right) P_{nn'}^{ll'}$$

THE MULTI-LEVEL ATOM (MLA)

Spontaneous Emission

- **Bound-free rate equation**

$$\dot{x}_{nl}^{bf} = \int dE_{\mathrm{e}} P_M(T_m, E_{\mathrm{e}}) n_H x_e x_p \left[1 + f(E_e - E_n)\right] \alpha_{nl}(E_{\mathrm{e}})$$
$$- \int dE_{\mathrm{e}}\ g(E_{\mathrm{E}} - E_n)\ x_{nl}\ f(E_{\mathrm{e}} - E_{nl}) \alpha_{nl}(E_{\mathrm{E}})/g_{nl}$$

- **Bound-bound rate equation**

$$\dot{x}_{nl}^{bb} = \sum_{n',l'=l\pm 1} \left(A_{nn'}^{ll'} (1 + f_{nn'}) x_{n',l'} - \frac{g_{n'l'}}{g_{nl}} f_{nn'} x_{nl} \right) P_{nn'}^{ll'}$$

THE MULTI-LEVEL ATOM (MLA)

- Two photon transitions between n=1 and n=2 are included:

$$\dot{x}_{2s\to1s,2\gamma} = -\dot{x}_{1s\to2s,2\gamma} = \Lambda_{2s}(-x_{2s} + x_{1s}e^{-E_{2s\to1s}/T_\gamma})$$

- Net recombination rate:

$$x_e \simeq 1 - x_{1s} \to \dot{x}_e \simeq -\dot{x}_{1s} = -\dot{x}_{1s\to2s}$$
$$+ \sum_{n,l>1s} A_{n1}^{l0} P_{n1}^{l0} \left\{ \frac{g_{nl}}{2} f_{n1}^+ x_{1s} - (1 + f_{n1}^+)x_{nl} \right\}$$

RATE COEFFICIENTS

- Bound-bound rates given by Fermi's golden rule and matrix element

$$\rho(n'l', nl) = \int_0^\infty u_{n'l'}(r) u_{nl}(r) r^3 dr = \mathcal{C} \times \left[F_{2,1}\left(-n + l + 1, -n' + l, 2l, \frac{-4nn'}{(n-n')^2} \right) \right.$$

$$\left. - \left(\frac{n-n'}{n+n'} \right)^2 F_{2,1}\left(-n + l - 1, -n' + l, 2l, \frac{-4nn'}{(n-n')^2} \right) \right]^2$$

- Power-series destabilizes at high-n, recursion relation used

- Bound-free rates at temperature T given by phase space integral of matrix element $g_{nl} = \int_0^\infty u_{nl}(r) f_{El}(r) r^3 dr$

- Rates are tabulated at all n and l of interest, at a variety of energies, and integrated at each time step

RATE COEFFICIENTS

- Rates are tabulated at all n and l of interest, at a variety of energies, and integrated at each time step $\rho(n'l', nl) = a_0 n^2 \int_{-\pi}^{\pi} d\tau e^{i\Omega\tau} (1 + cos\eta)$

$$\Omega = \omega_n - \omega_{n'}$$

$$r = r_{\max} (1 + \cos \eta) /2$$

$$\tau = \eta + \sin \eta$$

Fourier transform of classical orbit!
Application of correspondence principle!

- Similar WKB approximation can be used to check stability of BF matrix elements

Radiation Field: Black Body+

- Escape probability treated in Sobolev approx.

$$P_{n,n'}^{l,l'} = \frac{1 - e^{-\tau_s}}{\tau_s}$$

$$\tau_s = \frac{c^3 n_{\mathrm{H}}}{8\pi H \nu_{nn'}^3} A_{nn'}^{ll'} \left[\frac{g_{n'}^{l'}}{g_n^l} x_n^l - x_{n'}^{l'} \right]$$

$$\mathcal{R}(\nu, \nu') = \phi(\nu)\phi(\nu')$$

$$\frac{v_{\mathrm{th}}}{H(z)} \ll \lambda$$

- Excess line photons injected into radiation field

$$\left(\frac{8\pi \nu_{nn'}^3}{c^3 n_H} \right) \left(f_{nn'}^+ - f_{nn'}^- \right) = A_{nn'}^{ll'} P_{nn'}^{ll'} \left[x_n^l \left(1 + f_{nn'}^+ \right) - \frac{g_n^l}{g_{n'}^{l'}} x_{n'}^{l'} f_{nn'}^+ \right]$$

- Photons are conserved outside of line regions

$$f_{n1}^{+10} = f_{n+1,1}^{-10} \left[\frac{1 - (n+1)^{-2}}{1 - n^{-2}} (1 + z) - 1 \right]$$

Radiation Field: Black Body+

- Escape probability treated in Sobolev approx.

$$P_{n,n'}^{l,l'} = \frac{1 - e^{-\tau_s}}{\tau_s}$$

$$\tau_s = \frac{c^3 n_{\mathrm{H}}}{8\pi H \nu_{nn'}^3} A_{nn'}^{ll'} \left[\frac{g_{n'}^{l'}}{g_n^l} x_n^l - x_{n'}^{l'} \right]$$

$$\mathcal{R}(\nu, \nu') = \phi(\nu)\phi(\nu')$$

- Forbes, Hirata, and Ali-Haimoud are solving FP eqn. to obtain evolution of $f(\nu)$ more generally, including atomic recoil/diffusion, 2γ decay and full time-dependence of problem, coherent and incoherent scattering, overlap of higher-order Lyman lines

24

STEADY-STATE APPROXIMATION FOR EXCITED STATES

- Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

STEADY-STATE APPROXIMATION FOR EXCITED STATES

- Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

$$\vec{x} = \begin{pmatrix} \vec{x}_0 \\ \vec{x}_1 \\ \dots \\ \vec{x}_{n_{\max}-1} \end{pmatrix}$$

