

COSMOLOGICAL HYDROGEN RECOMBINATION: The effect of extremely high- n states and forbidden transitions

arXiv:0911.1359, submitted to Phys. Rev. D.

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in collaboration with Christopher M. Hirata

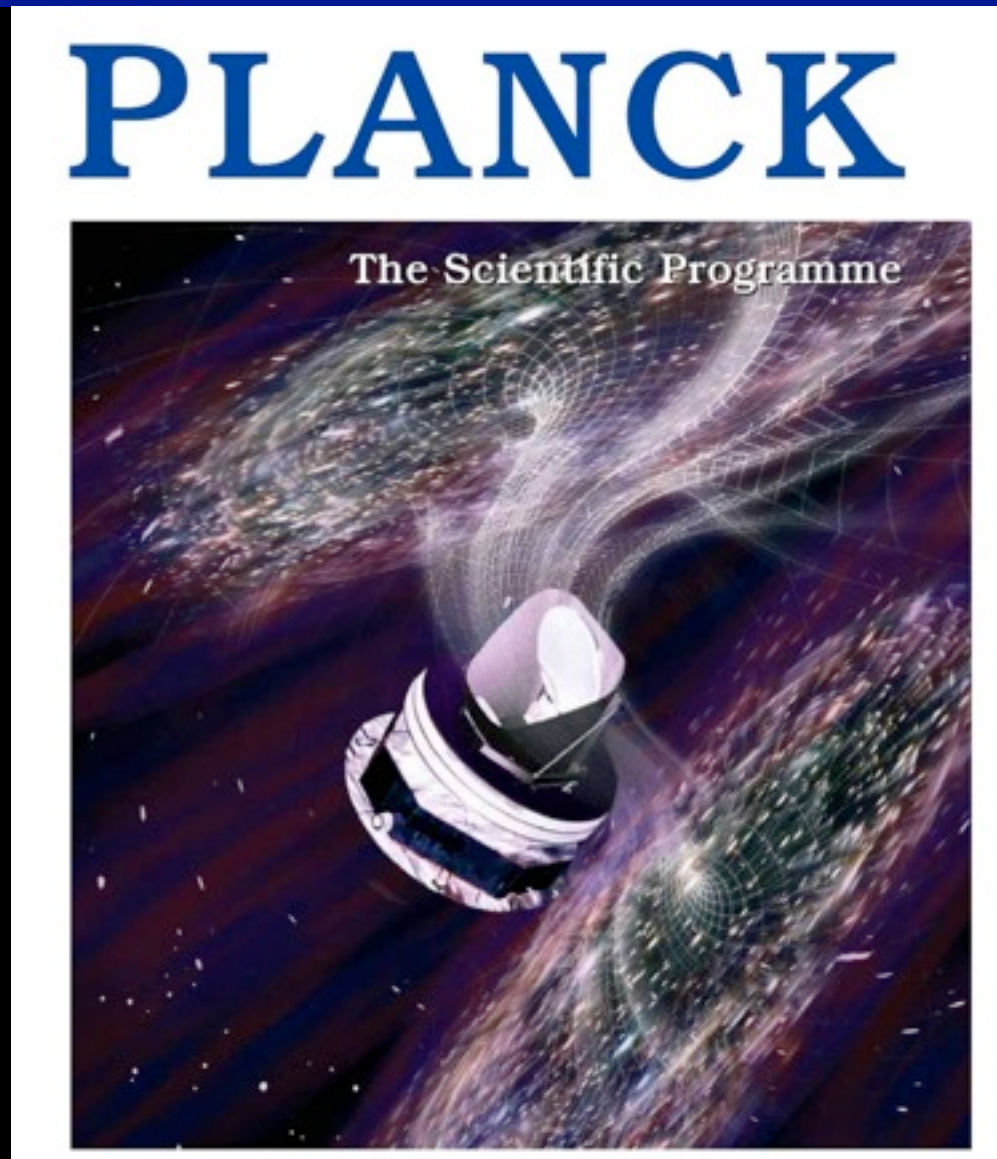
ITC Seminar

12/8/09

OUTLINE

- * Motivation: CMB anisotropies and recombination spectra
- * Recombination in a nutshell
- * Breaking the Peebles/RecFAST mold
- * **RecSparse**: a new tool for high- n states
- * Forbidden transitions
- * Results
- * Ongoing/future work

WALK THE PLANCK

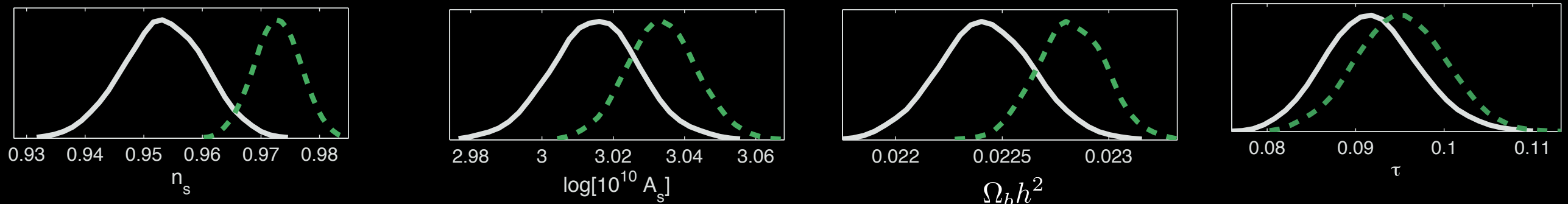


- * Planck (launched May 2009) will make cosmic-variance limited CMB anisotropy measurements up to $l \sim 2500$ (T), and $l \sim 1500$ (E)
- * Wong 2007 and Lewis 2006 show that $x_e(z)$ needs to be predicted to several parts in 10^4 accuracy for Planck data analysis

RECOMBINATION, INFLATION, AND REIONIZATION

$$P(k) = A_s (k\eta_0)^{n_s}$$

* Planck uncertainty forecasts using MCMC



- * Cosmological parameter inferences will be off if recombination is improperly modeled (Wong/Moss/Scott 2007)
- * Leverage on new physics comes from high l . Here the details of recombination matter!
- * Inferences about inflation will be wrong if recombination is improperly modeled

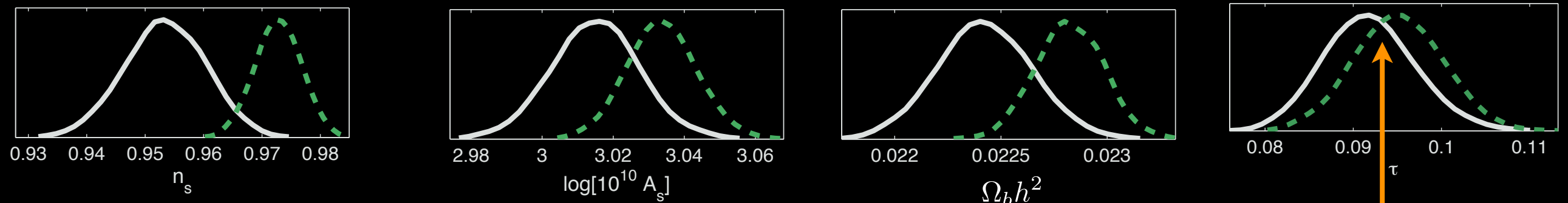
$$n_s = 1 - 4\epsilon + 2\eta \quad \epsilon = \frac{m_{\text{pl}}^2}{16\pi} \left[\frac{V'(\phi)}{V(\phi)} \right]^2 \quad A_s^2 = \frac{32}{75} \frac{V}{m_{\text{pl}}^4 \epsilon} \Big|_{k_{\text{pivot}} = aH}$$

CAVEAT EMPTOR: $3 \lesssim ? \lesssim 16$

Need to do eV physics right to infer anything about 10^7 GeV physics! 4

RECOMBINATION, INFLATION, AND REIONIZATION

* Planck uncertainty forecasts using MCMC



Bad recombination history yields biased inferences about reionization

PHYSICAL RELEVANCE FOR CMB: SMEARING AND MOVING THE SURFACE OF LAST SCATTERING (SLS)

- * Photons kin. decouple when Thompson scattering freezes out



- * Acoustic mode evolution influenced by visibility function

$$g = \dot{\tau} e^{-\tau} \qquad \tau(z) = \int_0^{\eta(z)} n_e \sigma_T a(\eta') d\eta'$$

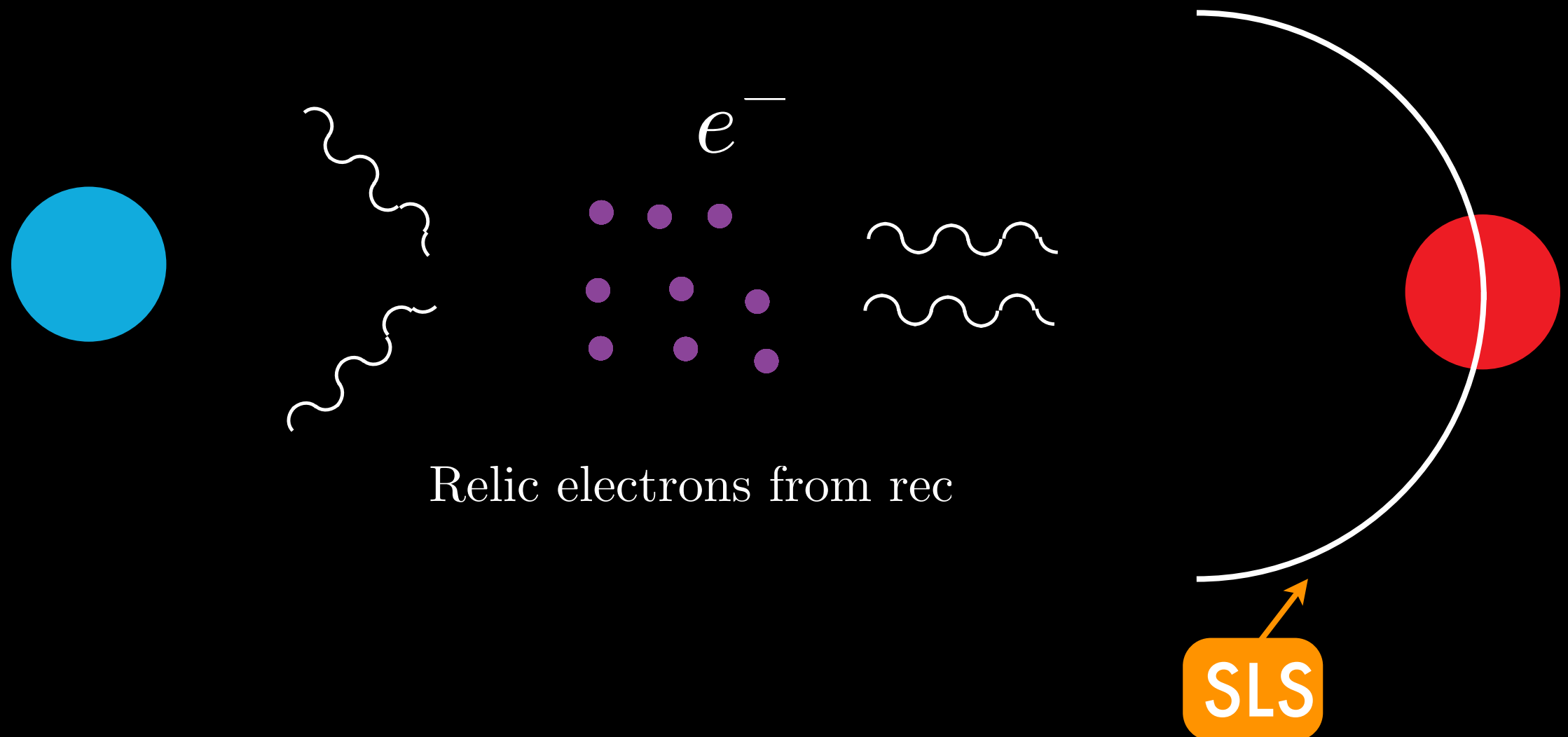
- * $z_{\text{dec}} \simeq 1100$: Decoupling occurs during recombination

$$C_l \rightarrow C_l e^{-2\tau(z)} \text{ if } l > \eta_{\text{dec}}/\eta(z)$$

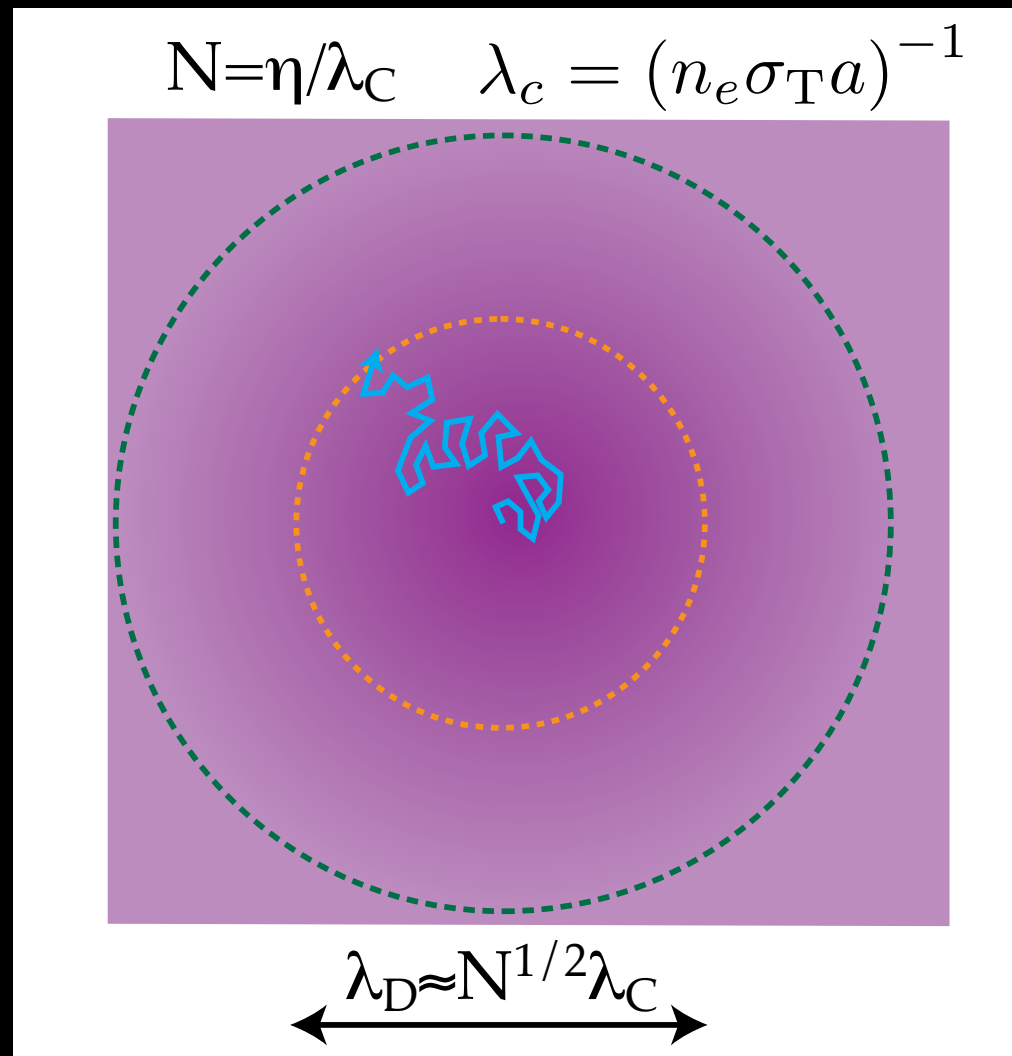
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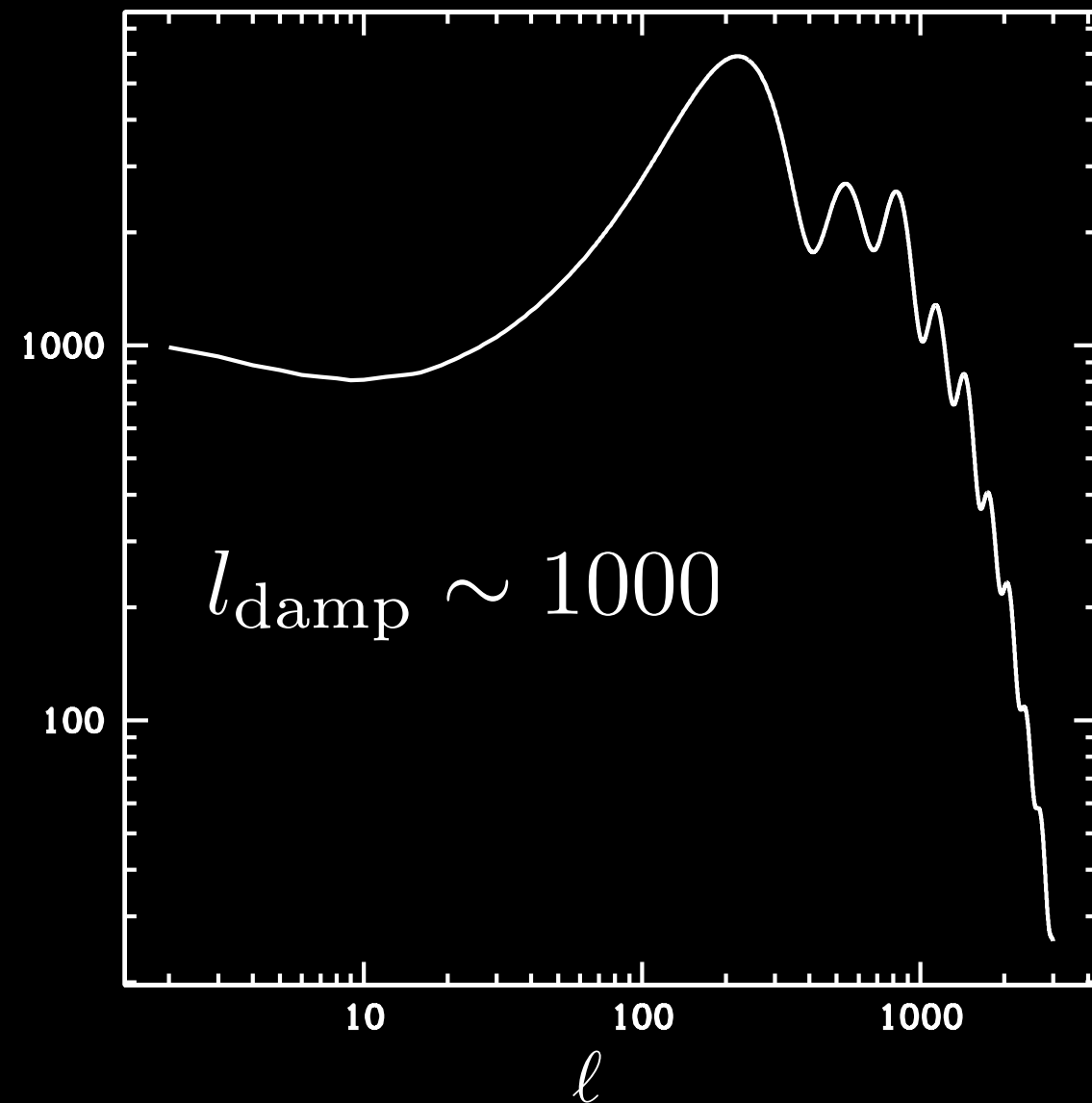
$$\gamma + e^{-} \Leftrightarrow \gamma + e^{-}$$



PHYSICAL RELEVANCE FOR CMB: THE SILK DAMPING TAIL



$$C_\ell^{\text{TT}} (\mu K^2)$$



✳ Inhomogeneities are damped for $\lambda < \lambda_D$

PHYSICAL RELEVANCE FOR CMB: POLARIZATION

Isotropic radiation

Quadrupole moment

No polarization

Polarization

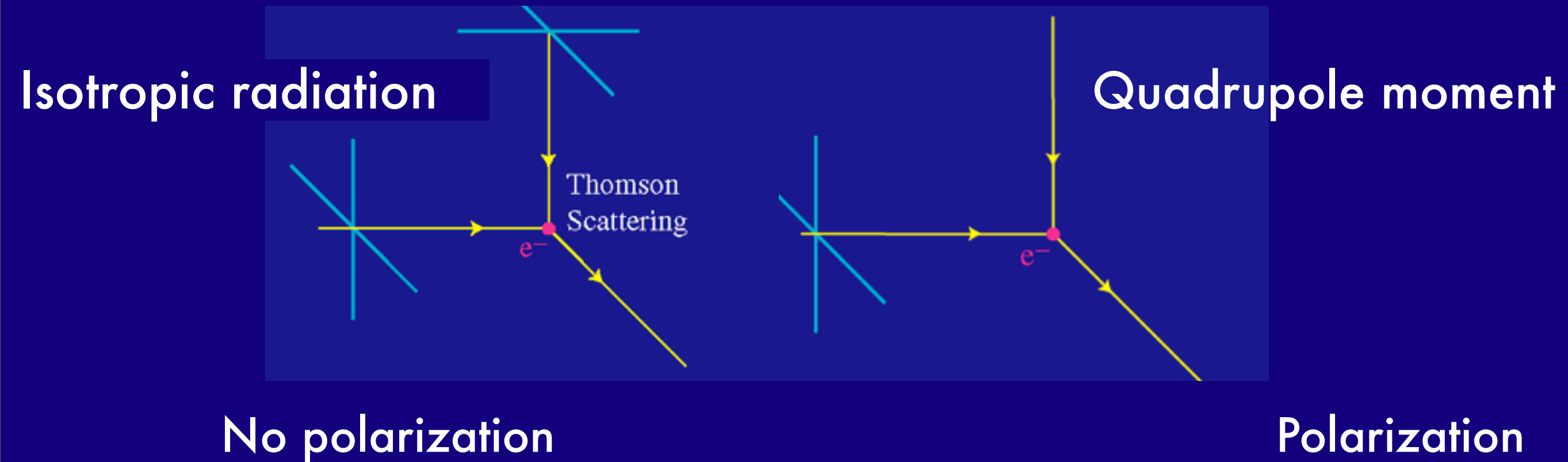
From Wayne Hu's website

* Need time to develop a quadrupole

$$\Theta_l(k\eta) \sim \frac{k\eta}{2\tau} \Theta_{l+1}(k\eta) \ll \Theta_{l+1}(k\eta) \text{ if } l \geq 2, \text{ in tight coupling regime}$$

* Need to scatter quadrupole to polarize CMB

PHYSICAL RELEVANCE FOR CMB: POLARIZATION



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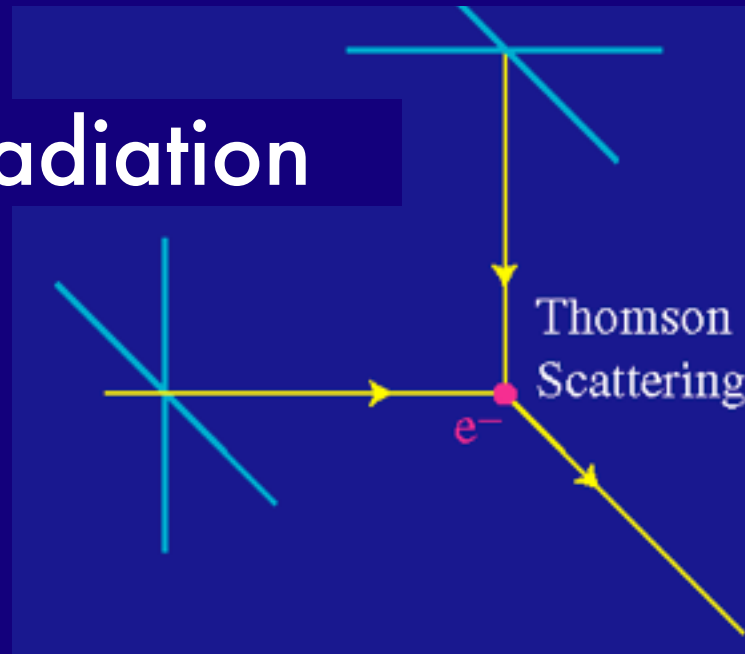
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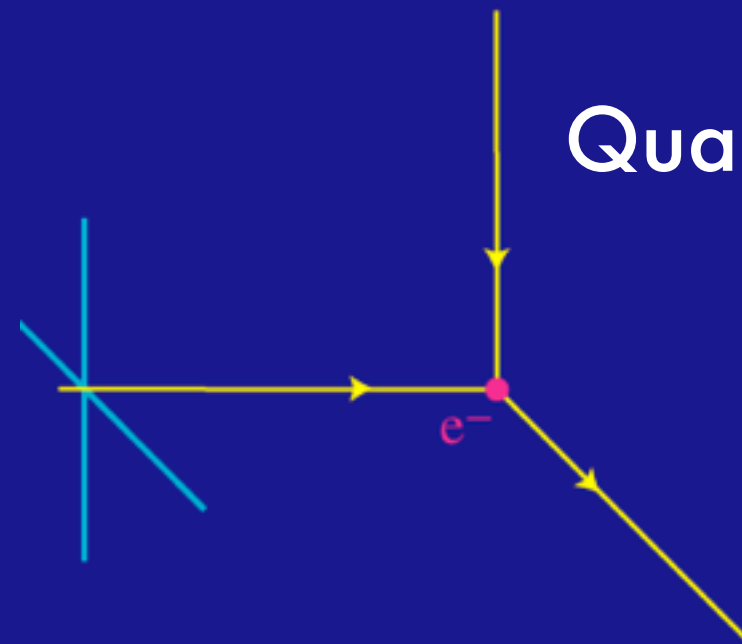
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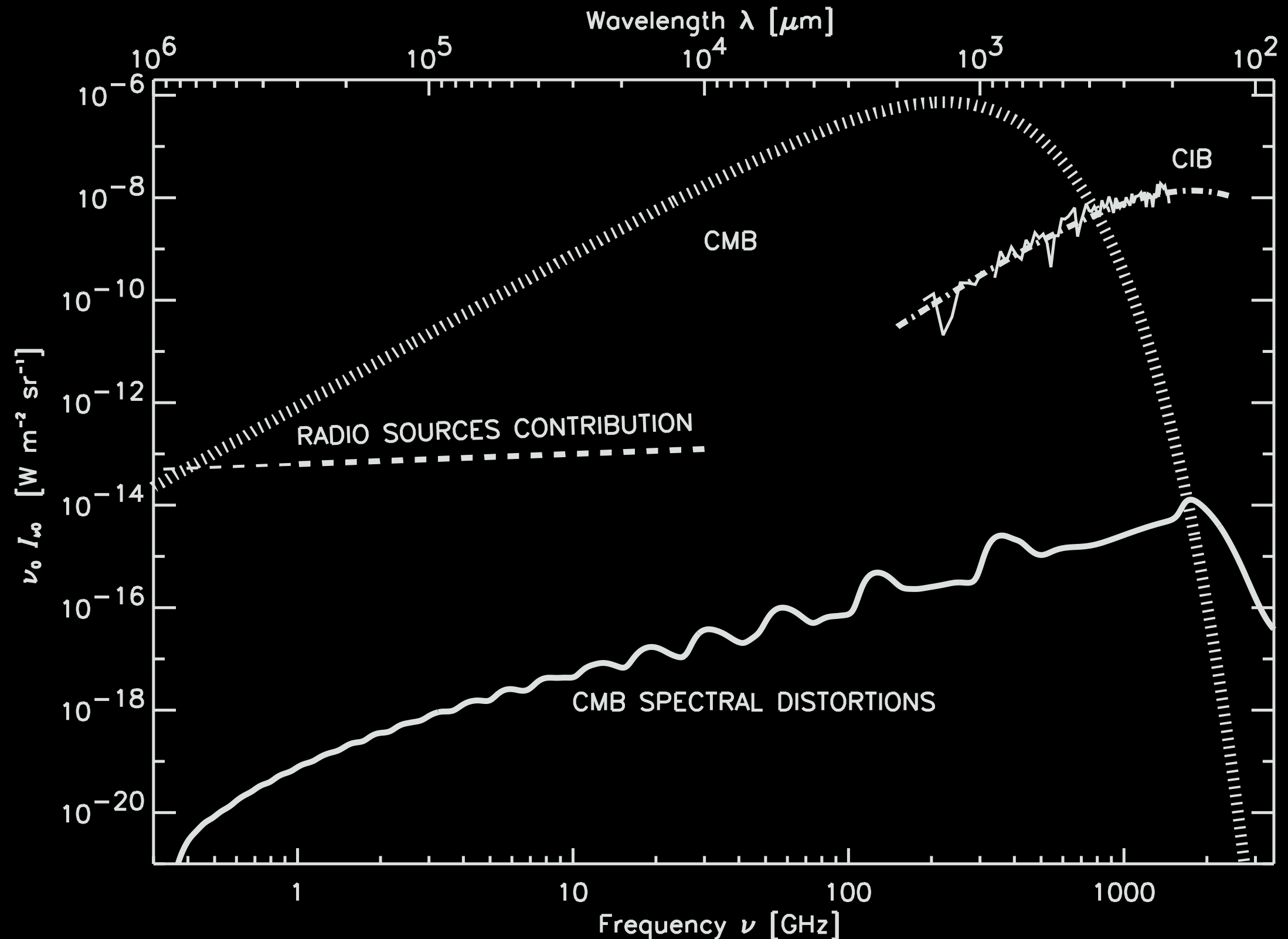
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PHYSICAL RELEVANCE FOR CMB: SPECTRAL DISTORTIONS FROM RECOMBINATION



SAHA EQUILIBRIUM IS INADEQUATE



- * Chemical equilibrium does reasonably well predicting “moment of recombination”

$$\frac{x_e^2}{1 - x_e} = \left(\frac{13.6}{T_{\text{eV}}} \right)^{3/2} e^{35.9 - 13.6/T_{\text{eV}}}$$

$$x_e = 0.5 \text{ when } T = T_{\text{rec}} \simeq 0.3 \text{ eV} \qquad z_{\text{rec}} \simeq 1300$$

- * Further evolution falls prey to reaction freeze-out

$$\Gamma < H \text{ when } T < T_{\text{F}} \simeq 0.25 \text{ eV}$$

BOTTLENECKS/ESCAPE ROUTES

BOTTLENECKS

- * Ground state recombinations are ineffective

$$\Gamma_{c \rightarrow 1s} = 10^{-1} \text{ s}^{-1} \gg H \simeq 10^{-12} \text{ s}^{-1}$$

- * Resonance photons are re-captured, e.g. Lyman α

$$\Gamma_{2p \rightarrow 1s} = 10^{-2} \text{ s}^{-1} \gg H \simeq 10^{-12} \text{ s}^{-1}$$

ESCAPE ROUTES (e.g. n=2)

- * Two-photon processes

$$H^{2s} \rightarrow H^{1s} + \gamma + \gamma \quad \Lambda_{2s \rightarrow 1s} = 8.22 \text{ s}^{-1}$$

- * Redshifting off resonance

$$R \sim (n_H \lambda_\alpha^3)^{-1} H$$

THE PEEBLES PUNCHLINE

- * Only $n=2$ bottlenecks are treated
- * Net Rate is suppressed by bottleneck vs. escape factor

$$-\frac{dx_e}{dt} = \mathcal{S} \sum_{n,l>1s} \alpha_{nl}(T) \{nx_e^2 - x_{1s}f(T)\}$$

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Recombination rate



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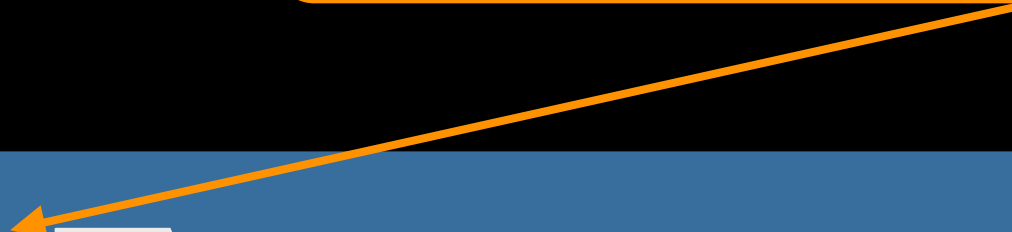
ionization rate



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$$-\frac{dx_e}{dt} = S \sum_{n,l>1s} \alpha_{nl}(T) \{nx_e^2 - x_{1s}f(T)\}$$


THE PEEBLES MODEL

✳ Net Rate is suppressed by bottleneck vs. escape factor

$$\mathcal{S} = \frac{\frac{8\pi}{\lambda_{\alpha}^3 n_{1s}} H + \Lambda_{2s \rightarrow 1s}}{\frac{8\pi}{\lambda_{\alpha}^3 n_{1s}} H + (\Lambda_{2s \rightarrow 1s} + \beta_c)}$$

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Redshifting term

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→ 2γ term

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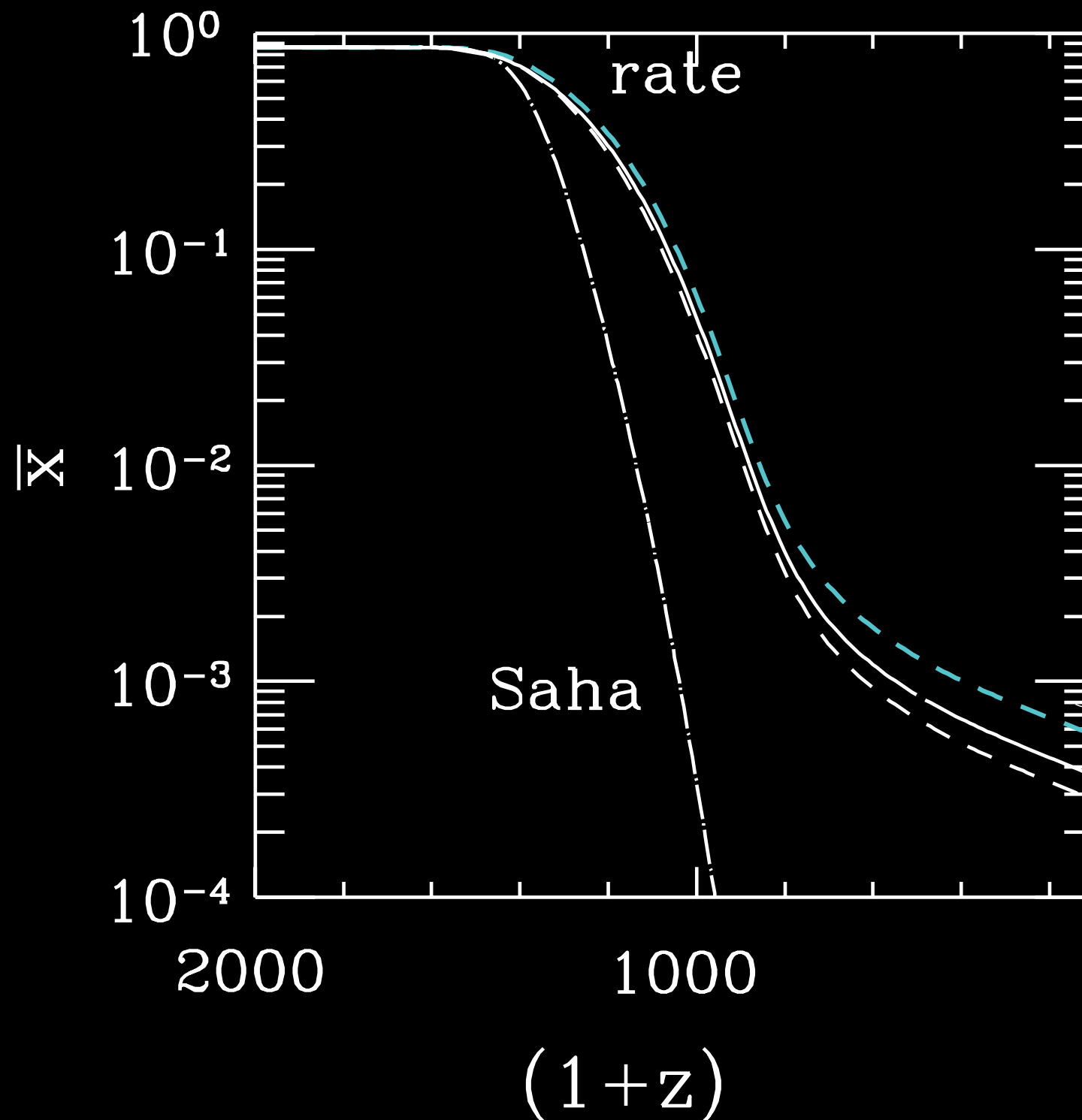
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$$\frac{\text{redshift term}}{2\gamma \text{ term}} \simeq 0.02 \frac{\Omega_m^{1/2}}{(1 - x_e[z]) \left(\frac{1+z}{1100}\right)^{3/2}}$$

2γ process dominates until late times ($z \lesssim 850$)

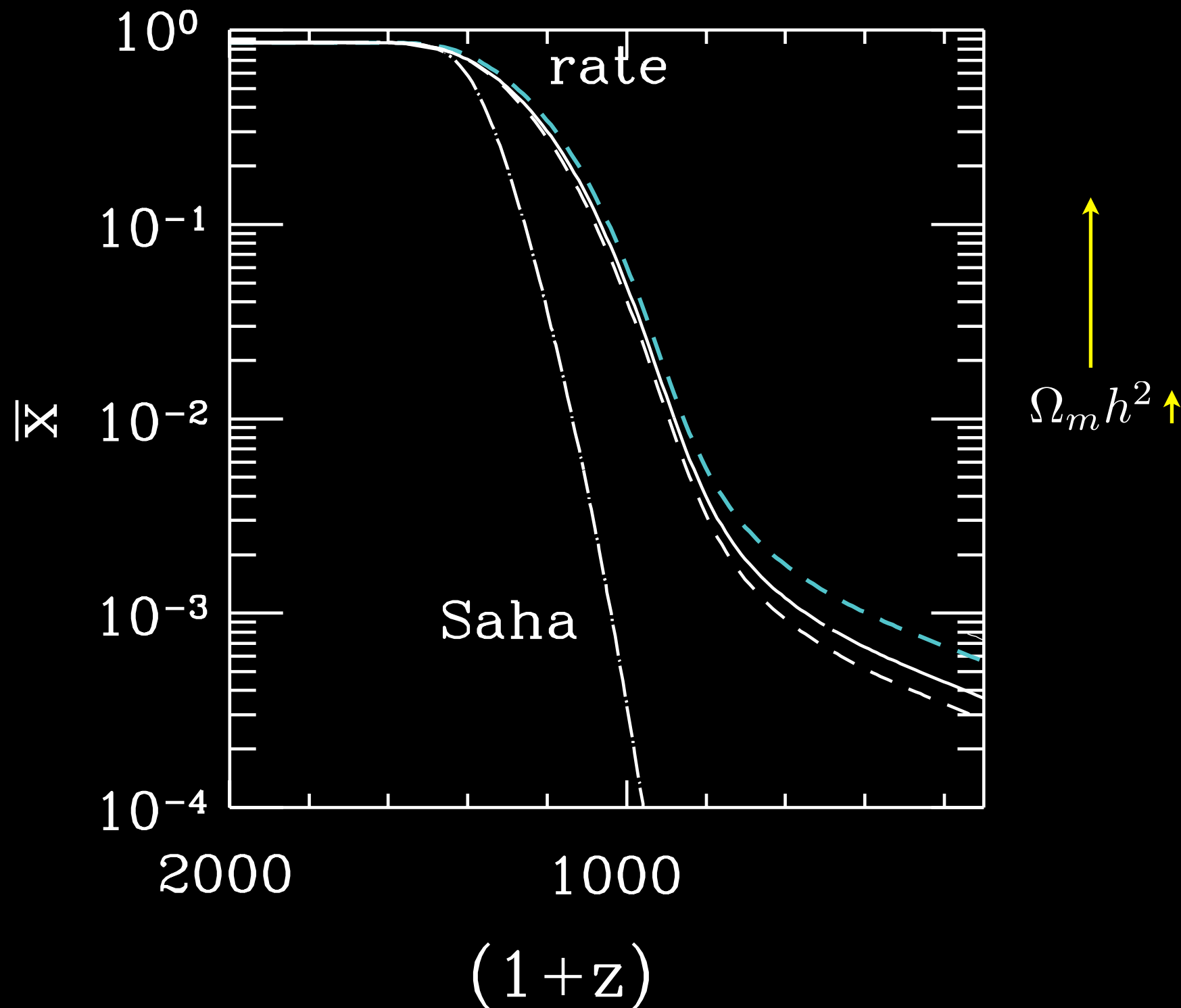
THE PEEBLES MODEL

✦ Peebles 1967: State of the Art for 30 years!



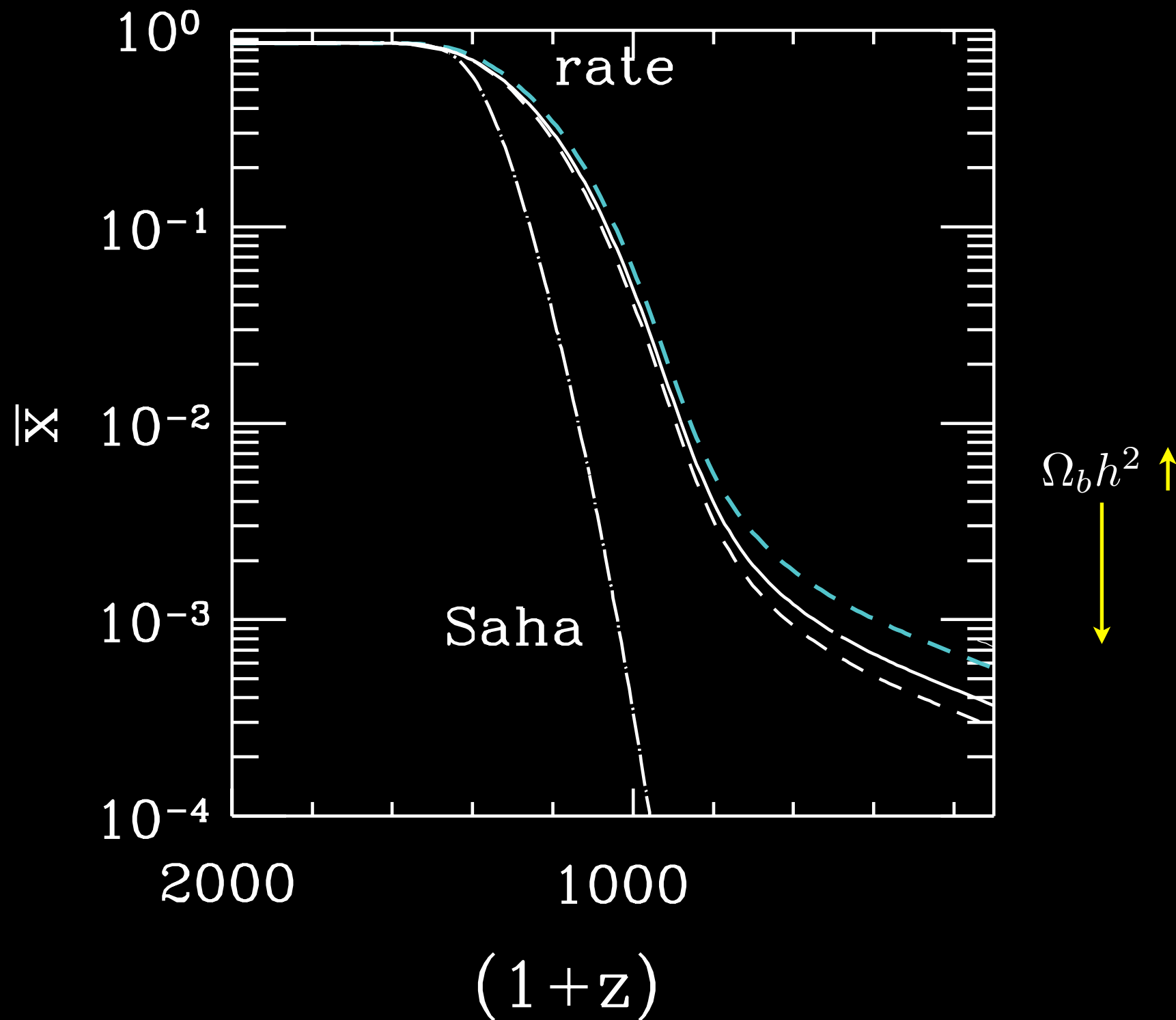
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EQUILIBRIUM ASSUMPTIONS

*Radiative/collisional eq. between different l

$$\mathcal{N}_{nl} = \mathcal{N}_n \frac{(2l + 1)}{n^2}$$

* Radiative eq. between different n-states

$$\mathcal{N}_n = \sum_l \mathcal{N}_{nl} = \mathcal{N}_2 e^{-(E_n - E_2)/T}$$

Non-eq rate equations

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Seager/Scott/Sasselov 2000/RECFAST!