STEADY-STATE APPROXIMATION FOR EXCITED STATES

- Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$
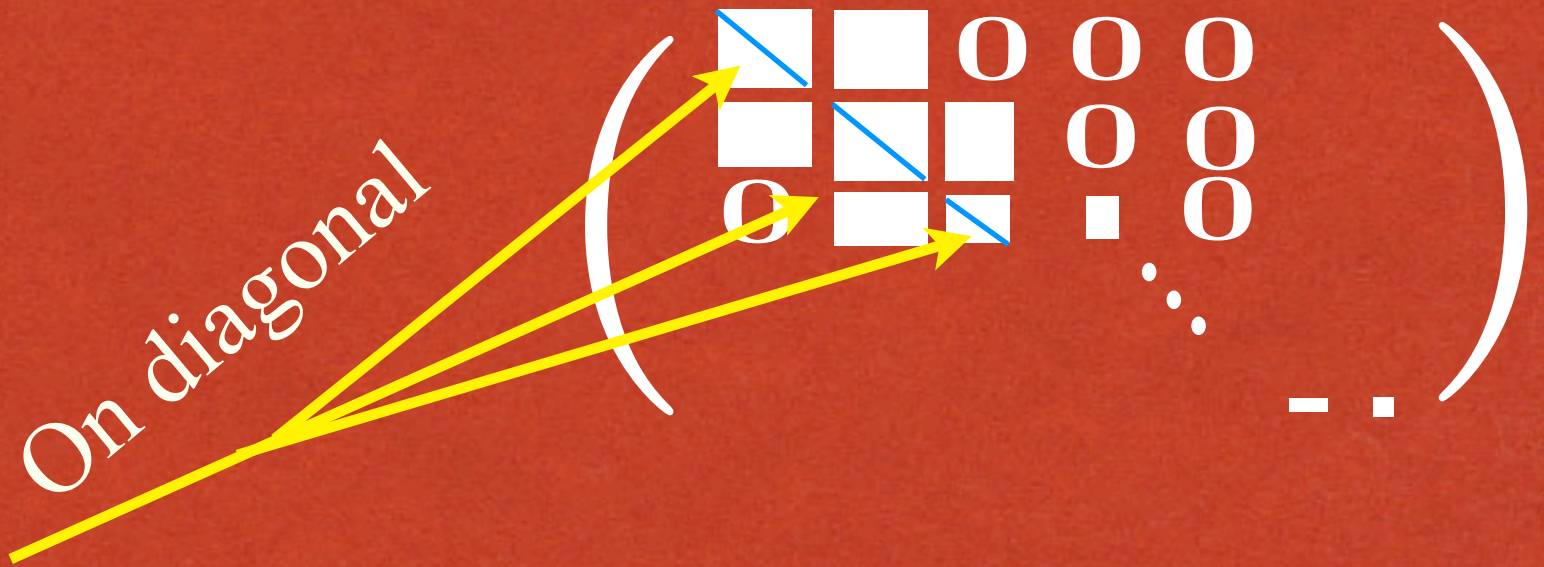
On diagonal

For state l, includes BB transitions out of l to all other l'', photo-ionization, 2γ transitions to ground state

STEADY-STATE APPROXIMATION FOR EXCITED STATES

- Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

Off diagonal

For state l, includes BB transitions into l from all other l'

STEADY-STATE APPROXIMATION FOR EXCITED STATES

- Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

- Includes recombination to 1,
  1 and 2γ transitions from ground state

STEADY-STATE APPROXIMATION FOR EXCITED STATES

- Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

- For n>1, $\boxed{t_{\text{rec}}^{-1} \sim 10^{-12} s^{-1} \ll \mathbf{R} \, , \vec{s} \to \vec{x} \simeq \mathbf{R}^{-1}\vec{s}}$

$$\mathbf{R} \lesssim 1 \text{ s}^{-1} \text{ (e.g. Lyman-}\alpha\text{)}$$

RAPID MATRIX INVERSION: SPARSITY TO THE RESCUE

- Matrix is $\sim n_{max}^2 \times n_{max}^2$

- Brute force would require $n_{max}^6 \sim 1000$ s for $n_{max} = 200$ for a single time step

- Sparsity to the rescue $\Delta l = \pm 1$

$$\boxed{\mathbf{M}_{l,l-1}\vec{x}_{l-1} + \mathbf{M}_{l,l}\vec{x}_l + \mathbf{M}_{l,l+1}\vec{x}_{l+1} = \vec{s}_l}$$

$$\vec{v}_l = \chi_l \left[\vec{s}_l - \mathbf{M}_{l,l+1}\vec{v}_l + \Sigma_{l'=l-1}^0 \sigma_{l,l'} \vec{s}_{l'} (-1)^{l'-l} \right]$$

$$\chi_l = \begin{cases} \mathbf{M}_{00}^{-1} & \text{if } l = 0 \\ \left(\mathbf{M}_{l+1,l+1} - \mathbf{M}_{l+1,l}\chi_l\mathbf{M}_{l,l+1} \right)^{-1} & \text{if } l > 0 \end{cases}$$