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Non-eq rate equations

BREAKING EQUILIBRIUM

- * Chluba et al. (2005,6) follow l , n separately, get to $n_{\max} = 100$
- * 0.1 %-level corrections to CMB anisotropies at $n_{\max} = 100$
- * Equilibrium between l states: $\Delta l = \pm 1$ bottleneck
- * Beyond this, testing convergence with n_{\max} is hard!

$$t_{\text{compute}} \sim \mathcal{O}(\text{years}) \text{ for } n_{\max} = 300$$

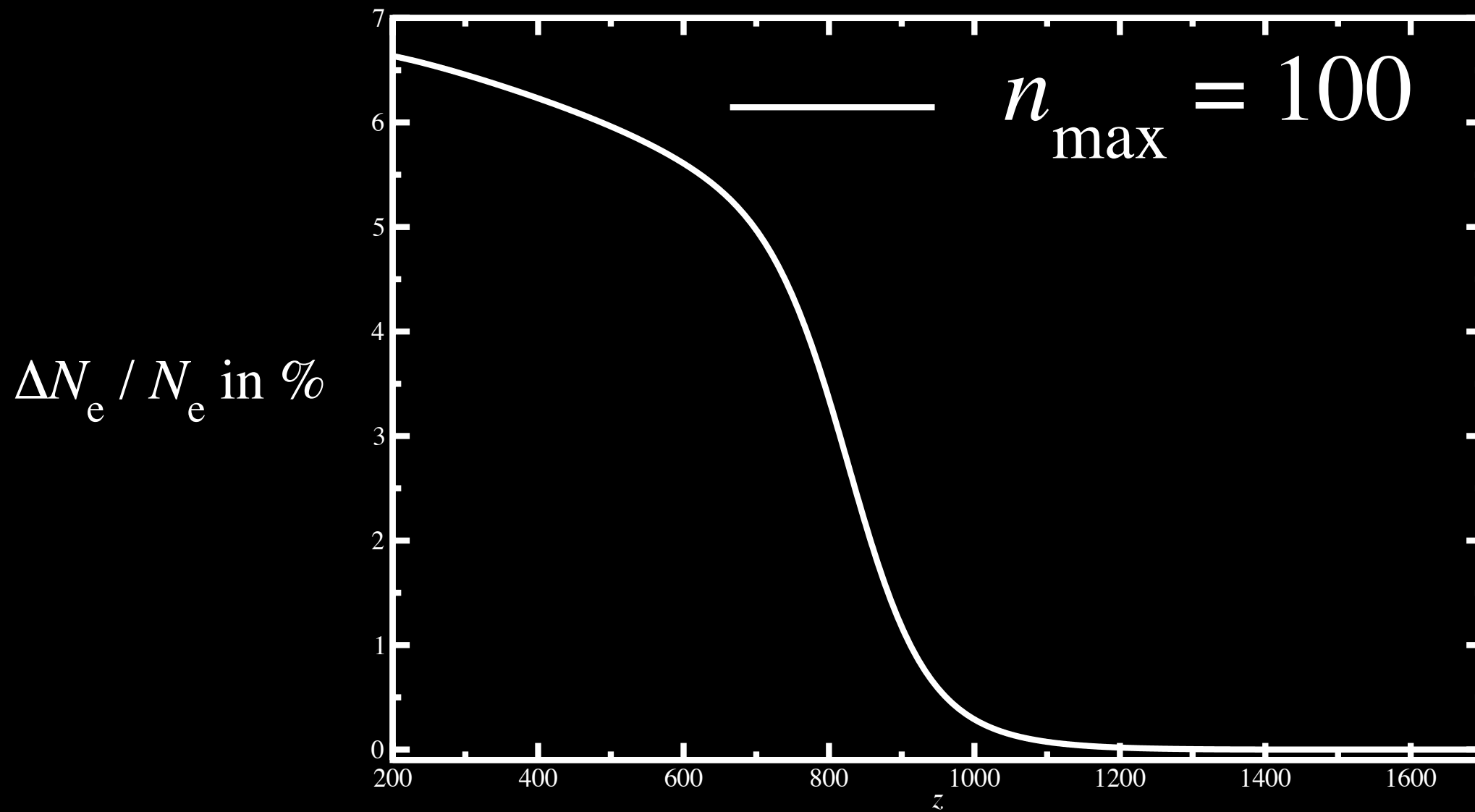
How to proceed if we want $\mathcal{O}(1) \times 10^{-4}$ accuracy in C_ℓ ?

THESE ARE REAL STATES

- * Still inside plasma shielding length for $n < 100000$
- * $r \sim a_0 n^2$ is as large as $2\mu\text{m}$ for $n_{\text{max}} = 200$
- * $\frac{\Delta E|_{\text{thermal}}}{E} < \frac{2}{n^3}$
- * Similarly high n are seen in emission line nebulae

THE EFFECT OF RESOLVING l - SUBSTATES

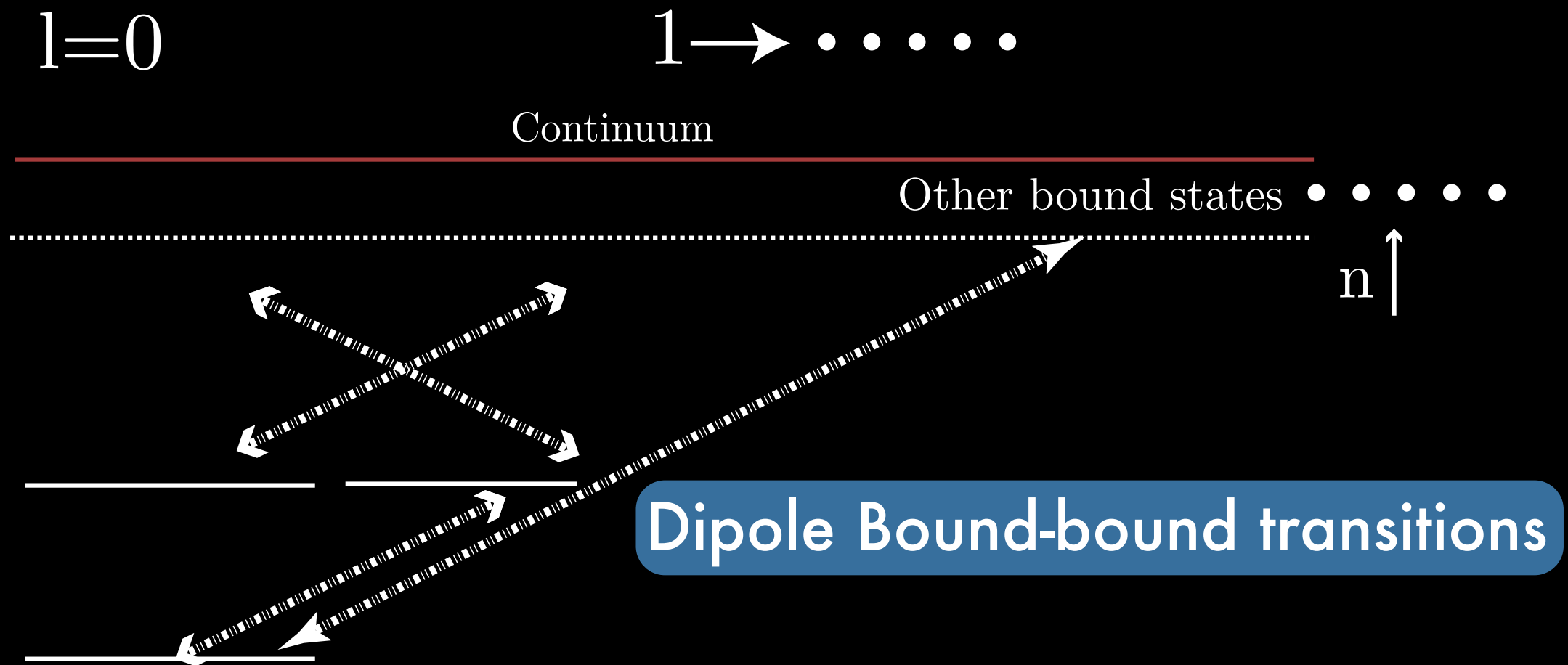
Resolved l vs unresolved l



From Chluba et al. 2006

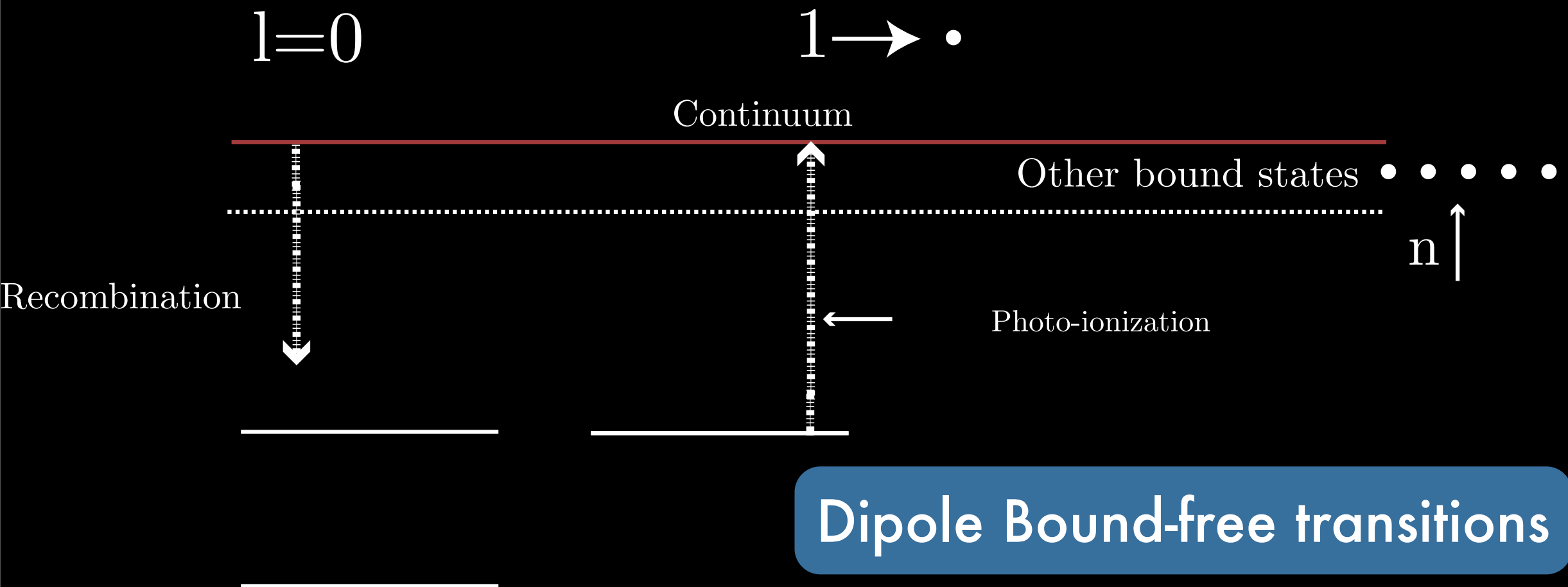
✳ 'Bottlenecked' l -substates decay slowly to 1s: Recombination is slower; Chluba et al. 2006

RECSPARSE AND THE MULTI-LEVEL ATOM



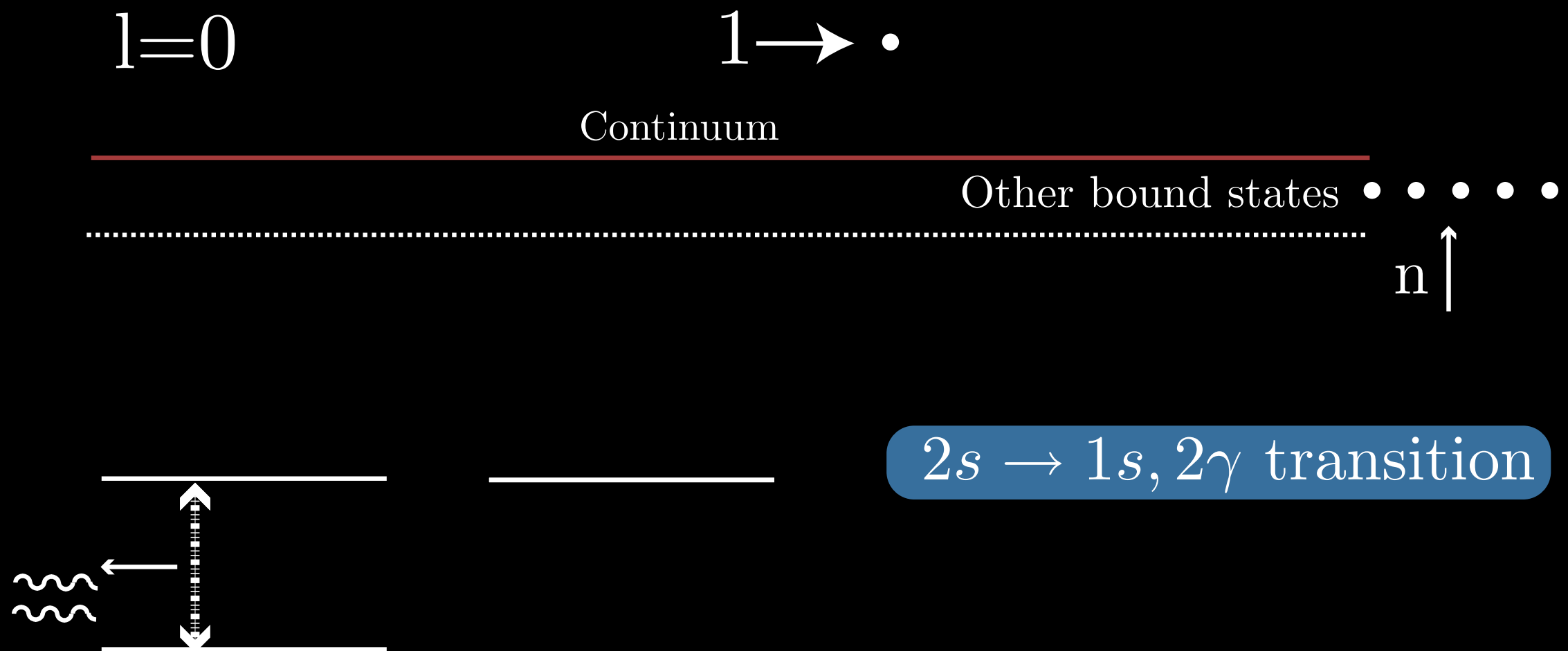
- * We implement a multi-level atom computation in a new code, **RecSparse!**
- * Boltzmann eq. solved for $T_m (T_\gamma)$
- * Spontaneous/stimulated emission/absorption included

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RECSPARSE AND THE MULTI-LEVEL ATOM

✳ Free electron fraction evolved according to

$$\begin{aligned}\dot{x}_e &= -\dot{x}_{1s} \\ &= -\Lambda_{2s \rightarrow 1s} \left(x_{2s} - x_{1s} e^{-E_{2s \rightarrow 1s}/T_\gamma} \right) + \sum_{n,l > 1s} A_{n1}^{l0} P_{n1}^{l0} \{g(T, n, l)\}\end{aligned}$$

2s-1s decay rate

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Lyman series current to ground state

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Einstein coeff.

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Escape probability



RADIATION FIELD: BLACK BODY +

- * Escape probability treated in Sobolev approx: depends on steady-state and incoherent scattering approximations

$$P_{n,n'}^{l,l'} = \frac{1 - e^{-\tau_s}}{\tau_s}$$

$$\tau_s \propto \frac{n_H x_n^l A_{nn'}^{ll'}}{H(z)} \quad n' > n$$

- * RecSparse includes radiative feedback
- * Ongoing work in field focuses on corrections to simple radiative transfer picture
- * Ultimate goal is to combine all new atomic physics effect in one fast recombination code

RADIATION FIELD: BLACK BODY +

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Resonant absorber density

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Einstein coefficient

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Cosmological expansion

$$\tau_s \propto \frac{n_{\text{H}} x_n^l A_{nn'}^{ll'}}{H(z)} \quad n' > n$$

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OTHER CORRECTIONS TO RECOMBINATION

- * Deviations from steady-state approx (Chluba/Sunyaev 2008)
- * Coherent scattering (Forbes and Hirata 2009, Switzer/Hirata 2007)
- * Atomic recoil (Forbes and Hirata 2009, Dubrovich and Grachev 2008)
- * Diffusion near resonance lines
- * Line overlap (Ali-Haimoud, Grin, Hirata in progress)
- * Feedback from hydrogen/helium (Chluba/Sunyaev 2007)
- * Higher-n two-photon processes (Chluba/Sunyaev 2007, Hirata 2008) in hydrogen and Helium (Switzer/Hirata 2007)
- * Deuterium
- * Additional effects in Helium (Switzer/Hirata 2007)

STEADY-STATE FOR EXCITED LEVELS

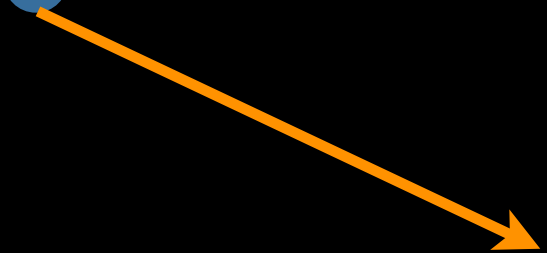
✱ Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

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$$\vec{x} =$$

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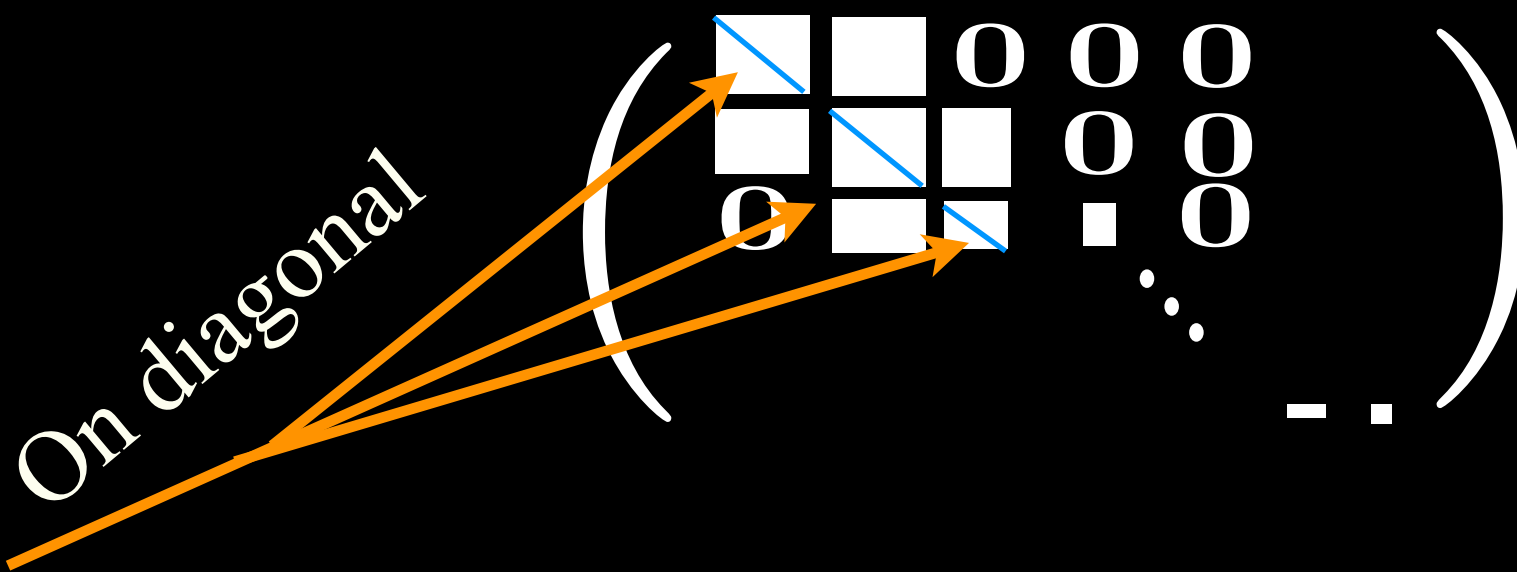
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$$\vec{x} = \begin{pmatrix} \vec{x}_0 \\ \vec{x}_1 \\ \dots \\ \vec{x}_l \\ \dots \\ \vec{x}_{l_{\max}} \end{pmatrix}$$
$$\vec{x}_l = \begin{pmatrix} x_{l,l+1} \\ \dots \\ x_{l,n_{\max}} \end{pmatrix}$$

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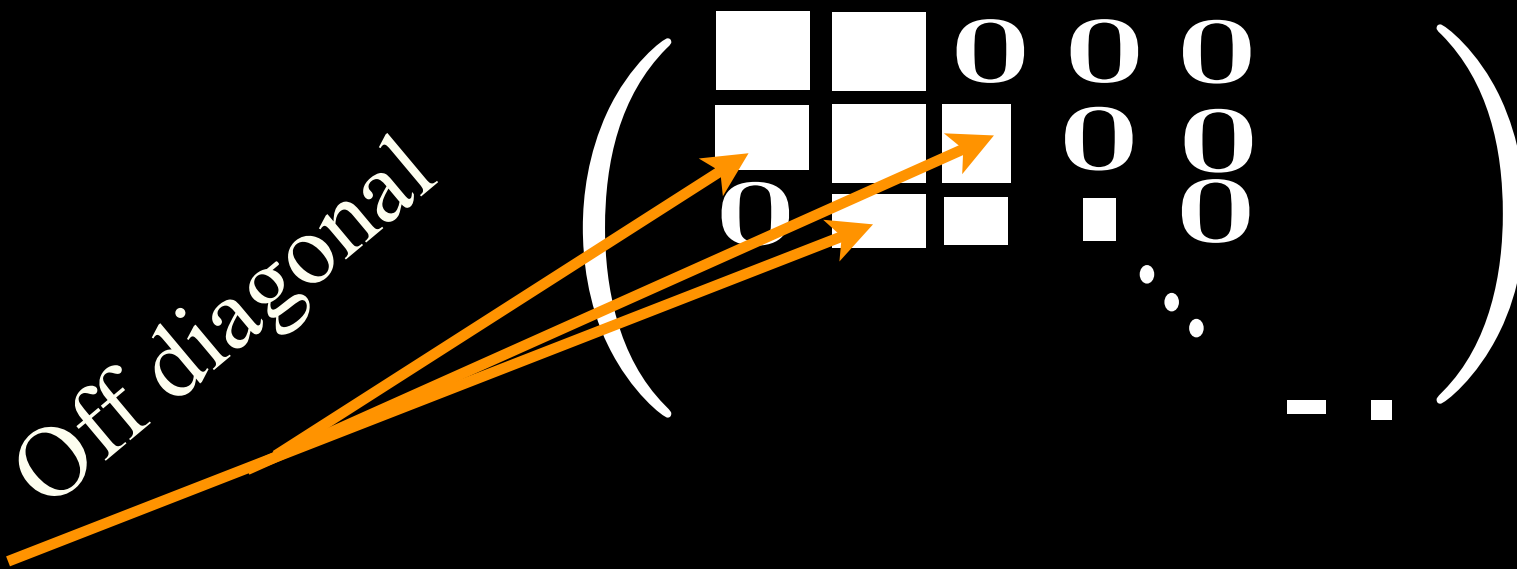


For state 1, includes BB transitions out of 1 to all other 1'',
photo-ionization, 2γ transitions to ground state

STEADY-STATE FOR EXCITED LEVELS

- * Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$




Off diagonal

The diagram shows a matrix \mathbf{R} enclosed in large parentheses. The matrix is represented by a grid of squares. The first three rows and columns are explicitly shown. The first row contains three squares, followed by three zeros. The second row contains three squares, followed by two zeros. The third row contains a zero, followed by three squares, followed by one zero. Below the third row, there are three dots and a dash, indicating the matrix continues. Three orange arrows originate from the text 'Off diagonal' and point to the off-diagonal elements: the square in the second row, first column; the square in the third row, second column; and the square in the third row, third column.

For state 1, includes BB transitions into 1 from all other 1'

STEADY-STATE FOR EXCITED LEVELS

- * Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$


Includes recombination to 1,
1 and 2γ transitions from ground state

STEADY-STATE FOR EXCITED LEVELS

- * Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

For $n > 1$, $\mathbf{R}, \vec{s} \geq 1 \text{ s}^{-1}$ e.g. Lyman- α

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$$t_{\text{rec}}^{-1} \sim 10^{-12} \text{ s}^{-1}$$

For $n > 1$, $\mathbf{R}, \vec{s} \geq 1 \text{ s}^{-1}$ e.g. Lyman- α

STEADY-STATE FOR EXCITED LEVELS

* Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt}$$

$$= \mathbf{R}\vec{x} + \vec{s}$$

LHS \ll RHS

$$\vec{x} \simeq -\mathbf{R}^{-1}\vec{s}$$

$$t_{\text{rec}}^{-1} \sim 10^{-12} \text{ s}^{-1}$$

For $n > 1$, $\mathbf{R}, \vec{s} \geq 1 \text{ s}^{-1}$ e.g. Lyman- α

RAPID MATRIX INVERSION: SPARSITY TO THE RESCUE

* Matrix is $\sim n_{max}^2 \times n_{max}^2$

* Brute force would require $An_{max}^6 \sim 10^5$ s for $n_{max} = 200$ for a single time step

* Dipole selection rules: $\Delta l = \pm 1$

$$M_{l,l-1}\vec{x}_{l-1} + M_{l,l}\vec{x}_l + M_{l,l+1}\vec{x}_{l+1} = \vec{s}_l$$

$$\begin{pmatrix} \blacksquare & \blacksquare & 0 & 0 & 0 \\ \blacksquare & \blacksquare & \blacksquare & 0 & 0 \\ 0 & \blacksquare & \blacksquare & \blacksquare & 0 \\ & & \ddots & \ddots & \ddots \\ & & & \ddots & \ddots \end{pmatrix} \begin{pmatrix} \vec{x}_0 \\ \vec{x}_1 \\ \dots \\ \vec{x}_{n_{max}-1} \end{pmatrix} = \vec{s}_l$$

* Physics imposes sparseness on the problem. Solved in closed form to yield algebraic $\vec{x}_{l_{max}}$, then \vec{x}_l in terms of \vec{x}_{l+1}

RAPID MATRIX INVERSION: SPARSITY TO THE RESCUE

- * Einstein coefficients to states with $n > n_{\max}$ are set $A = 0$: more later!
- * **RecSparse** generates rec. history with computation time $\sim n_{\max}^{2.5}$: Huge improvement!
- * Case of $n_{\max} = 100$ runs in less than a day, $n_{\max} = 200$ takes ~ 4 days.

FORBIDDEN TRANSITIONS AND RECOMBINATION

- * Higher- n 2γ transitions in H important at $7\text{-}\sigma$ for Planck (TT/EE) data analysis (Hirata 2008, Kholupenko 2006)
- * Some forbidden transitions are important in Helium recombination (Dubrovich 2005, Lewis 2006) and would bias cosmological parameter estimation.
- * *Are other forbidden transitions in hydrogen important, particularly for Planck data analysis? How about electric quadrupole (E2) transitions?*

QUADRUPOLE TRANSITIONS AND RECOMBINATION

- * Ground-state electric quadrupole (E2) lines are optically thick!

$$P_{n,n'}^{l,l'} = \frac{1 - e^{-\tau_s}}{\tau_s}$$

$$R \propto AP \propto A/\tau \text{ if } \tau \gg 1$$

$$\tau \propto A \rightarrow R \rightarrow A/A \rightarrow \text{const}$$

- * Coupling to ground state will overwhelmingly dominate:

$$\frac{A_{n,2 \rightarrow 1,0}^{\text{quad}}}{A_{n,2 \rightarrow m,0}^{\text{quad}}} \propto \frac{\omega_{n1}^5}{\omega_{nm}^5} \geq 10^3 \text{ if } m \geq 2$$

QUADRUPOLE TRANSITIONS AND RECOMBINATION

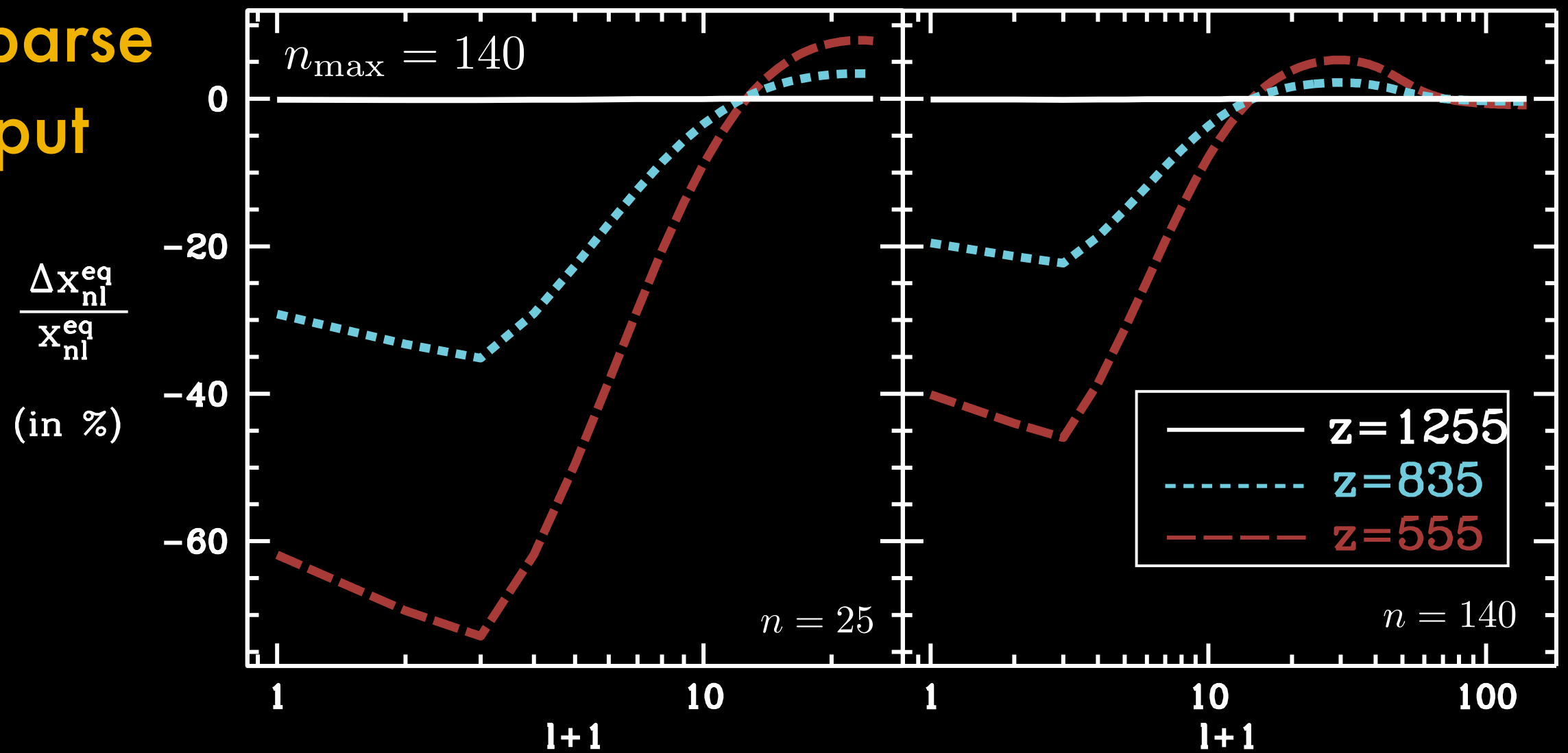
- * Lyman lines are optically thick, so $nd \rightarrow 1s$ immediately followed by $1s \rightarrow np$, so this can be treated as an effective $d \rightarrow p$ process with rate $A_{nd \rightarrow 1s} x_{nd}$.
- * Same sparsity pattern of rate matrix, similar to l-changing collisions
- * Detailed balance yields net rate

$$R_{nd \rightarrow np}^{\text{quad}} = A_{nd \rightarrow 1s} \left(x_{nd} - \frac{5}{3} x_{np} \right)$$

RESULTS: STATE OF THE GAS

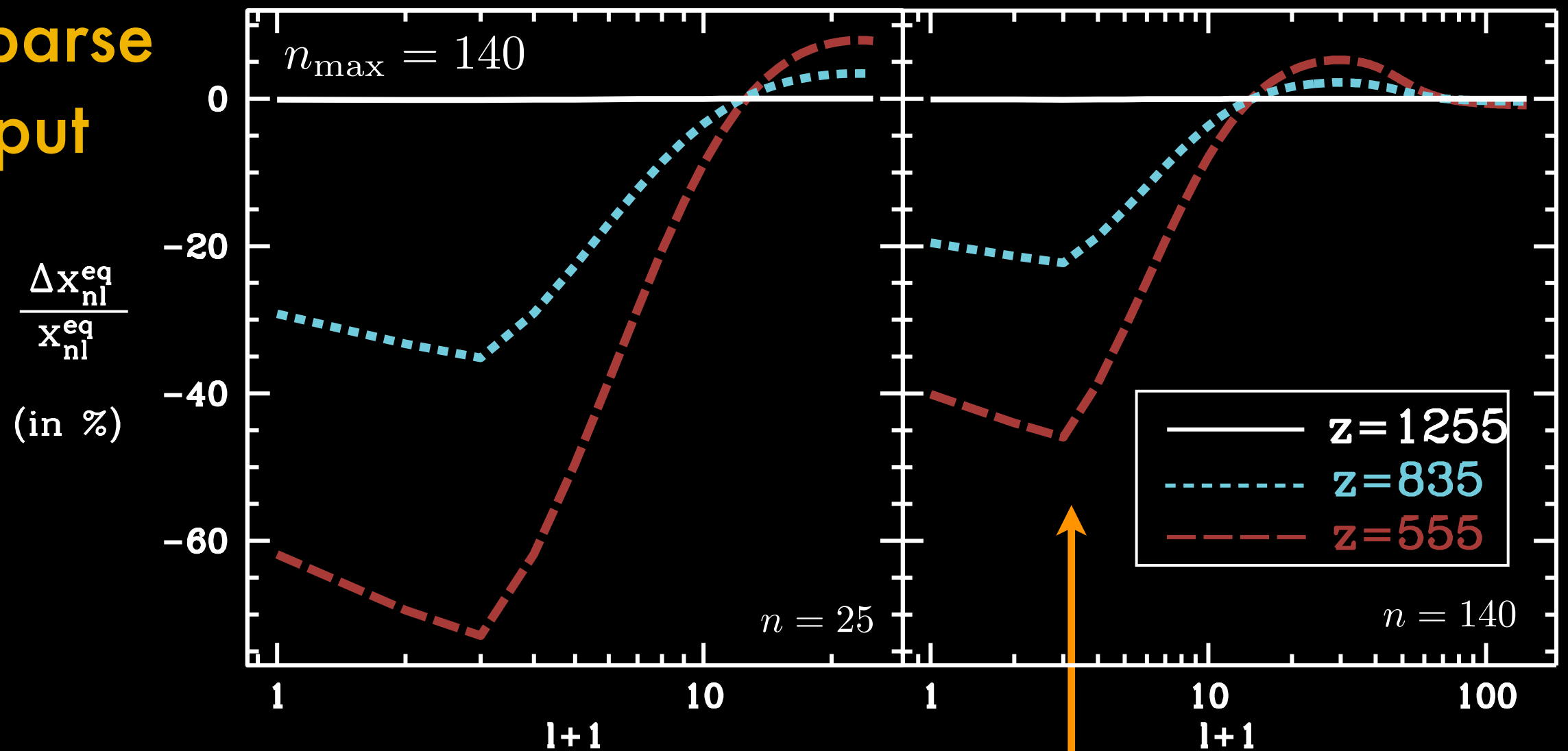
DEVIATIONS FROM BOLTZMANN EQ: l -SUBSTATES