$$\sigma_{l,l-1} = \mathbf{M}_{l,l-1}\chi_{l-1}$$
$$\sigma_{l,i} = \sigma_{l,i+1}\mathbf{M}_{i+1,i}\chi_i$$
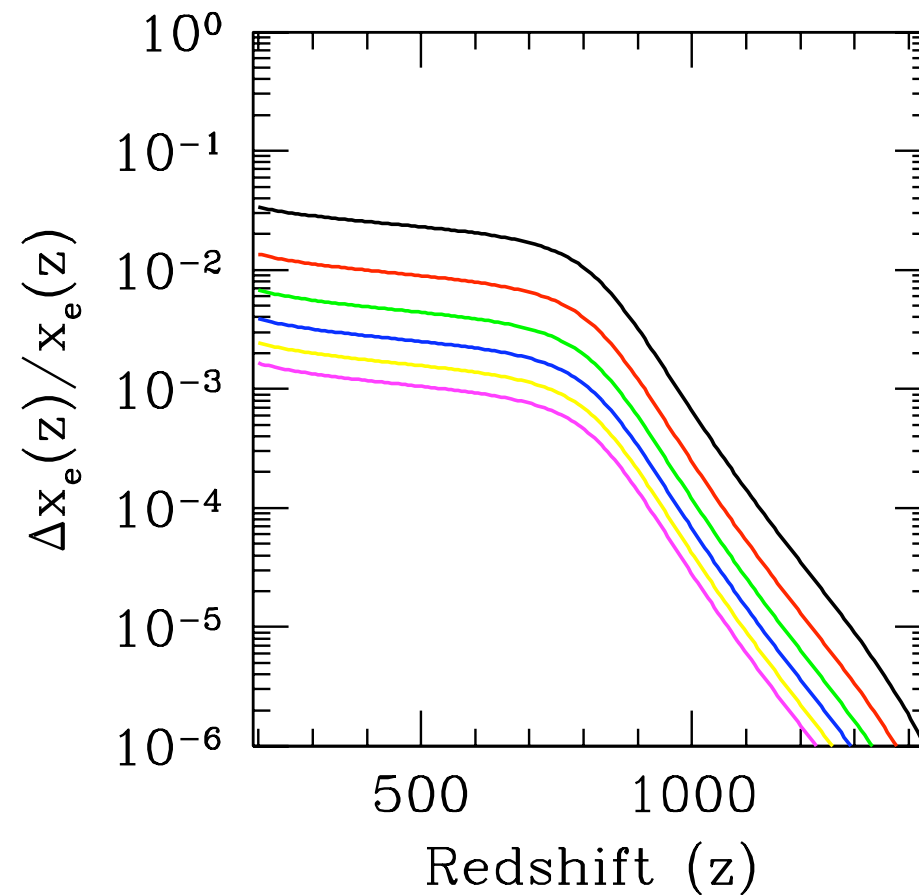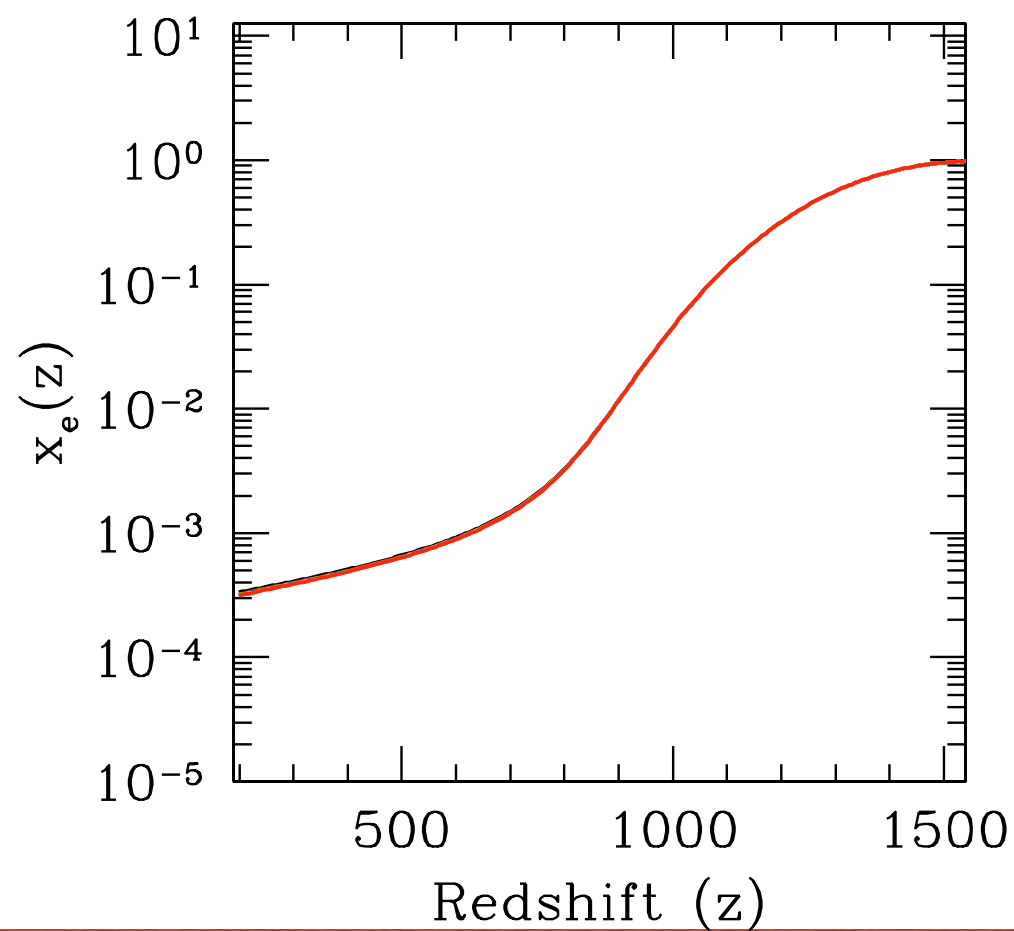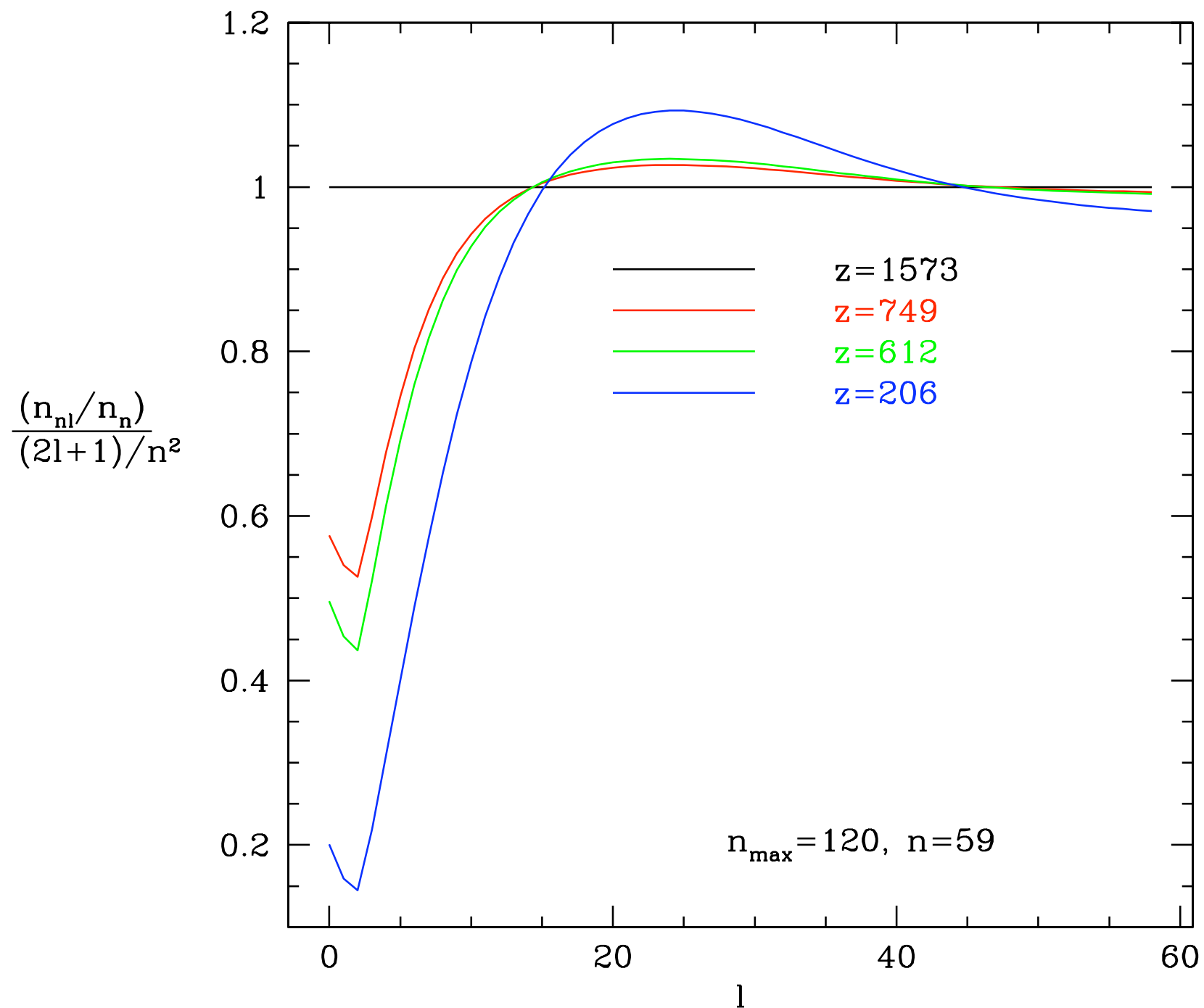
- $x_e(z)$ falls with increasing $n_{\max} = 10 \to 200$, as expected.

- Rec Rate>downward BB Rate> Ionization, upward BB rate

- For $n_{max} = 100$, code computes in only 2 hours
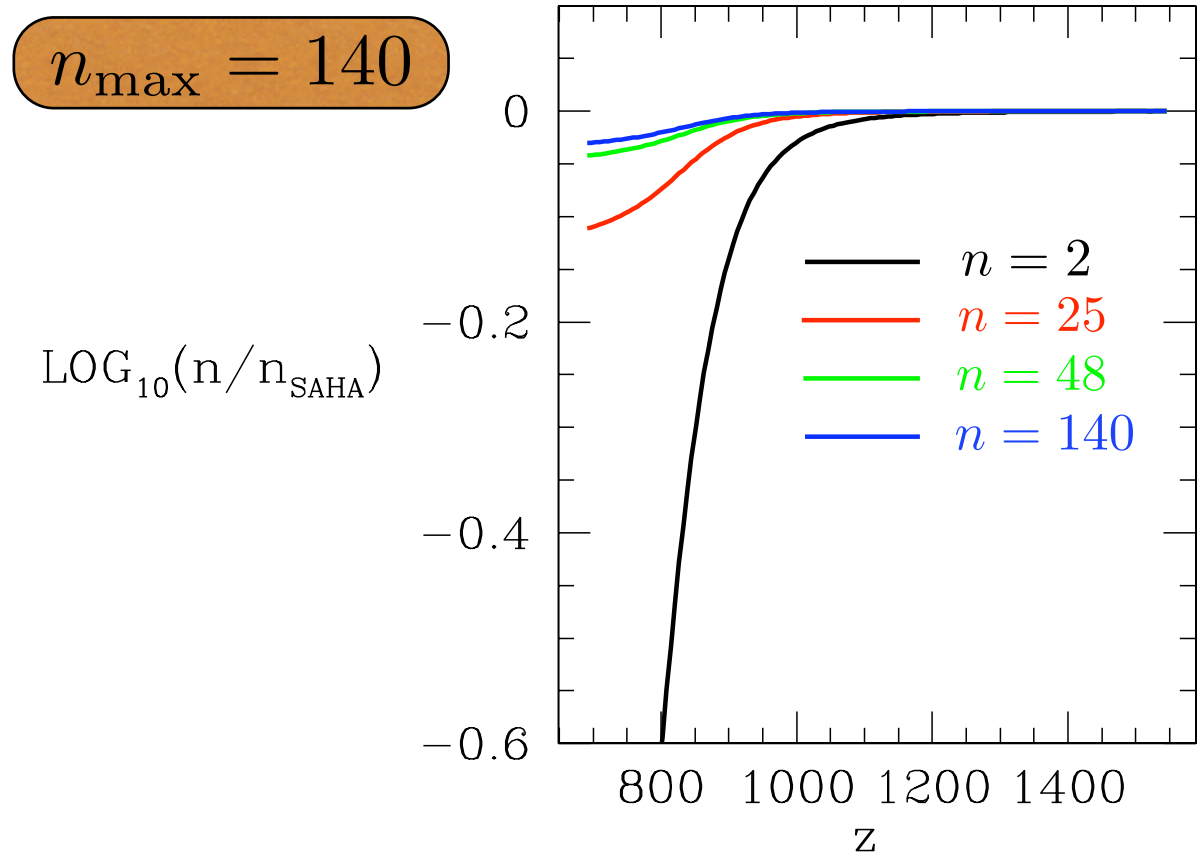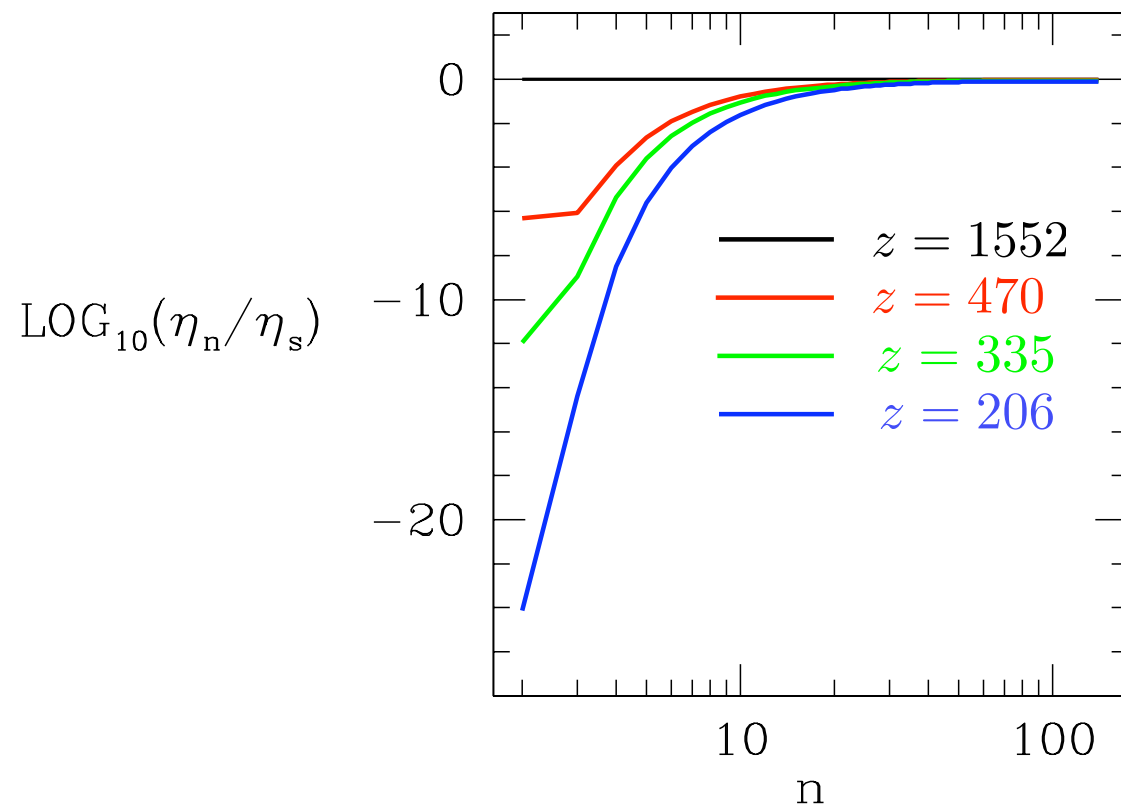
Deviations from Boltzmann EQ: high-n