RecSparse
output



DEVIATIONS FROM BOLTZMANN EQ: l -SUBSTATES

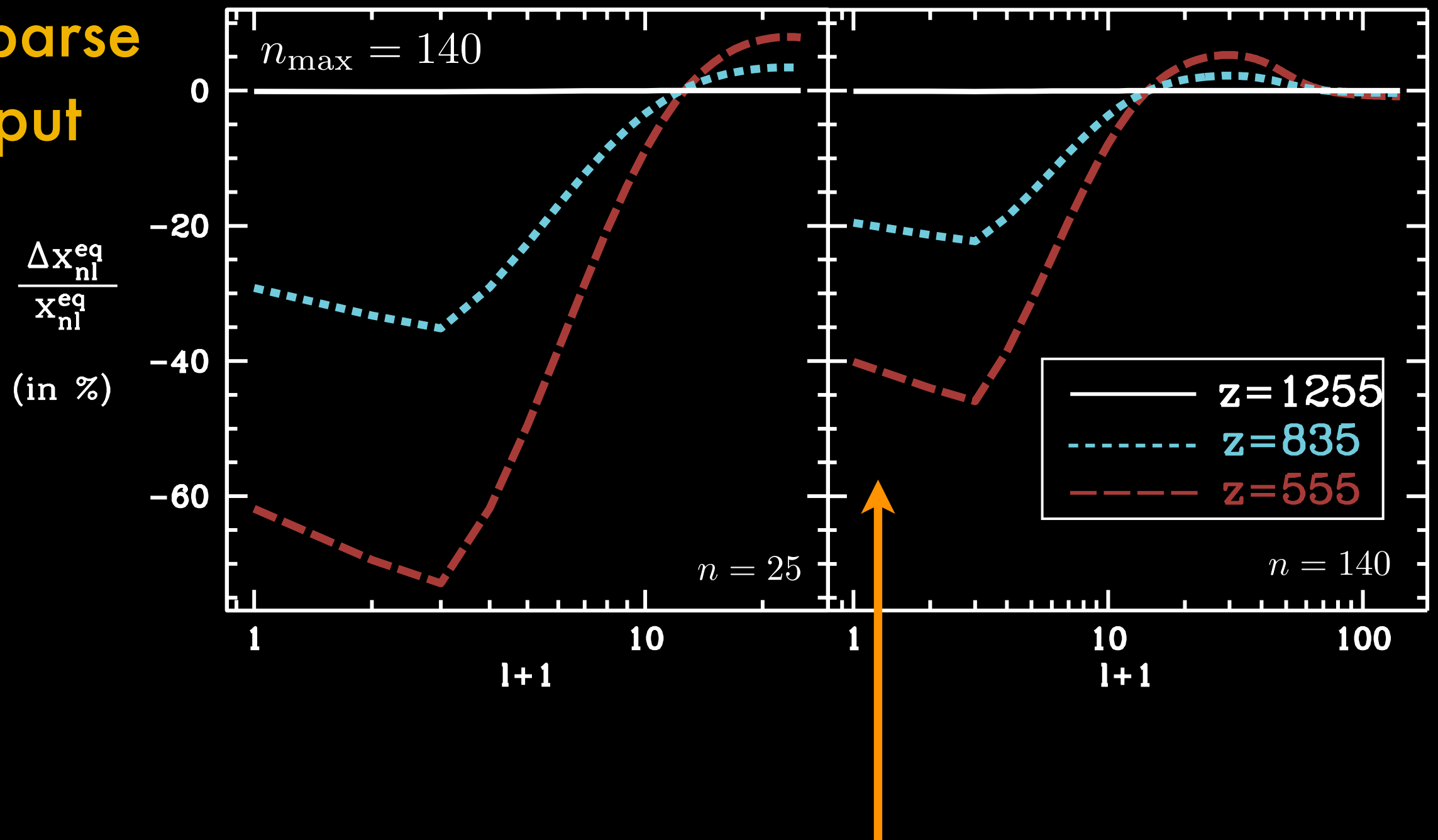
RecSparse
output



Lower l states can easily cascade down,
and are relatively under-populated

DEVIATIONS FROM BOLTZMANN EQ: l -SUBSTATES

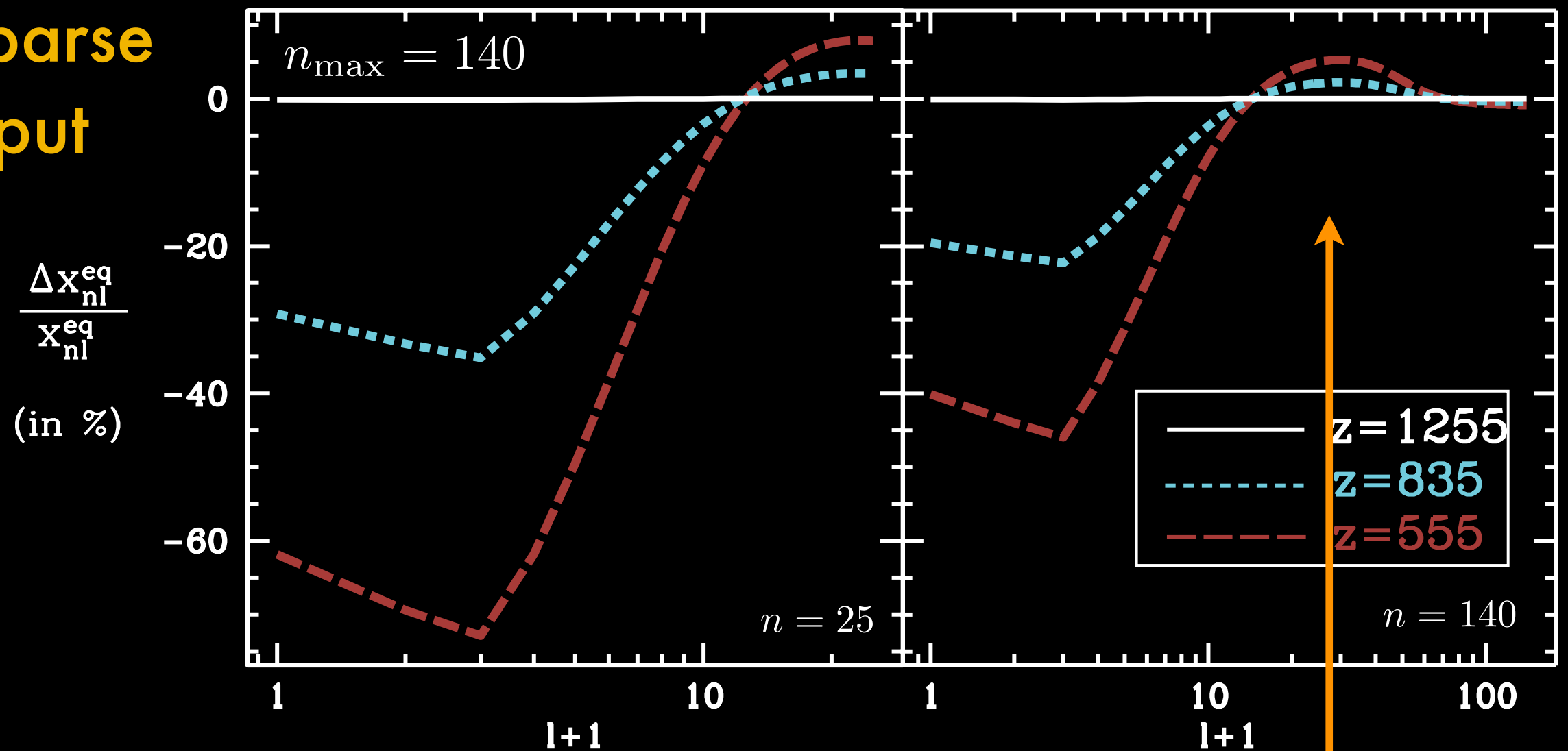
RecSparse
output



$l=0$ can't cascade down, so s states are not as under-populated

DEVIATIONS FROM BOLTZMANN EQ: l -SUBSTATES

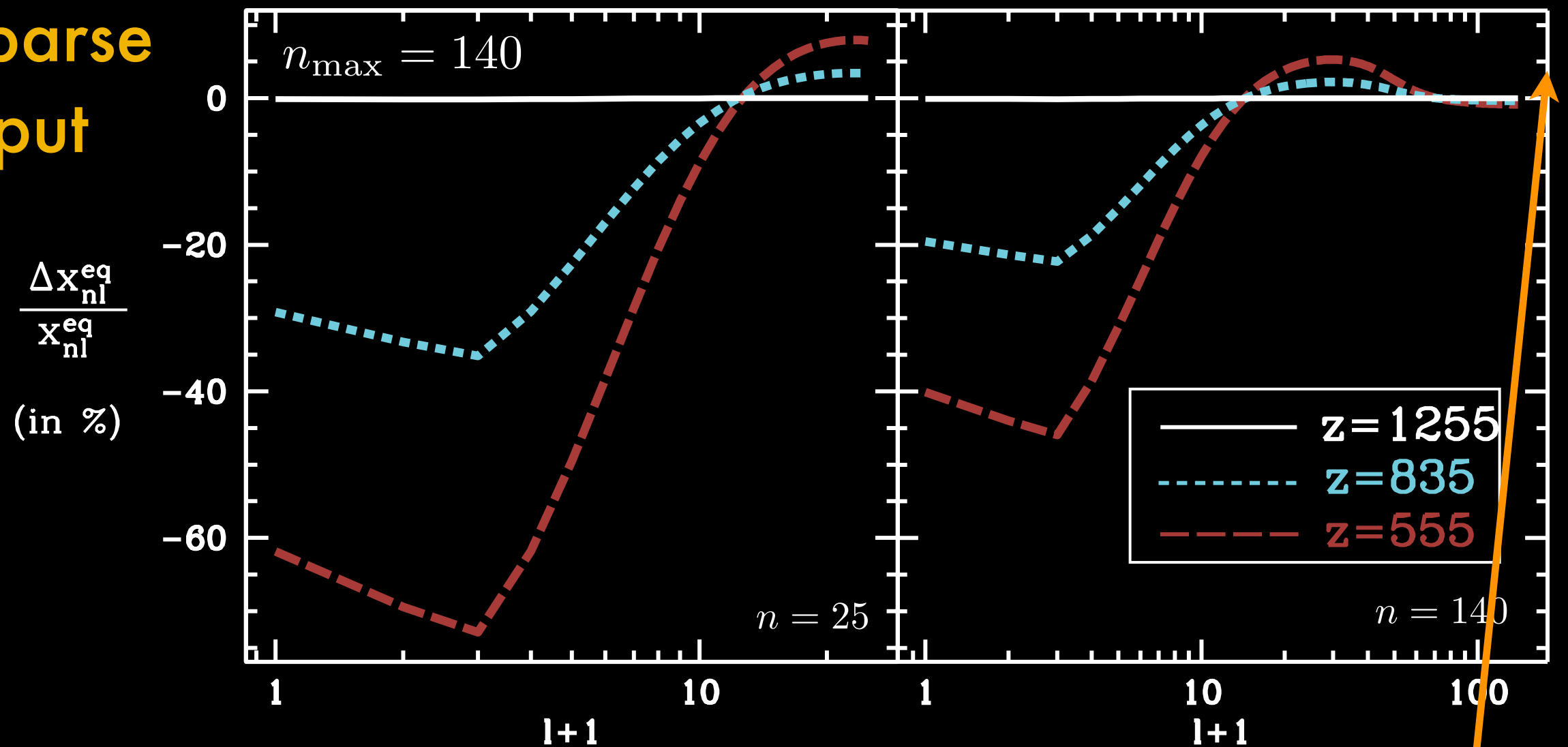
RecSparse
output



Higher l are bottlenecked by $\Delta l = \pm 1$ (over-pop)

DEVIATIONS FROM BOLTZMANN EQ: l -SUBSTATES

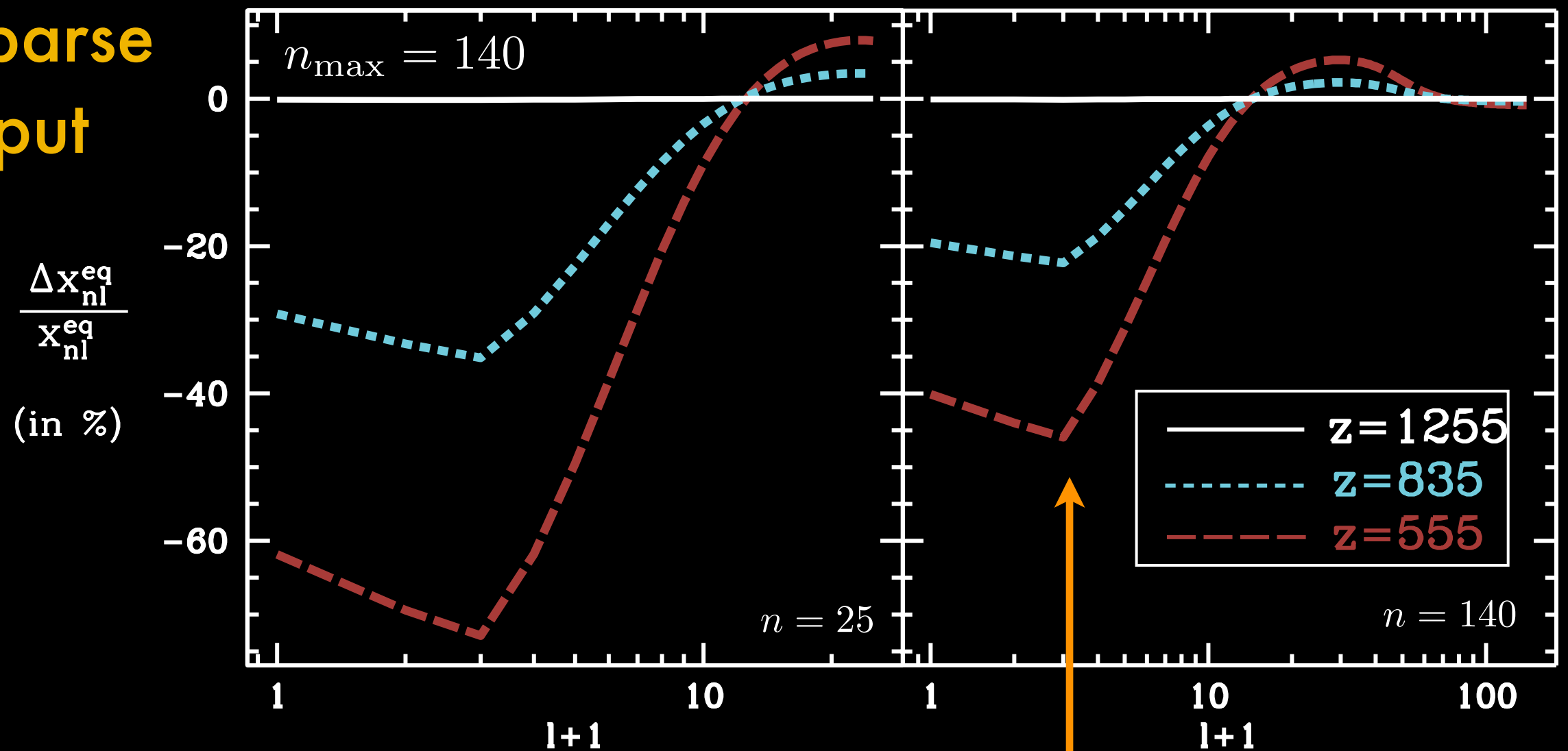
RecSparse
output



Highest l states recombine inefficiently, and are under-populated

DEVIATIONS FROM BOLTZMANN EQ: l -SUBSTATES

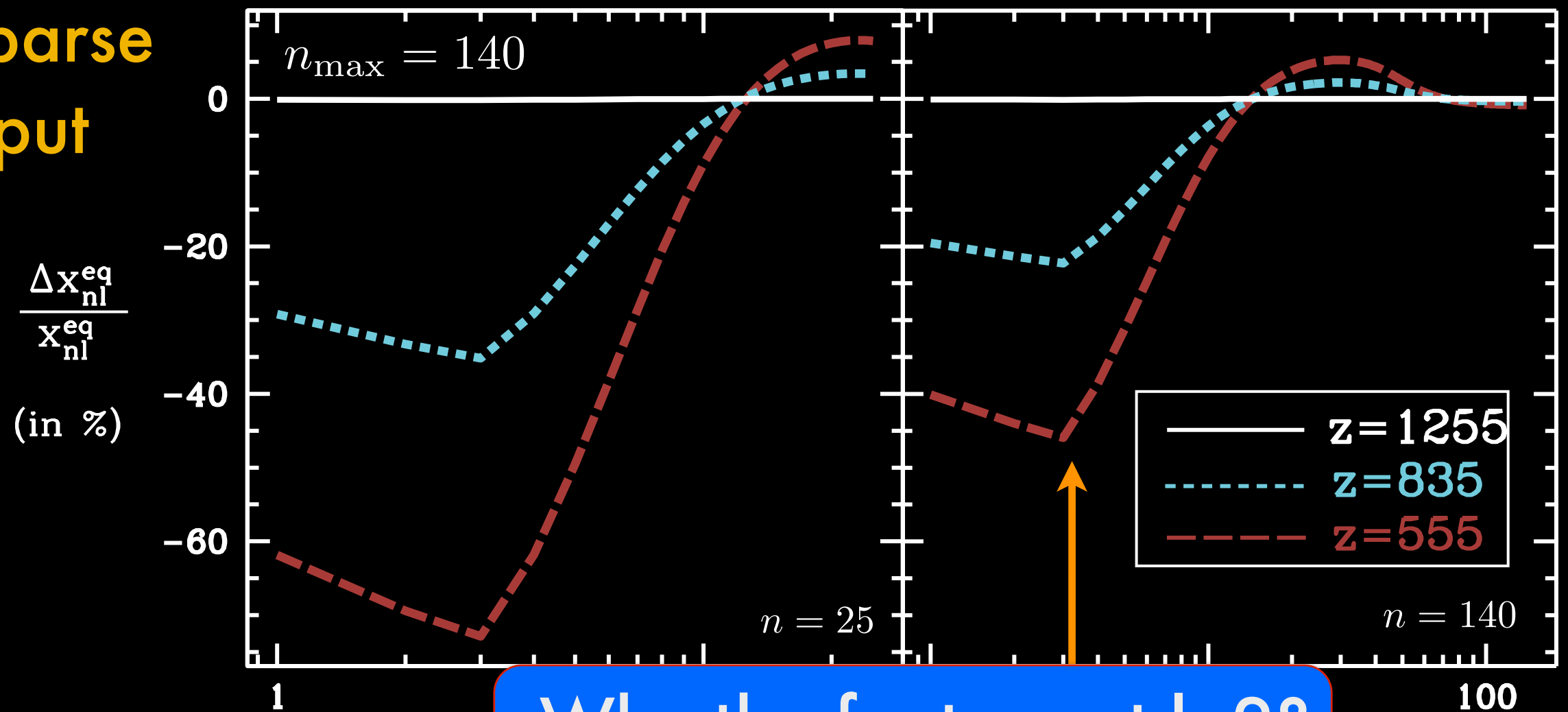
RecSparse
output



l -substates are highly out of Boltzmann eqb'm at late times

DEVIATIONS FROM BOLTZMANN EQ: l -SUBSTATES

RecSparse
output



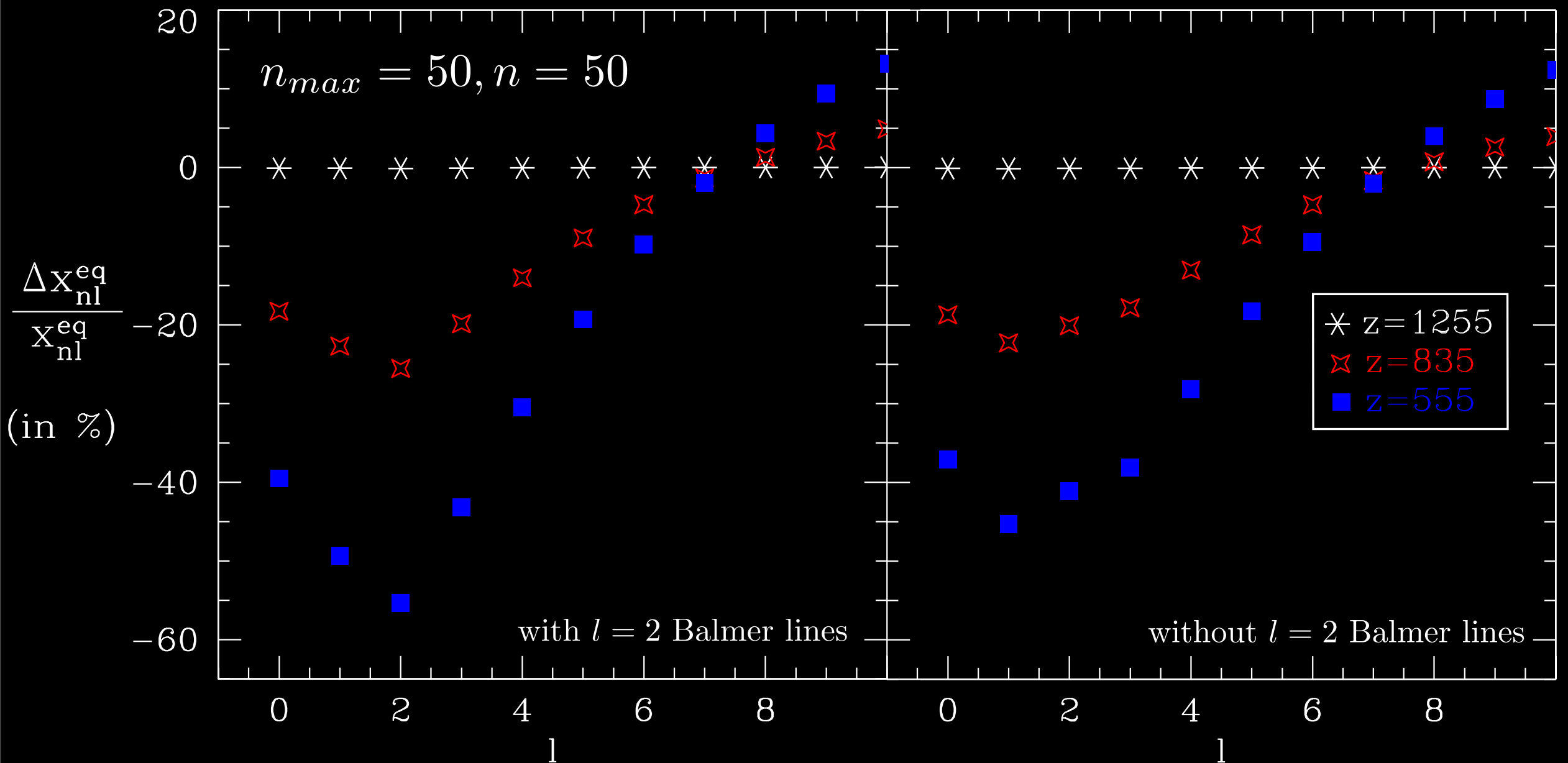
Why the feature at $l=2$?

WHAT IS THE ORIGIN OF THE $l=2$ DIP?

$$A_{nd \rightarrow 2p} > A_{np \rightarrow 2s} > A_{ns \rightarrow 2p}$$

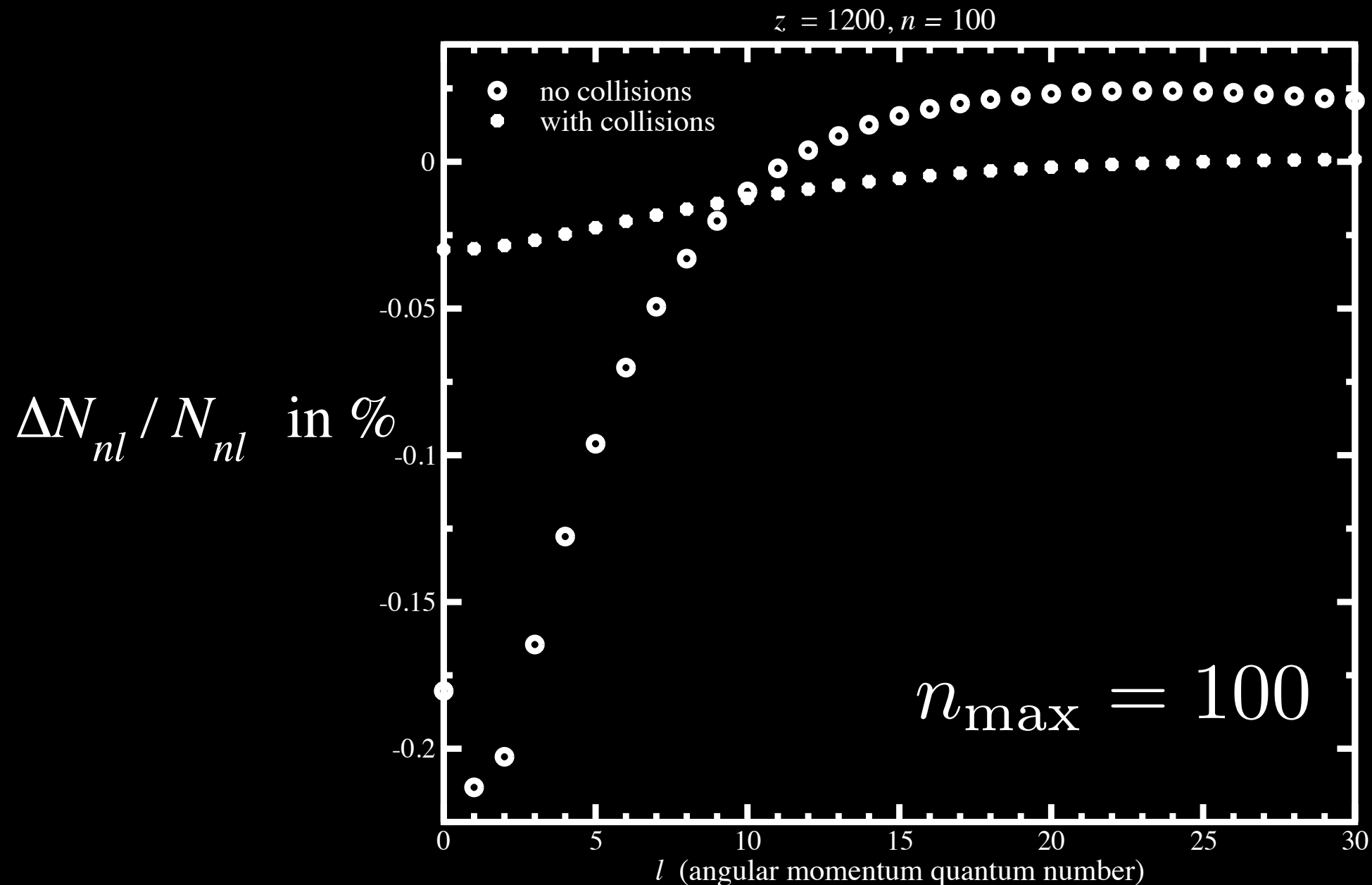
- * $l=2$ depopulates more rapidly than $l=1$ for higher ($n>2$) excited states
- * We can test if this explains the dip at $l=2$ by running the code with these Balmer transitions the blip should move to $l=1$

L-SUBSTATE POPULATIONS, BALMER LINES OFF



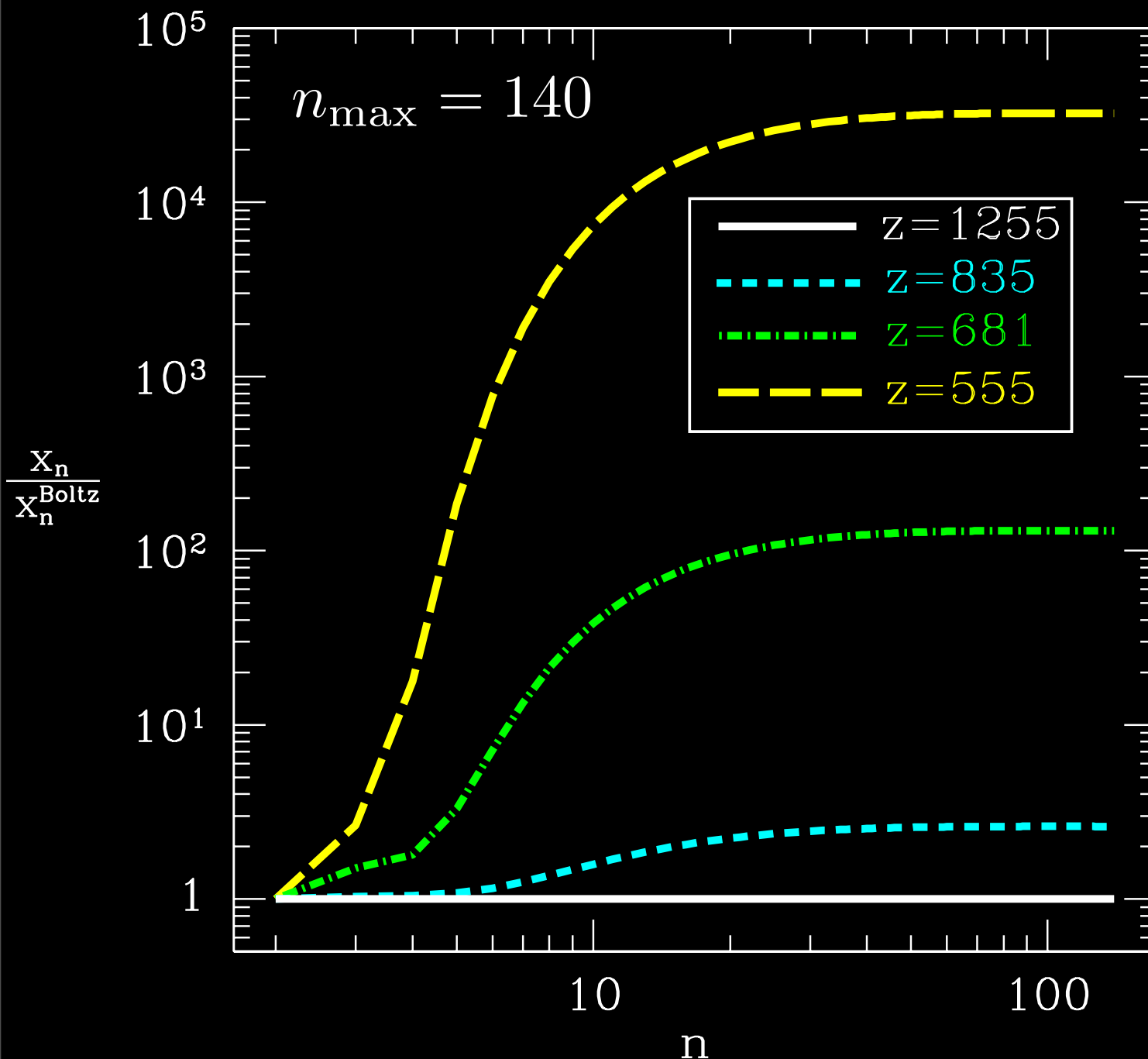
Dip moves as expected when Balmer lines are off!