- $\alpha n \gtrsim A_{\rm bb,down}$.
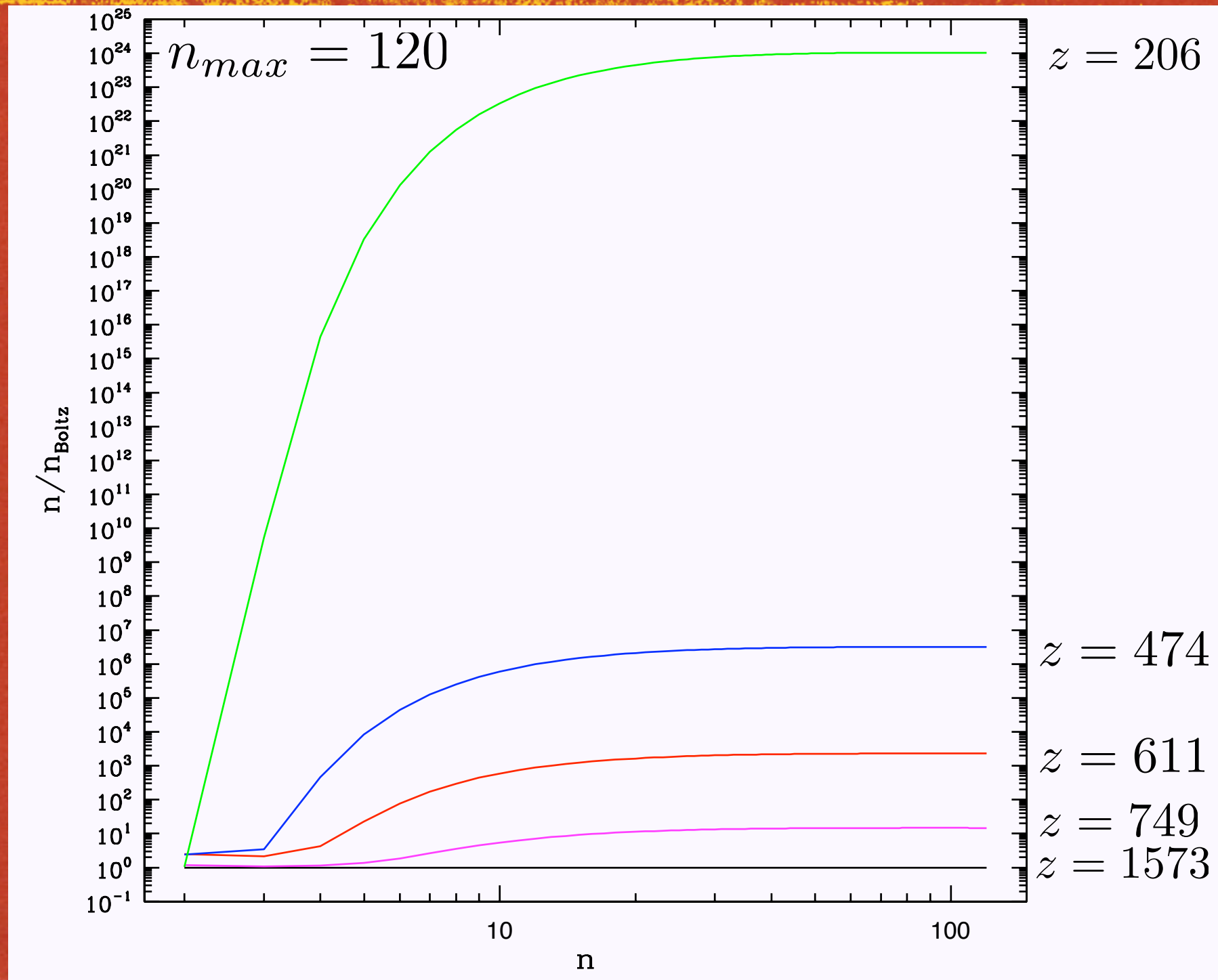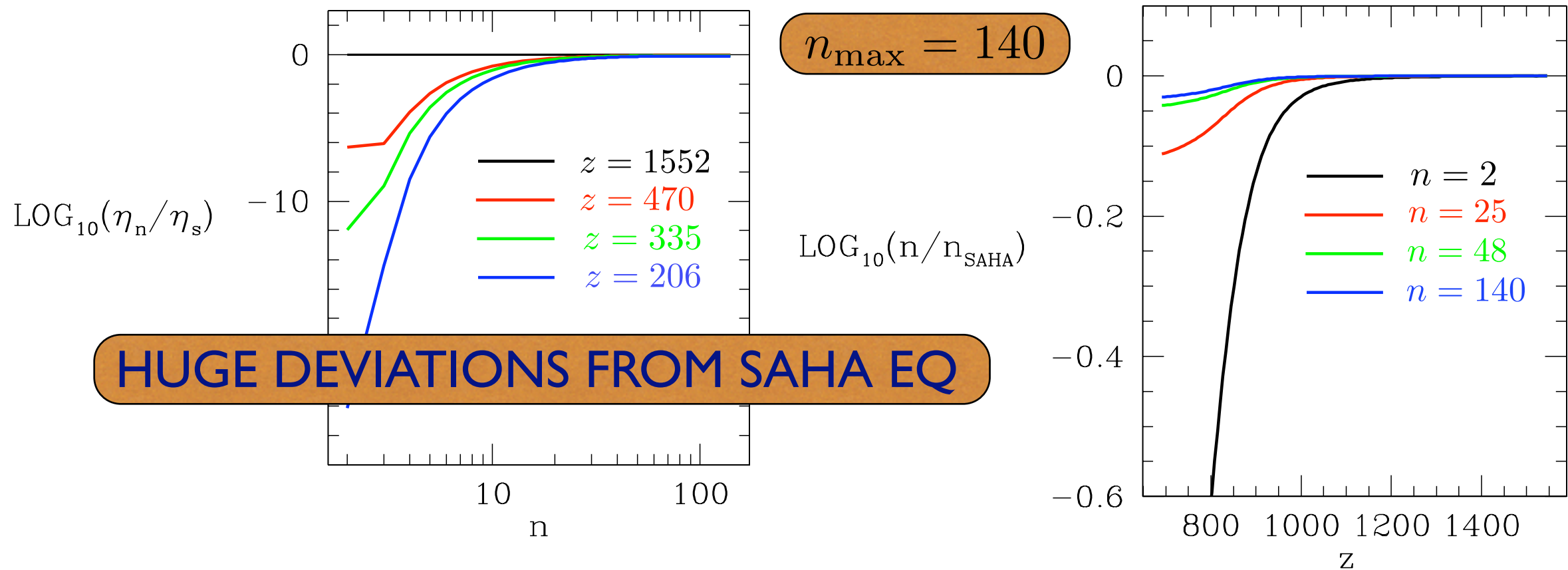
DEVIATIONS FROM BOLTZMANN EQ: RESOLVING l

DEVIATIONS FROM SAHA EQUILIBRIUM

- n=1 suppressed due to freeze-out of x_e

- Remaining levels 'try' to remain in Boltzmann eq. with n=2

- Super-Boltz effects and two-γ transitions (n=1 $\longrightarrow$ n=2) yield less suppression for n>1

- Problem gets worse at late times (low z) as rates fall
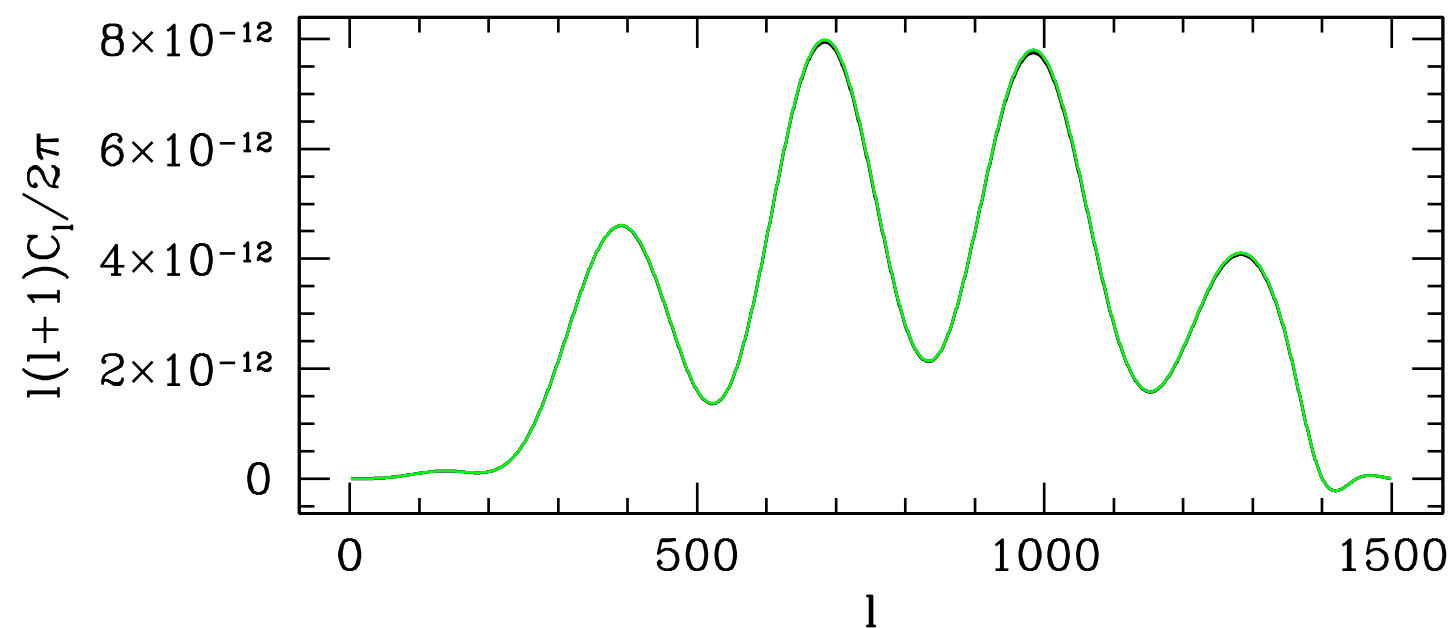
DEVIATIONS FROM SAHA EQUILIBRIUM

$n_{\max} = 140$

$\mathrm{LOG}_{10}(\eta_n/\eta_s)$

$z = 1552$
$z = 470$
$z = 335$
$z = 206$

$\mathrm{LOG}_{10}(n/n_{\mathrm{SAHA}})$

HUGE DEVIATIONS FROM SAHA EQ

$n = 2$
$n = 25$
$n = 48$
$n = 140$

- n=1 suppressed due to freeze-out of x_e

- Remaining levels 'try' to remain in Boltzmann eq. with n=2

- Super-Boltz effects and two-γ transitions (n=1 $\longrightarrow$ n=2) yield less suppression for n>1