ATOMIC COLLISIONS



- * l-changing collisions bring l-substates closer to statistical equilibrium (SE) (Chluba, Rubino Martin, Sunyaev 2006)
- * Theoretical collision rates unknown to factors of 2!

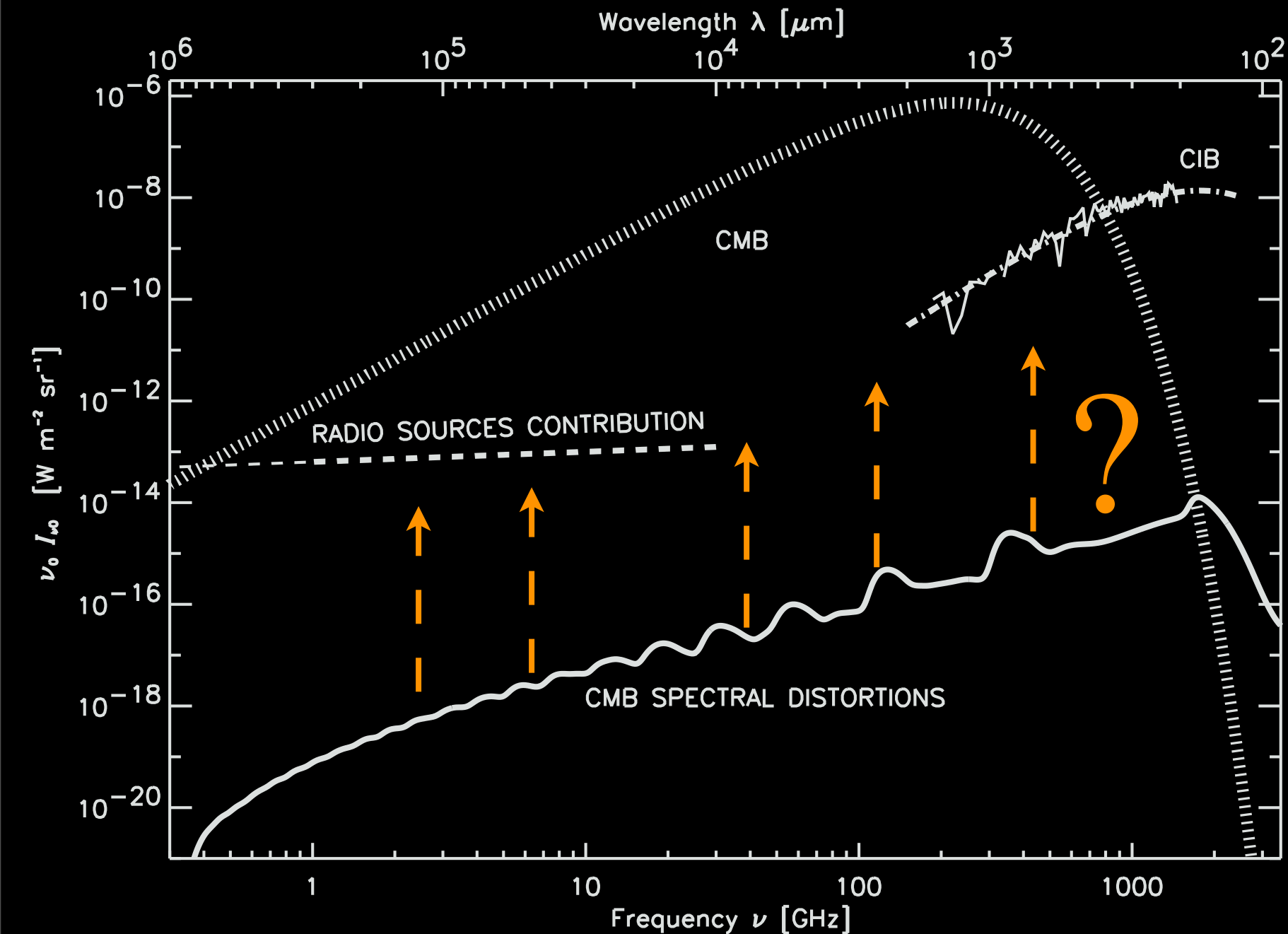
DEVIATIONS FROM BOLTZMANN EQUILIBRIUM: DIFFERENT n -SHELLS



$$\alpha_n n_e > \sum_{n'l}^{n' < n} A_{nn'}^{ll \pm 1}$$

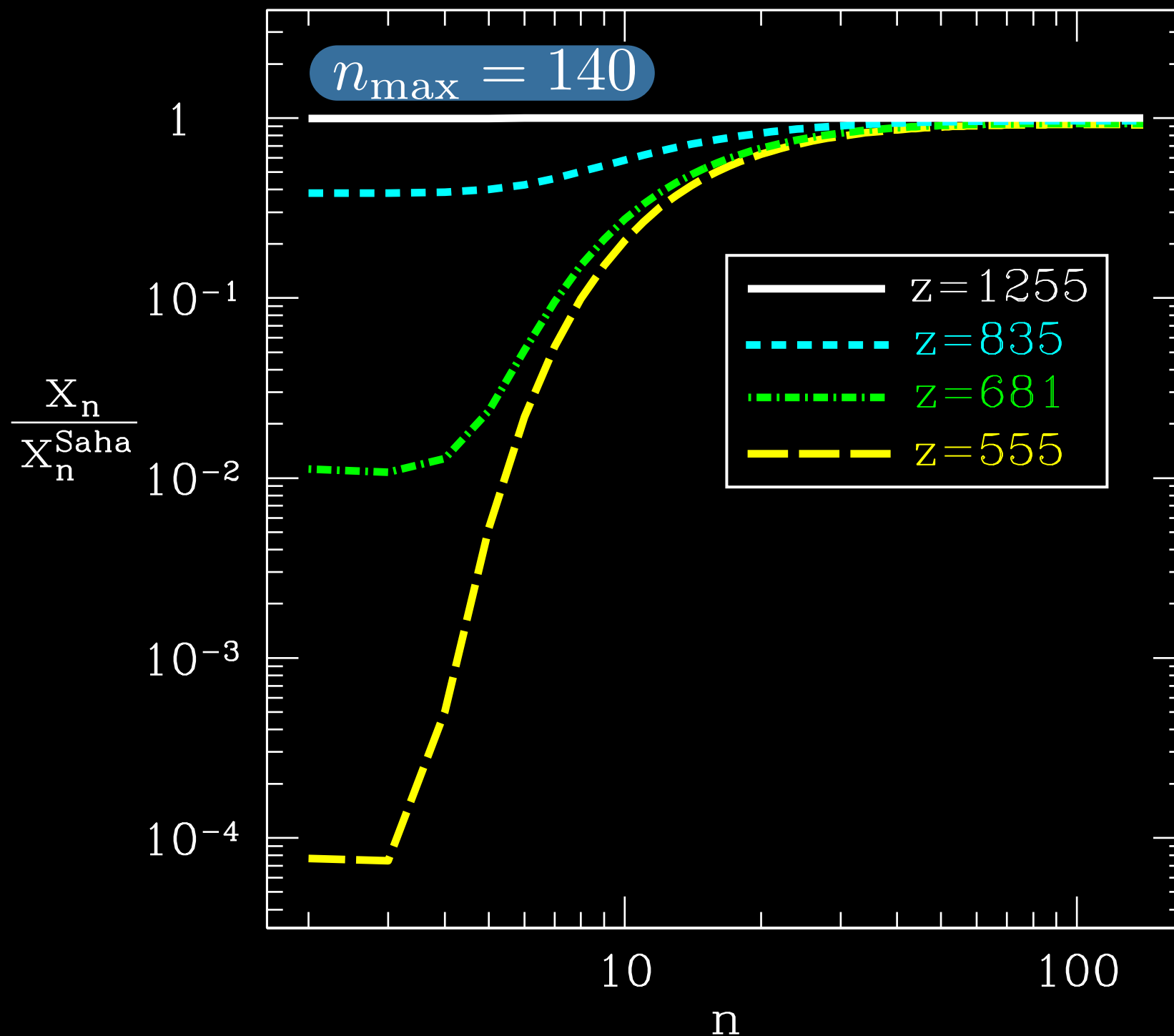
- * No inversion relative to $n=2$ (just over-population)
- * Population inversion seen between some excited states: Does radiation stay coherent? Does recombination make?

DEVIATIONS FROM BOLTZMANN EQUILIBRIUM: DIFFERENT n -SHELLS



**Masing could make spectral
distortions detectable!**

DEVIATIONS FROM SAHA EQUILIBRIUM

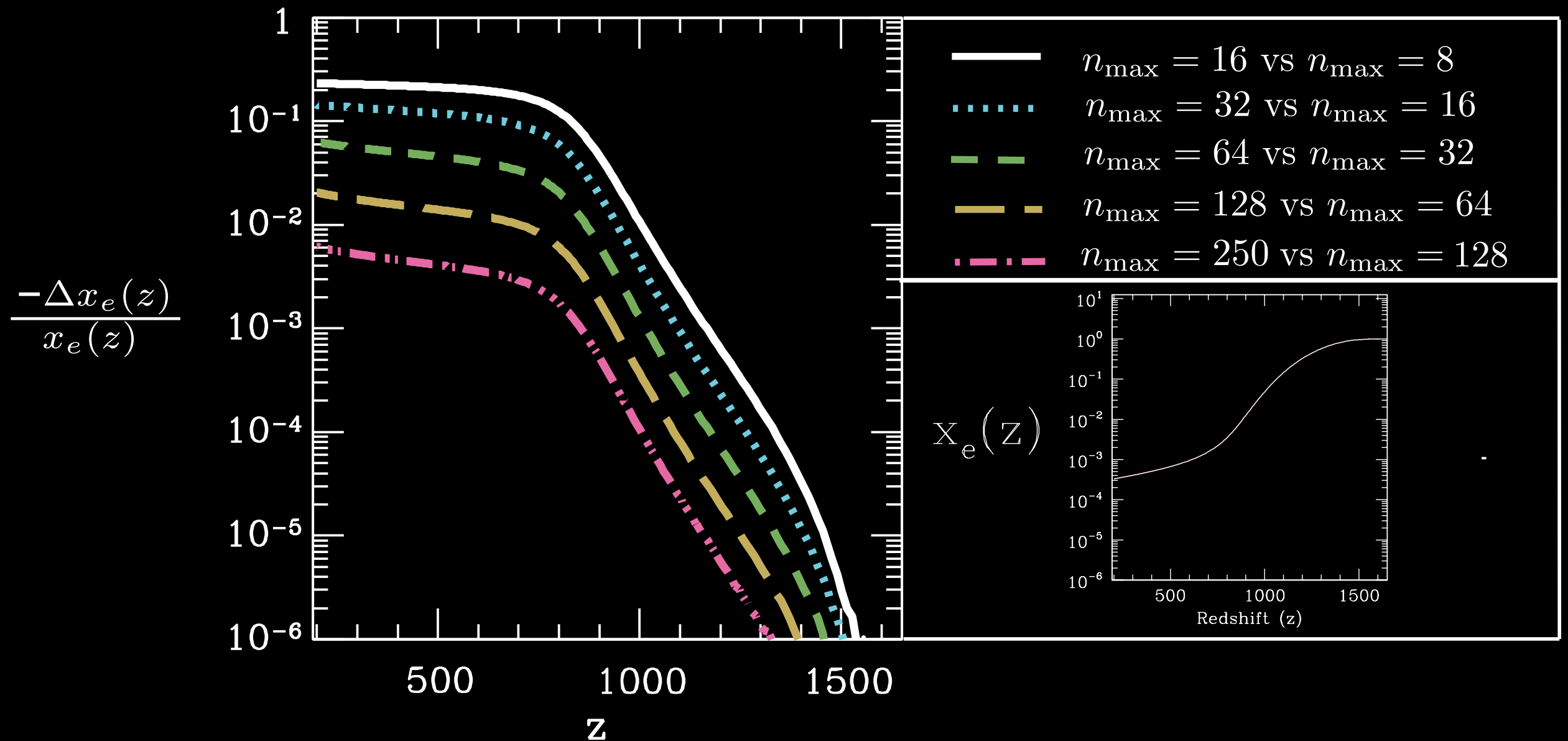


HUGE DEVIATIONS
FROM SAHA EQ!

- ✳ Effect of states with $n > n_{\max}$ could be approximated using asymptotic Einstein coeffs. and Saha eq, but Saha is elusive at high n /late times.
- ✳ At $z=200$, $n_{\max} \sim 1000$ needed, unless collisions included

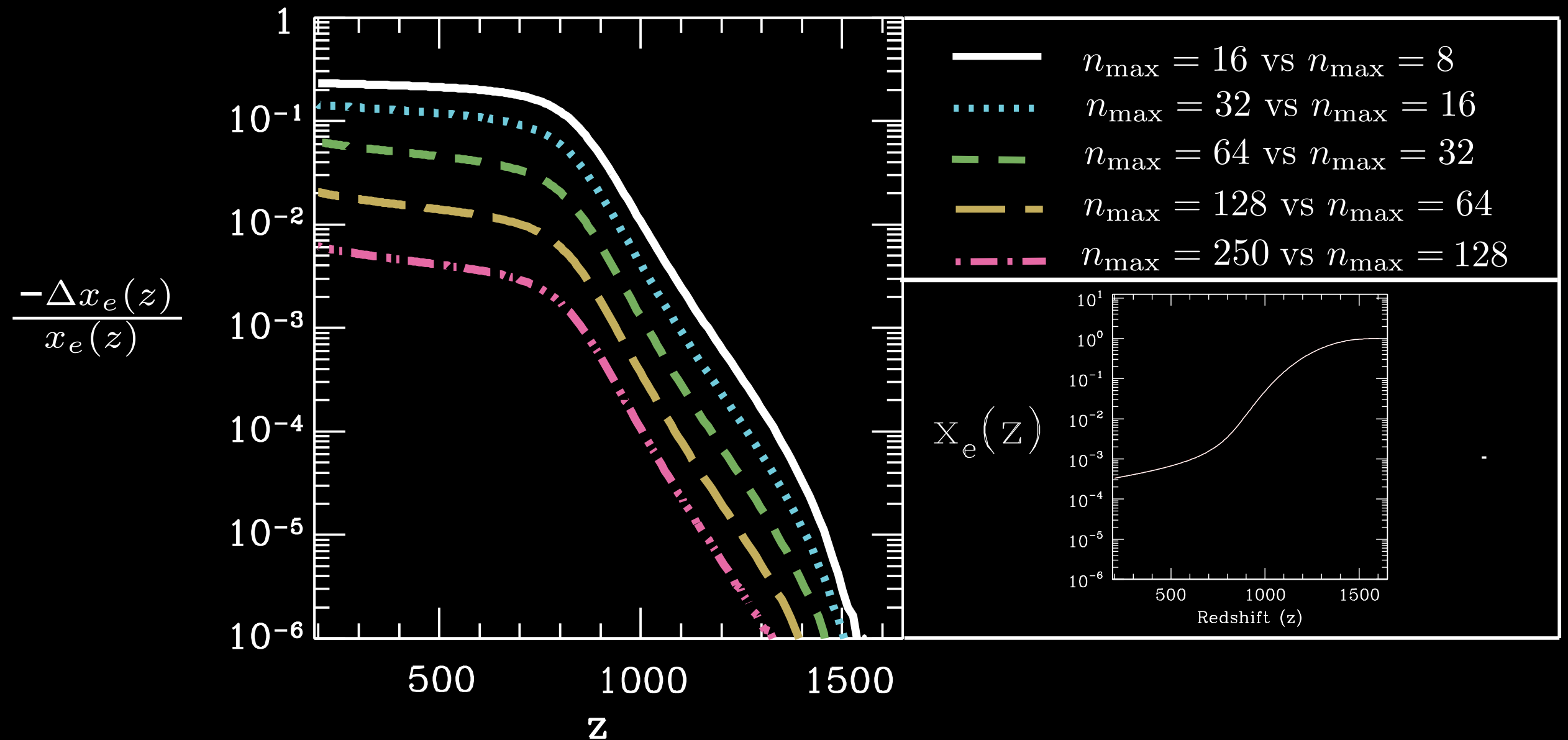
RESULTS: RECOMBINATION HISTORIES

RESULTS: RECOMBINATION HISTORIES INCLUDING HIGH- n



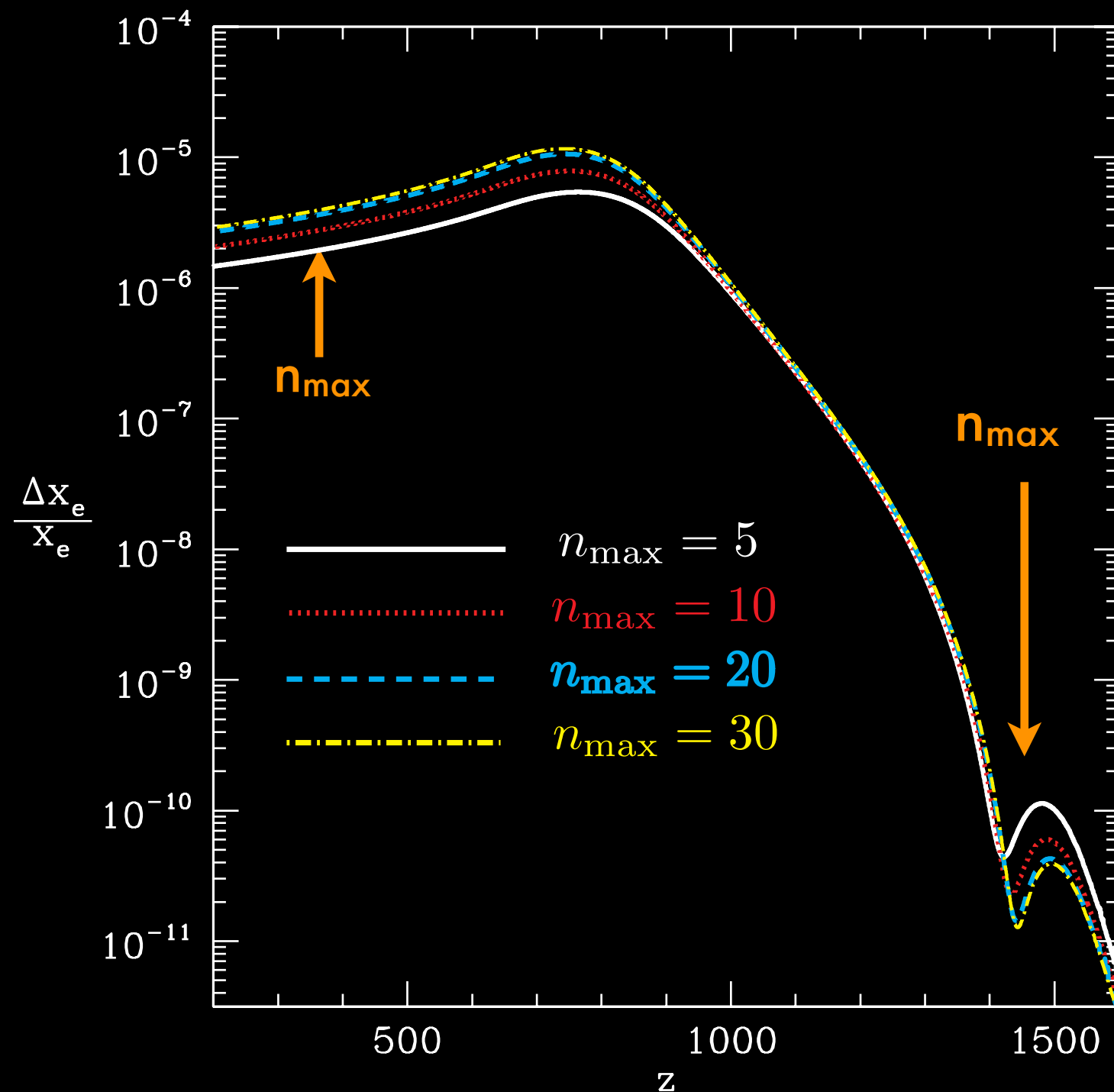
- * $x_e(z)$ falls with increasing $n_{\max} = 10 \rightarrow 250$, as expected.
- * Rec Rate > downward BB Rate > Ionization, upward BB rate
- * For $n_{\max} = 100$, code computes in only 2 hours

RESULTS: RECOMBINATION HISTORIES INCLUDING HIGH- n



- * Relative convergence is not the same thing as absolute convergence: Want to see Saha asymptote and impose well-motivated cutoff! Collisions could help
- * These are lower limits to the actual error
- * $n_{\max}=300$ just completed