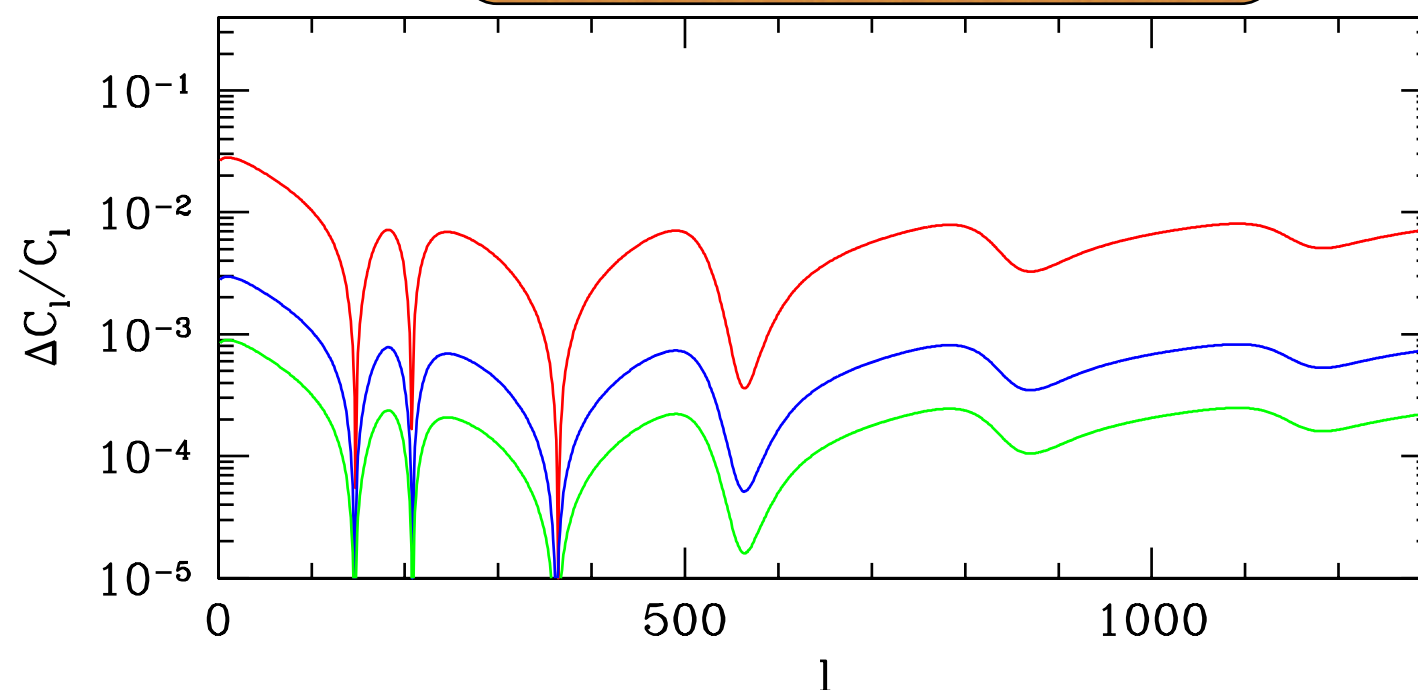- Problem gets worse at late times (low z) as rates fall

TEMPERATURE $C_l s$
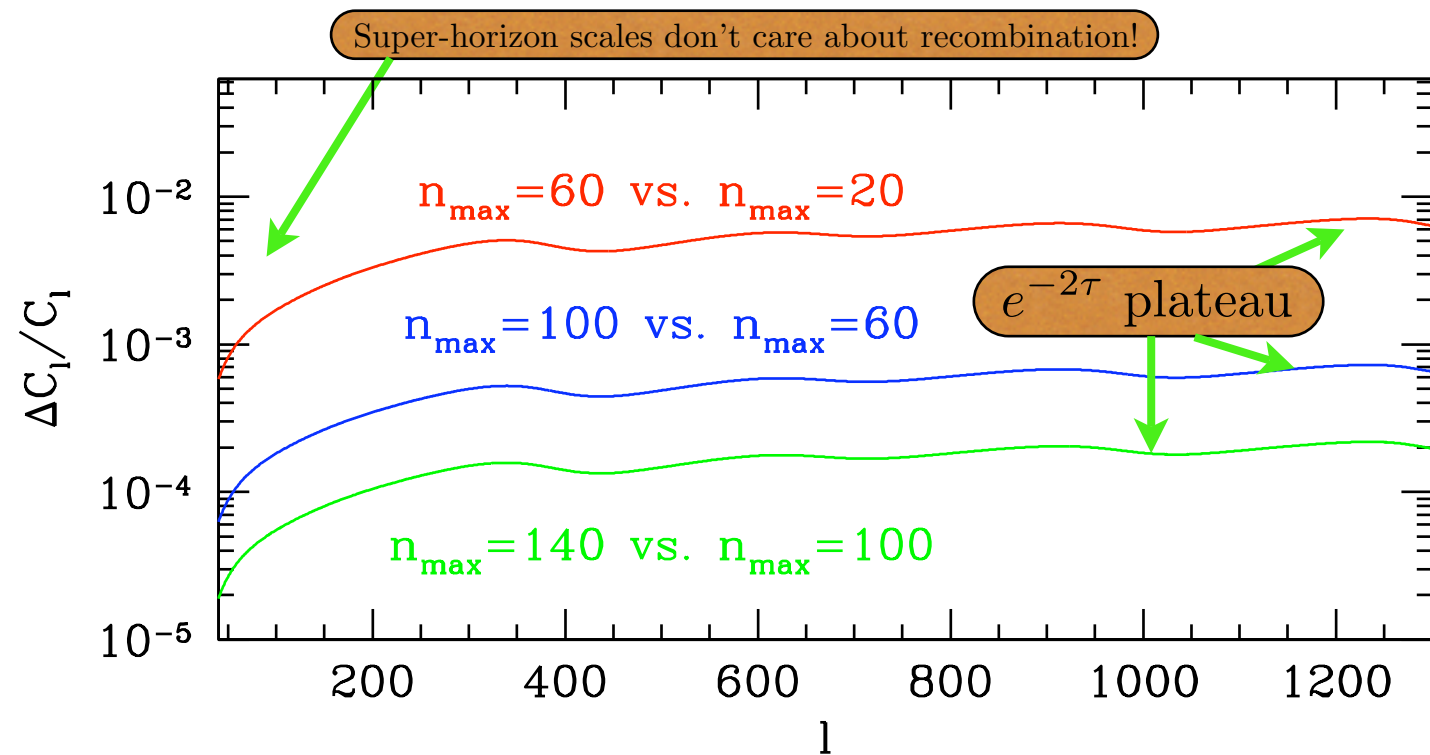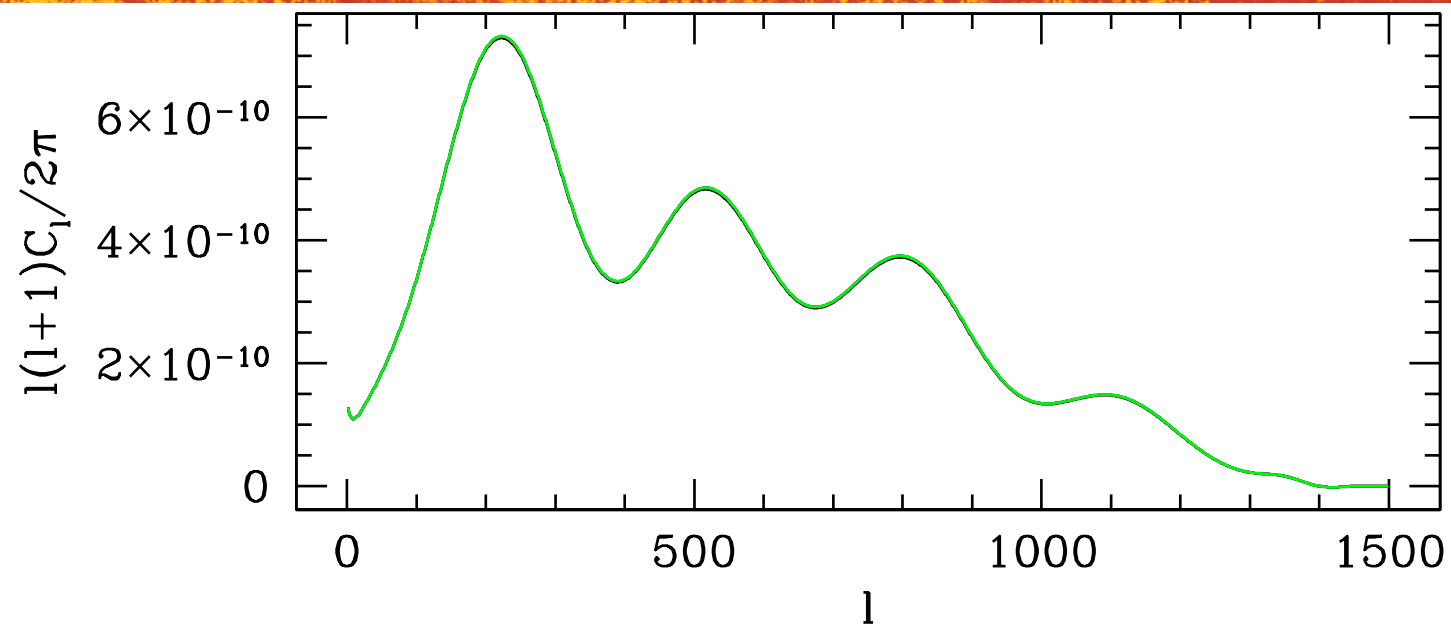
Lower τ_{LSS} trump $\Delta\eta_{\text{LSS}}$ effects