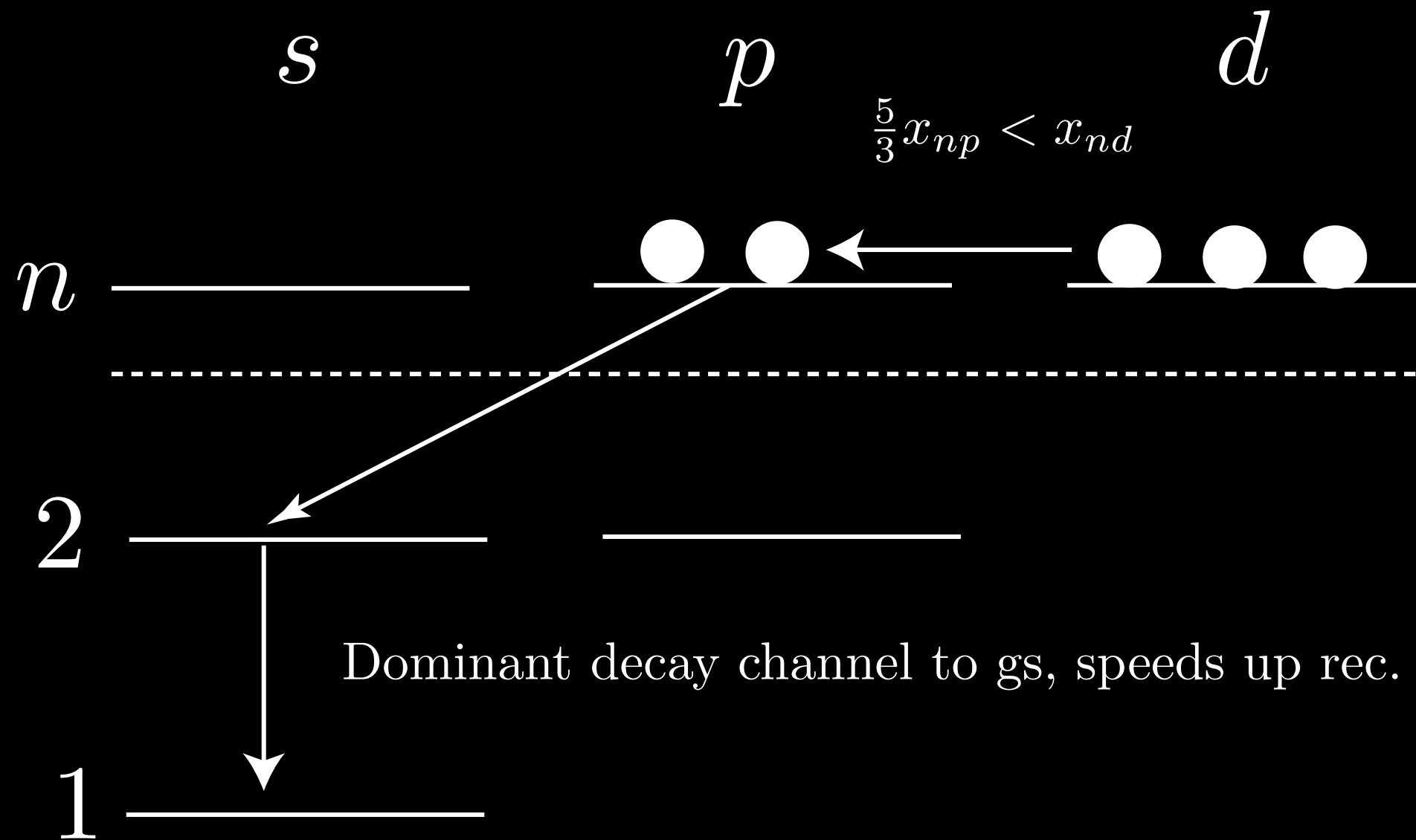
RESULTS: RECOMBINATION WITH HYDROGEN



$$\Delta x_e \equiv x_e|_{\text{no } E2 \text{ transitions}} - x_e|_{\text{with } E2 \text{ transitions}}$$

Negligible for Planck!

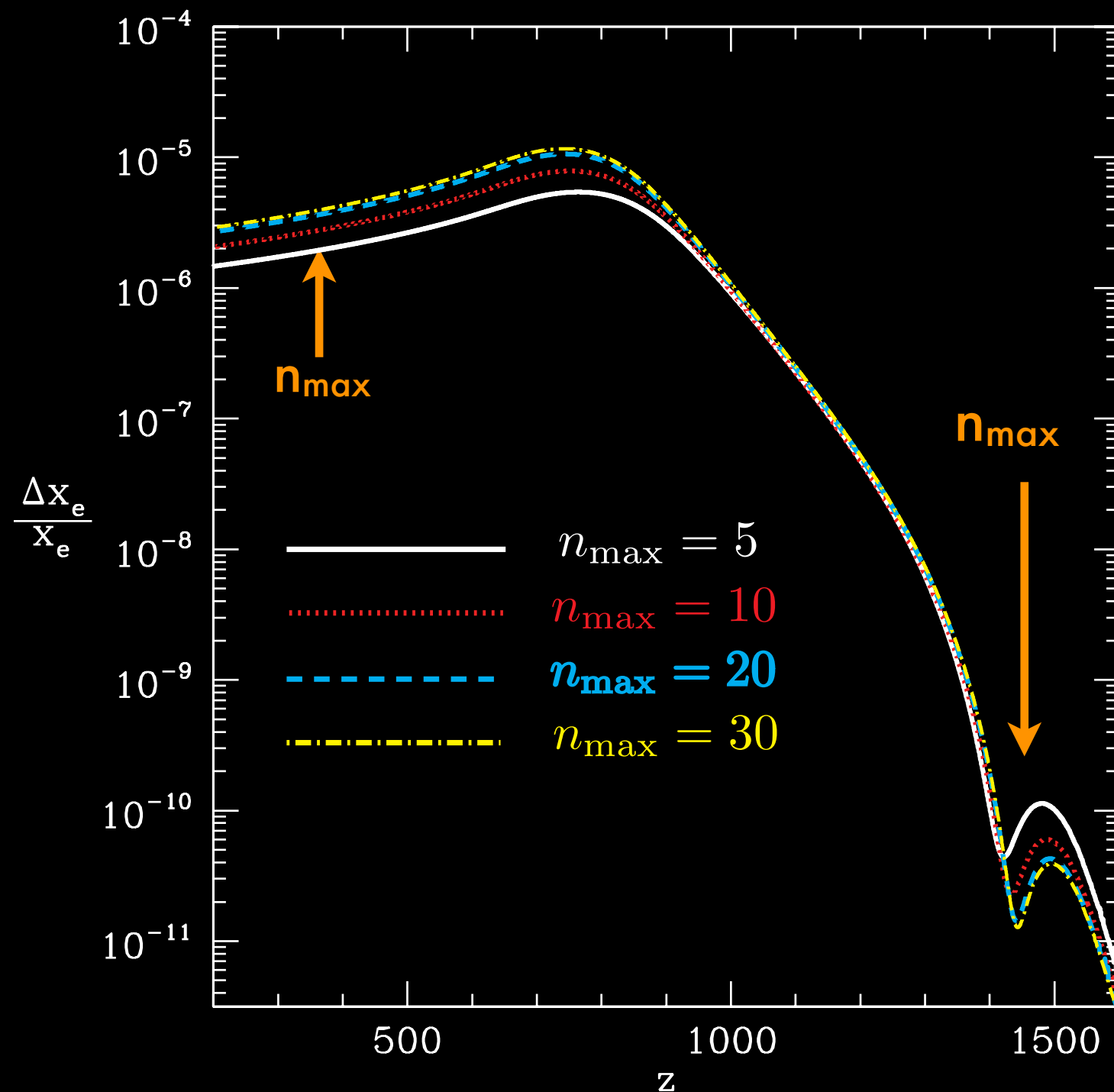
RESULTS: RECOMBINATION WITH HYDROGEN



$$R_{nd \rightarrow np}^{\text{quad}} = A_{nd \rightarrow 1s} \left(x_{nd} - \frac{5}{3}x_{np} \right)$$

$n < 5$, early times

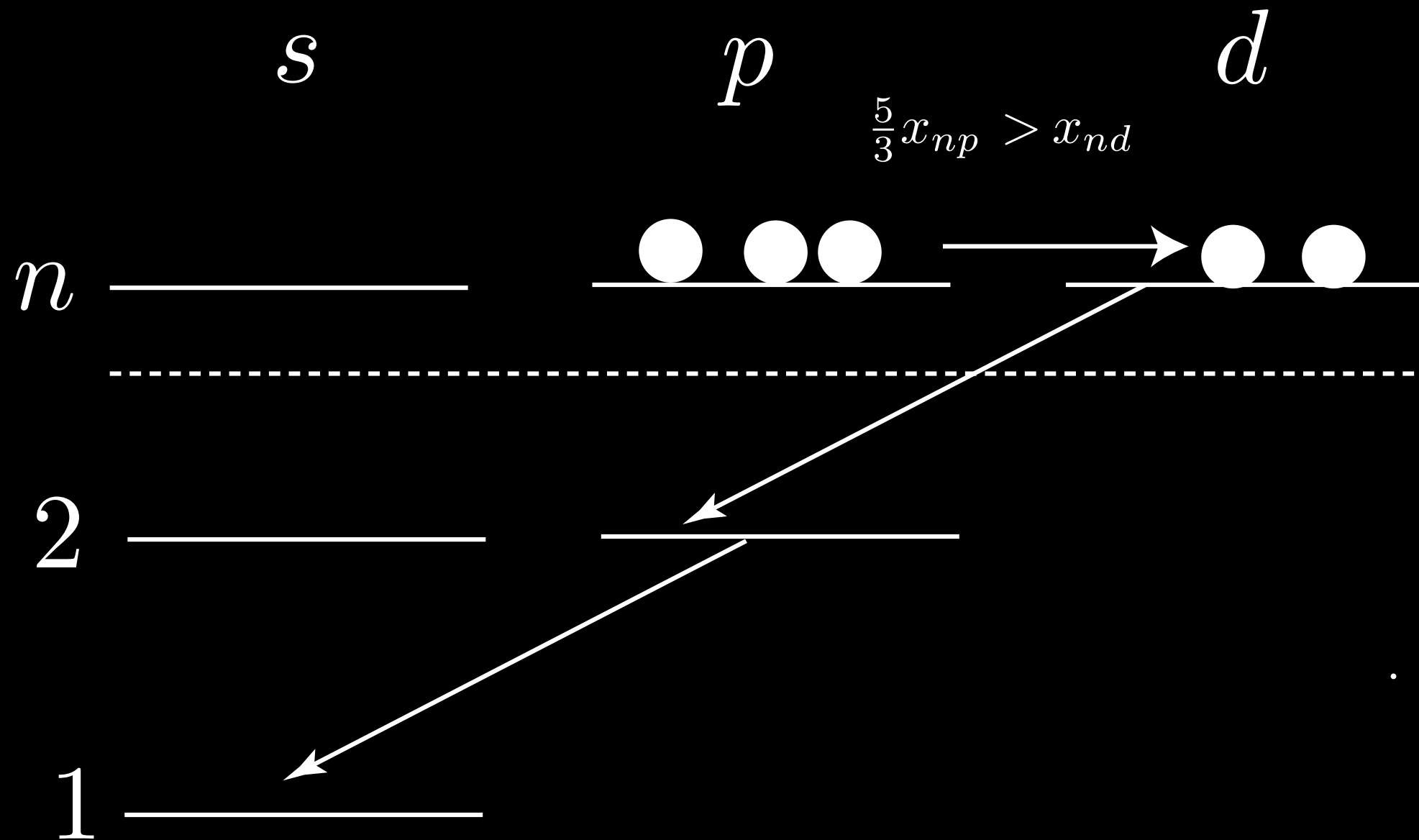
RESULTS: RECOMBINATION WITH HYDROGEN



$$\Delta x_e \equiv x_e|_{\text{no } E2 \text{ transitions}} - x_e|_{\text{with } E2 \text{ transitions}}$$

Negligible for Planck!

RESULTS: RECOMBINATION WITH HYDROGEN

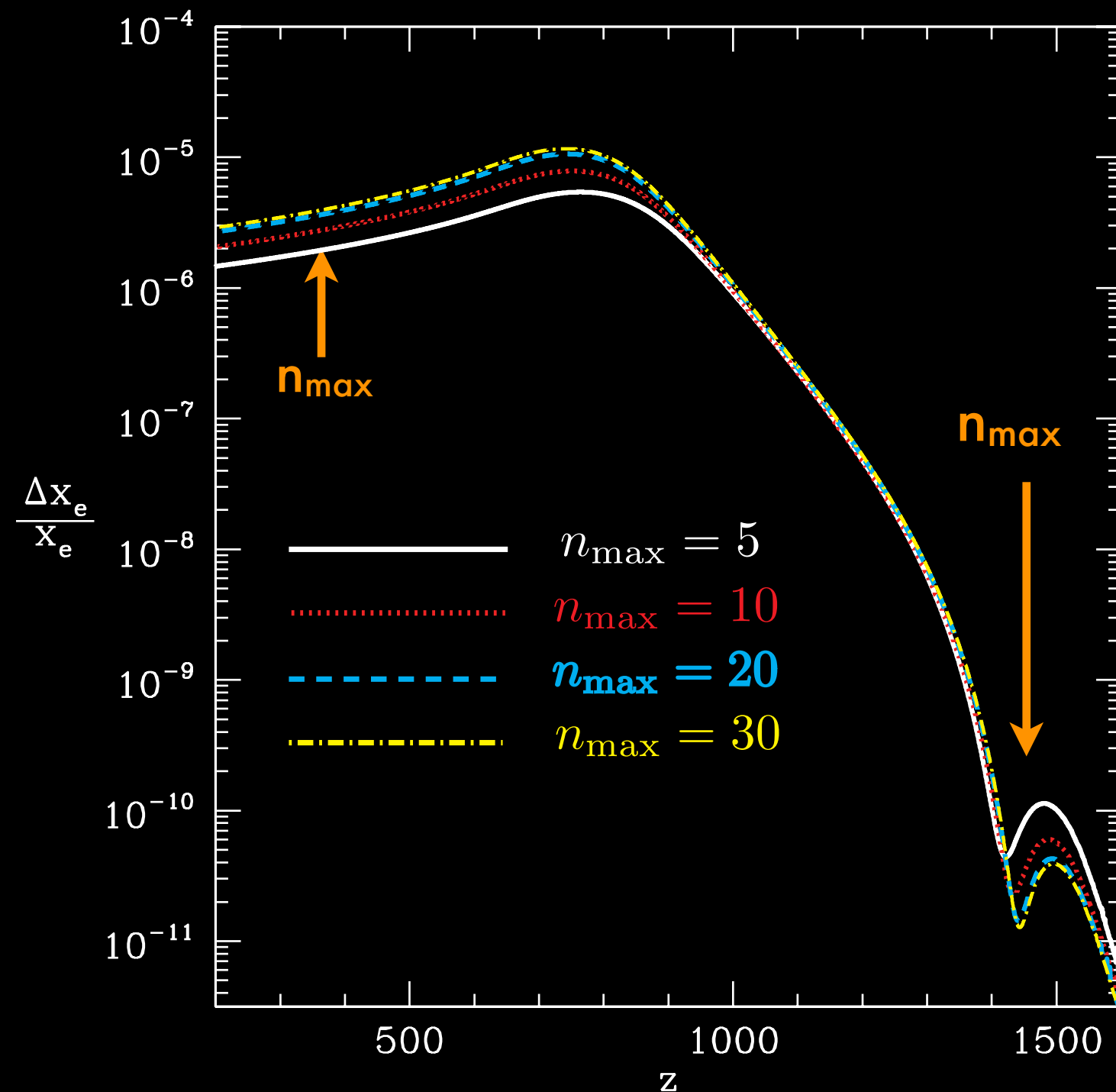


Sub-Dominant decay channel to gs, slows rec down rel. to $n < 5$

$$R_{nd \rightarrow np}^{\text{quad}} = A_{nd \rightarrow 1s} \left(x_{nd} - \frac{5}{3}x_{np} \right)$$

$n \geq 5$, early times

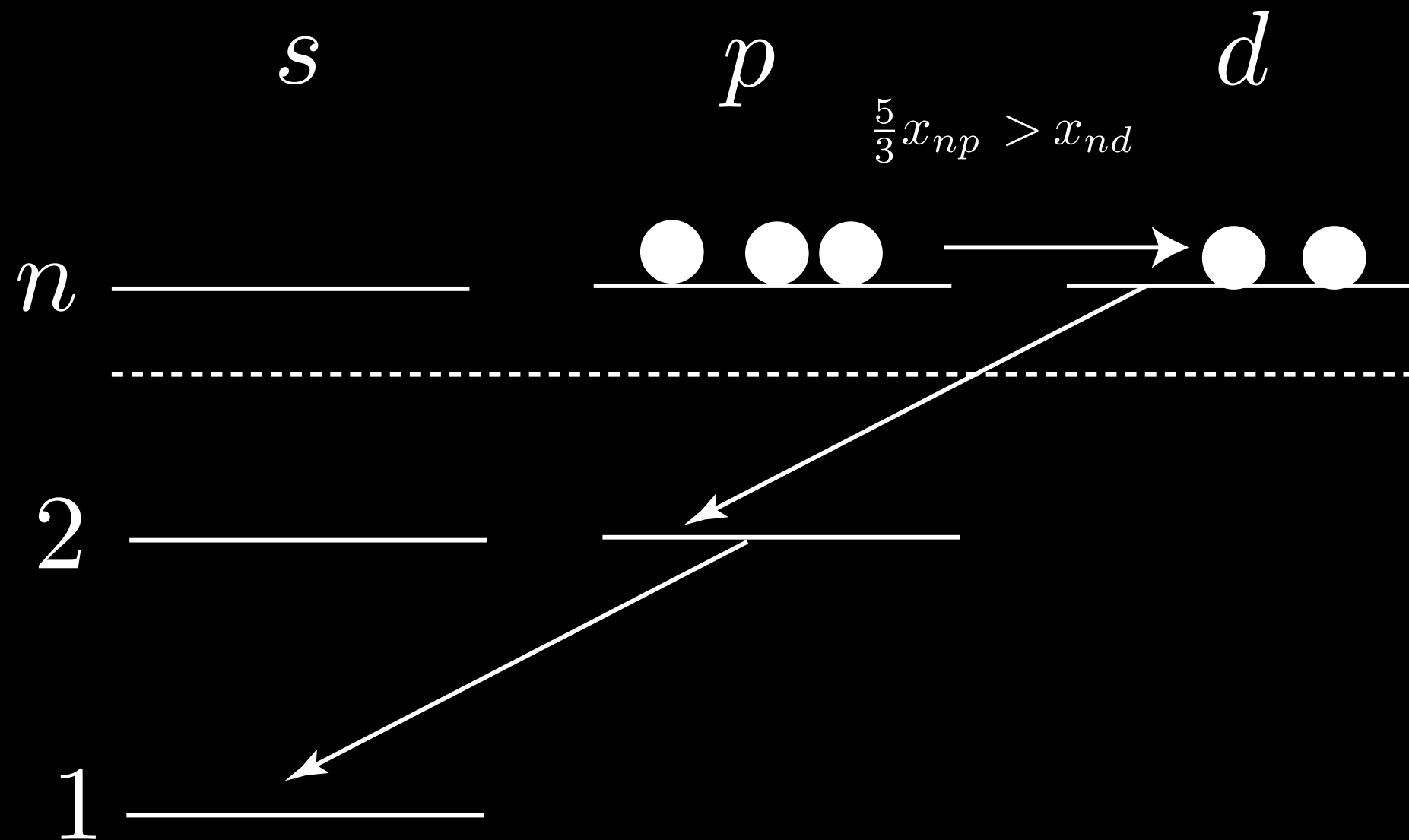
RESULTS: RECOMBINATION WITH HYDROGEN



$$\Delta x_e \equiv x_e|_{\text{no } E2 \text{ transitions}} - x_e|_{\text{with } E2 \text{ transitions}}$$

Negligible for Planck!