$n_{\text{max}}=60$ vs. $n_{\text{max}}=40$

$n_{\text{max}}=100$ vs. $n_{\text{max}}=60$

$n_{\text{max}}=140$ vs. $n_{\text{max}}=100$

ATOMIC COLLISIONS

- For fixed n, l-changing collisions bring different-l substates closer to statistical equilibrium (SE)

- Chluba et al. (2007) claim n-changing collisions irrelevant for n<100

- Being closer to SE speeds up rec. by mitigating high-l bottleneck

- Theoretical collision rates unknown to factors of 2!

  - $b < a_0 n^2 \to$ multi-body QM!
  - $t_{\text{pass}} < t_{\text{orbit}} \to$ Impulse approximation breaks down!

- Order-of-magnitude inclusion under way to determine if better theory needed for rec.

$n_{\max} = 100$
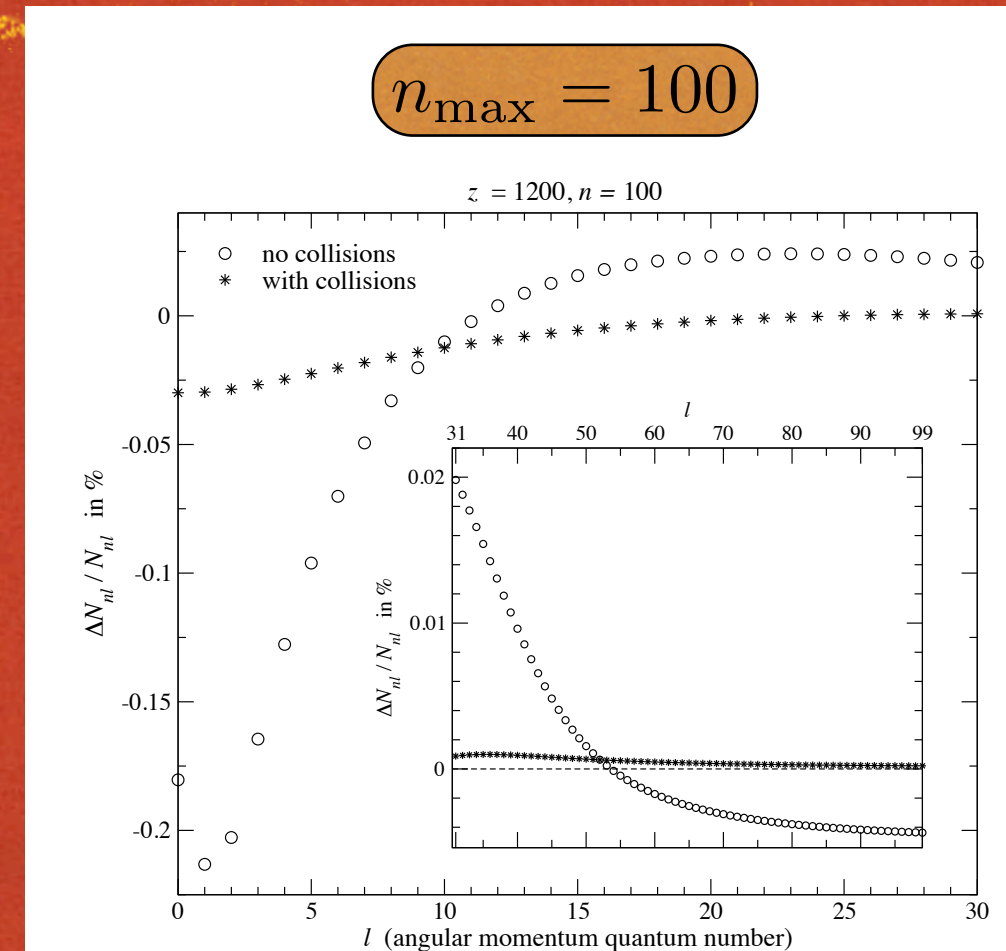
QUADRAPOLE TRANSITIONS

- $\Delta l = \pm 1$ transitions may also play a role

- Rates are given by $A_{nn'}^{ll',m} = C\omega^5 \langle f|r^2 Y_{2m}|i\rangle$

- Moments may be evaluated with radial wf. raising/lowering operators

- Transitions to/from 1s will dominate

- Transitions from nd to 1s will immediately be followed by transitions up to mp, etc...

- Rate can thus be rewritten as an effective $\Delta l = \pm 1$ transition rate, thus respecting our sparsity pattern

WRAPPING UP

- To do:

  - Line feedback via iterative procedure

  - Collisions

  - Quadrapole transitions

  - Effective source term for omitted higher levels- near Saha eq., should be tractable

  - Full incorporation into CMBFAST/CAMB and analysis of errors/degeneracies with cosmo. parameters