RESULTS: RECOMBINATION WITH HYDROGEN

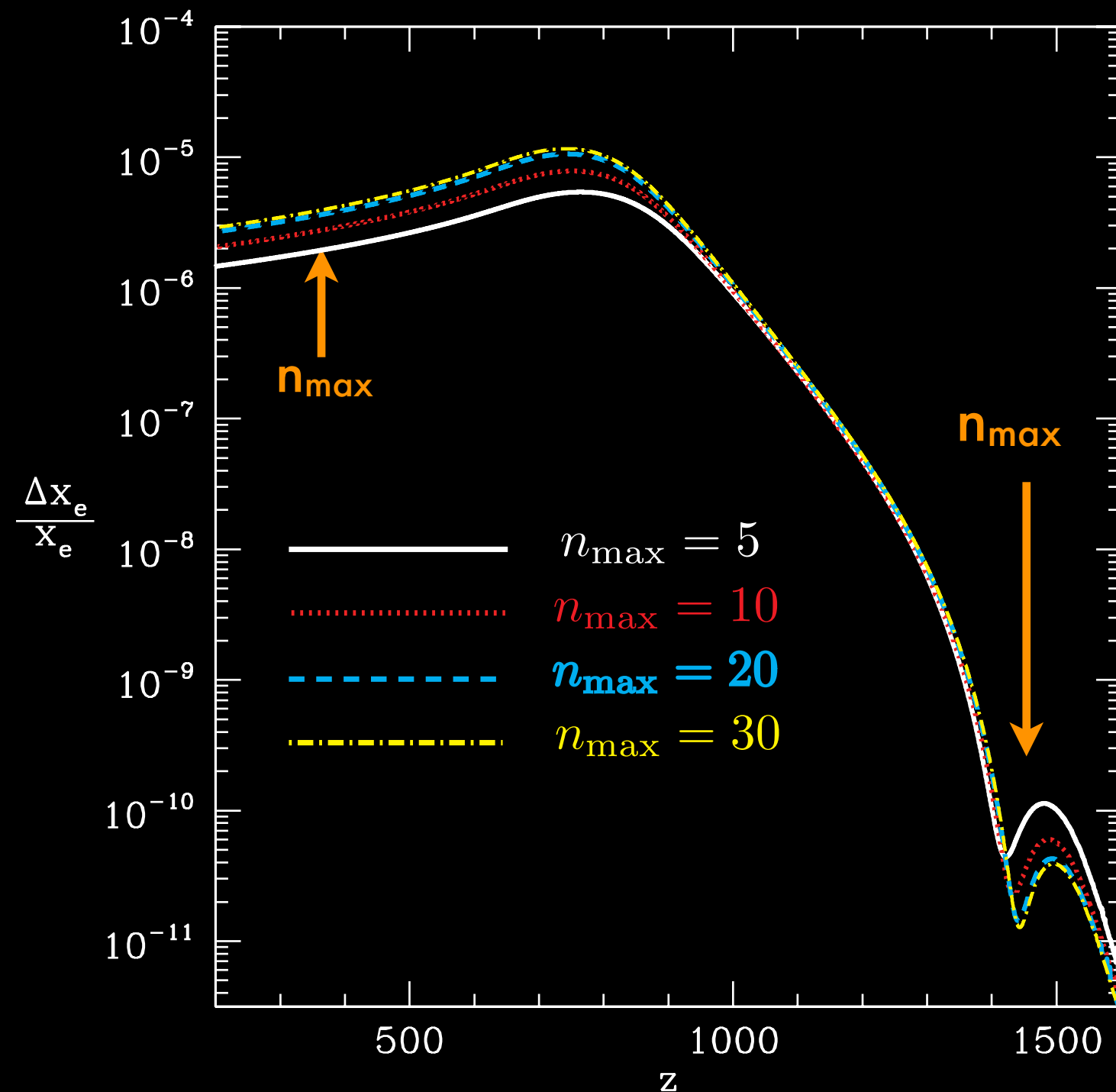


Dominant decay channel to gs, speeds up rec

$$R_{nd \rightarrow np}^{\text{quad}} = A_{nd \rightarrow 1s} \left(x_{nd} - \frac{5}{3}x_{np} \right)$$

All n , late times

RESULTS: RECOMBINATION WITH HYDROGEN

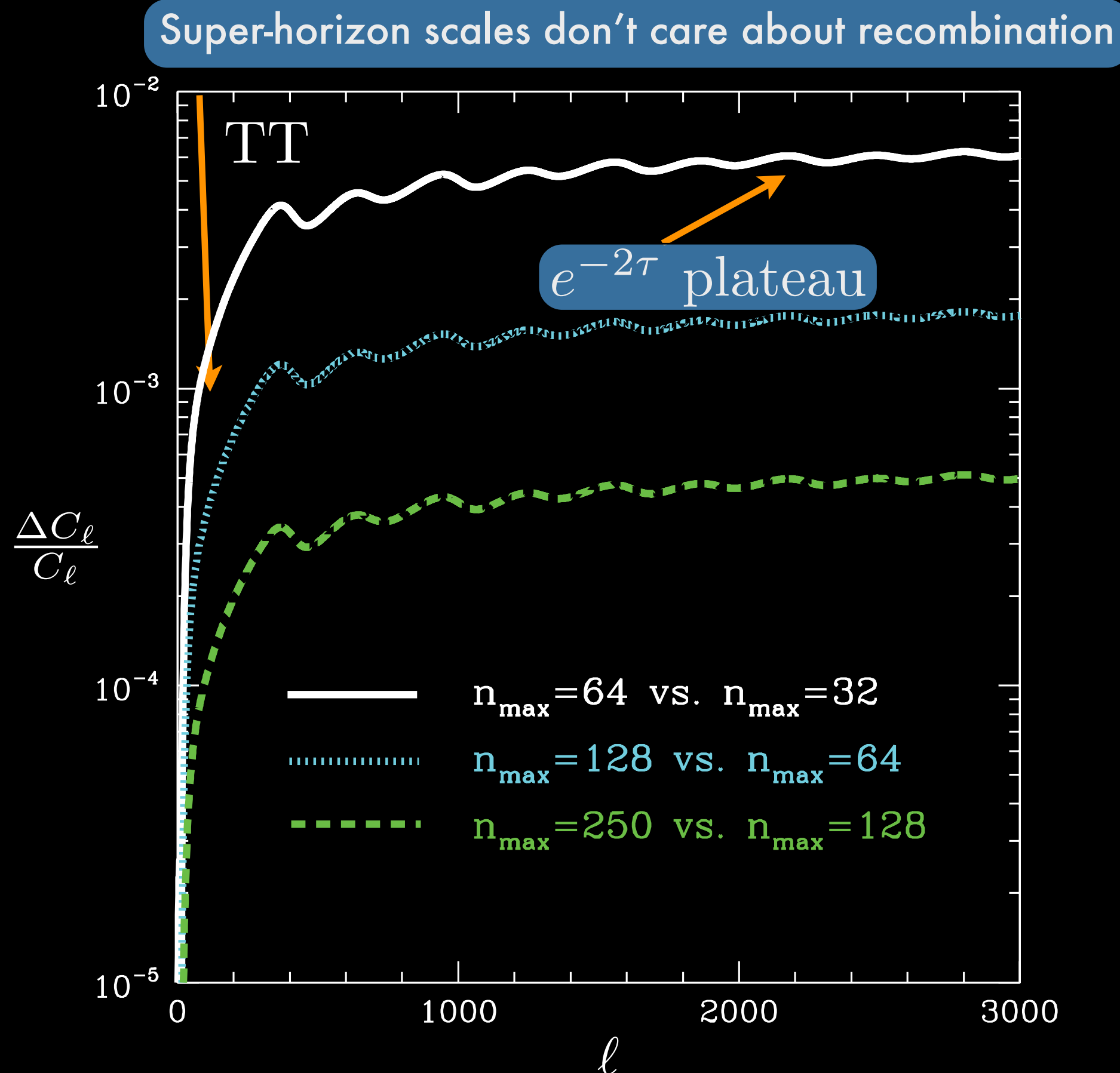


$$\Delta x_e \equiv x_e|_{\text{no } E2 \text{ transitions}} - x_e|_{\text{with } E2 \text{ transitions}}$$

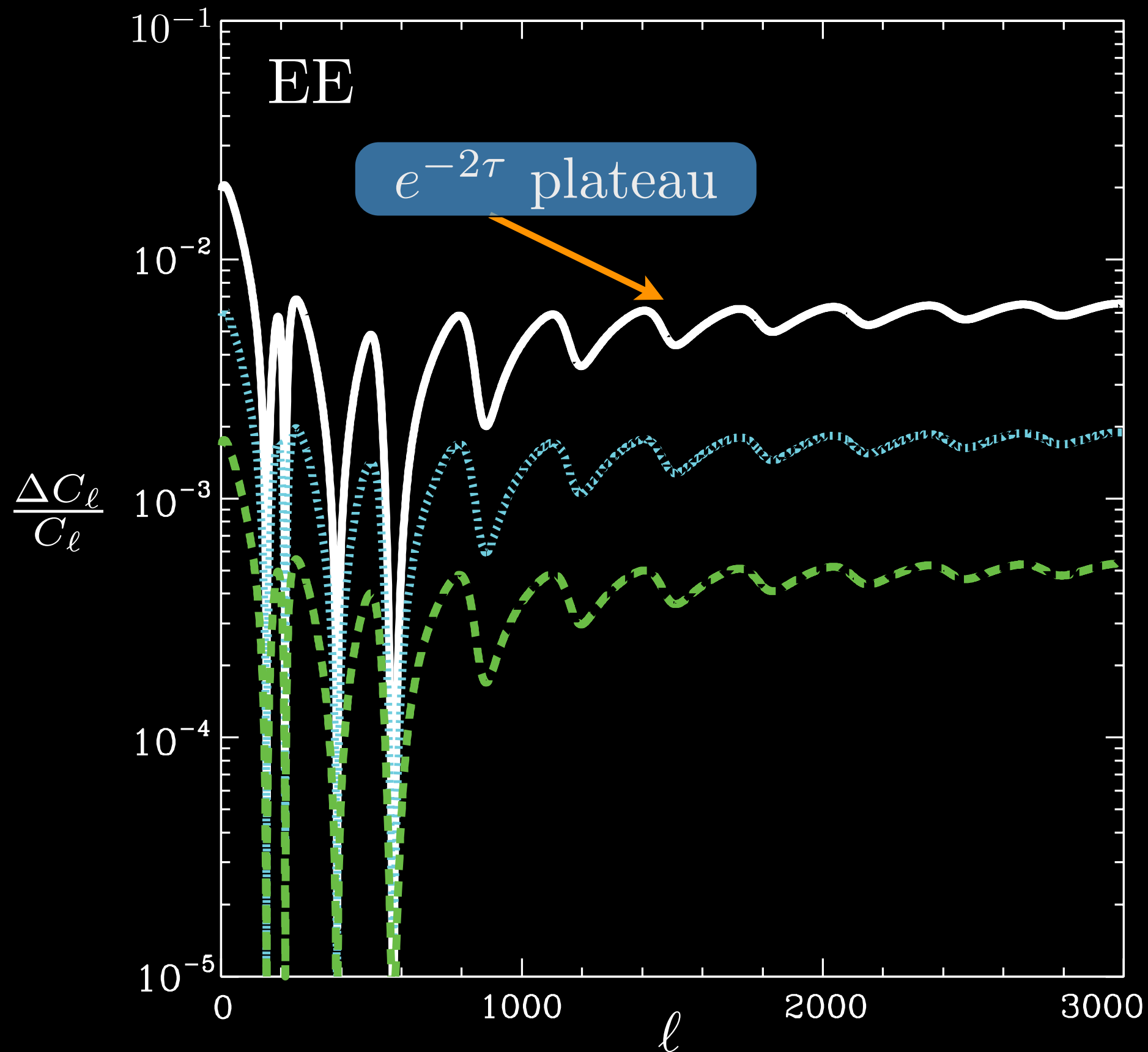
Negligible for Planck!

RESULTS: CMB ANISOTROPIES

RESULTS: TT C_ℓ s WITH HIGH-N STATES

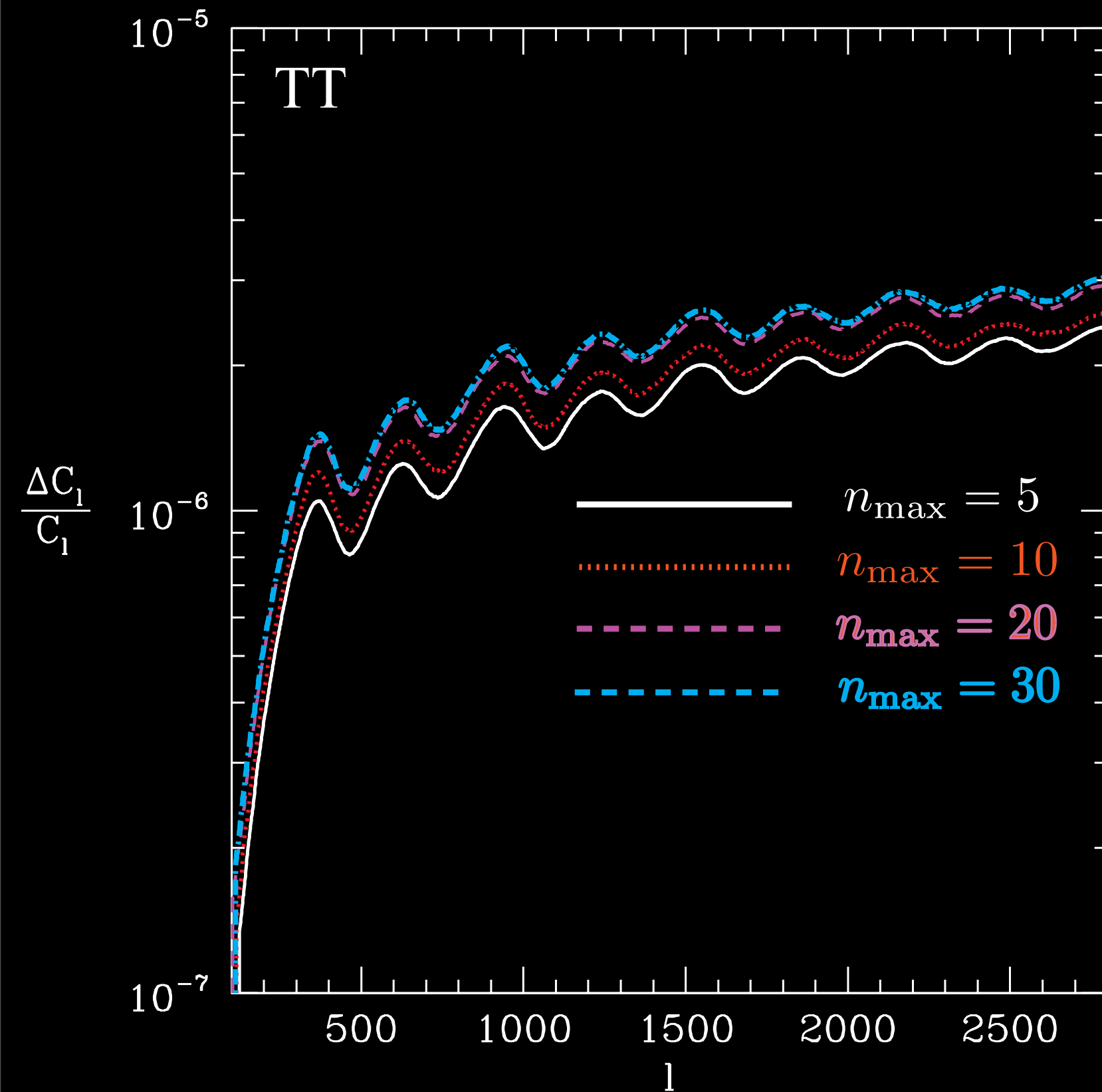


RESULTS: EE C_ℓ s WITH HIGH-N STATES



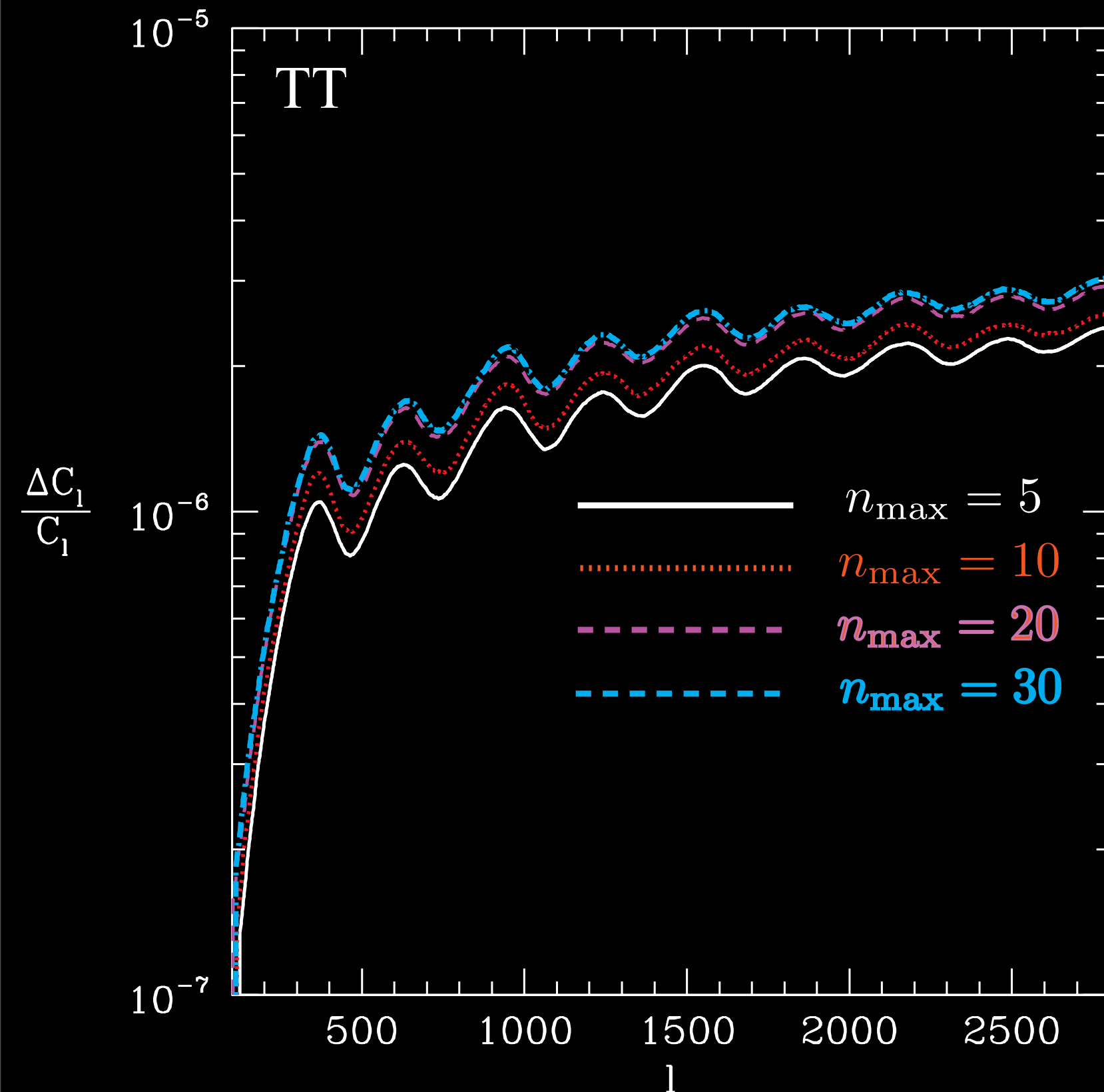
RESULTS: TEMPERATURE (TT) C_l s WITH HYDROGEN QUADRUPOLES,

Bulk of integral from late times, higher $n_{\text{max}} \rightarrow$ lower $x_e \rightarrow$ lower $\tau \rightarrow$ higher $e^{-2\tau} \rightarrow$ higher C_l



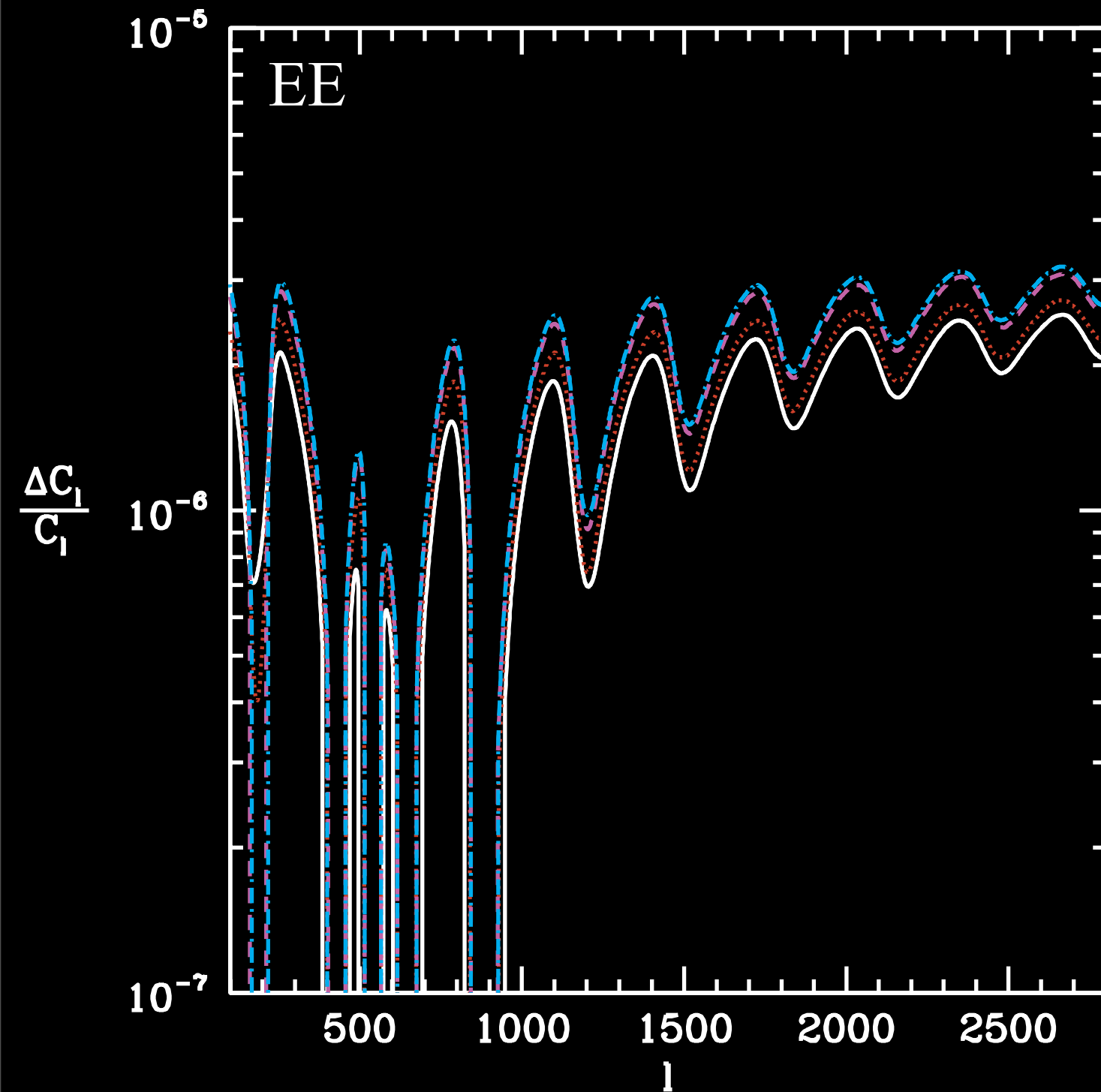
RESULTS: TEMPERATURE (TT) C_l s WITH HYDROGEN QUADRUPOLES,

Bulk of integral from late times, higher $n_{\text{max}} \rightarrow$ lower $x_e \rightarrow$ lower $\tau \rightarrow$ higher $e^{-2\tau} \rightarrow$ higher C_l



Overall effect is negligible for CMB experiments!

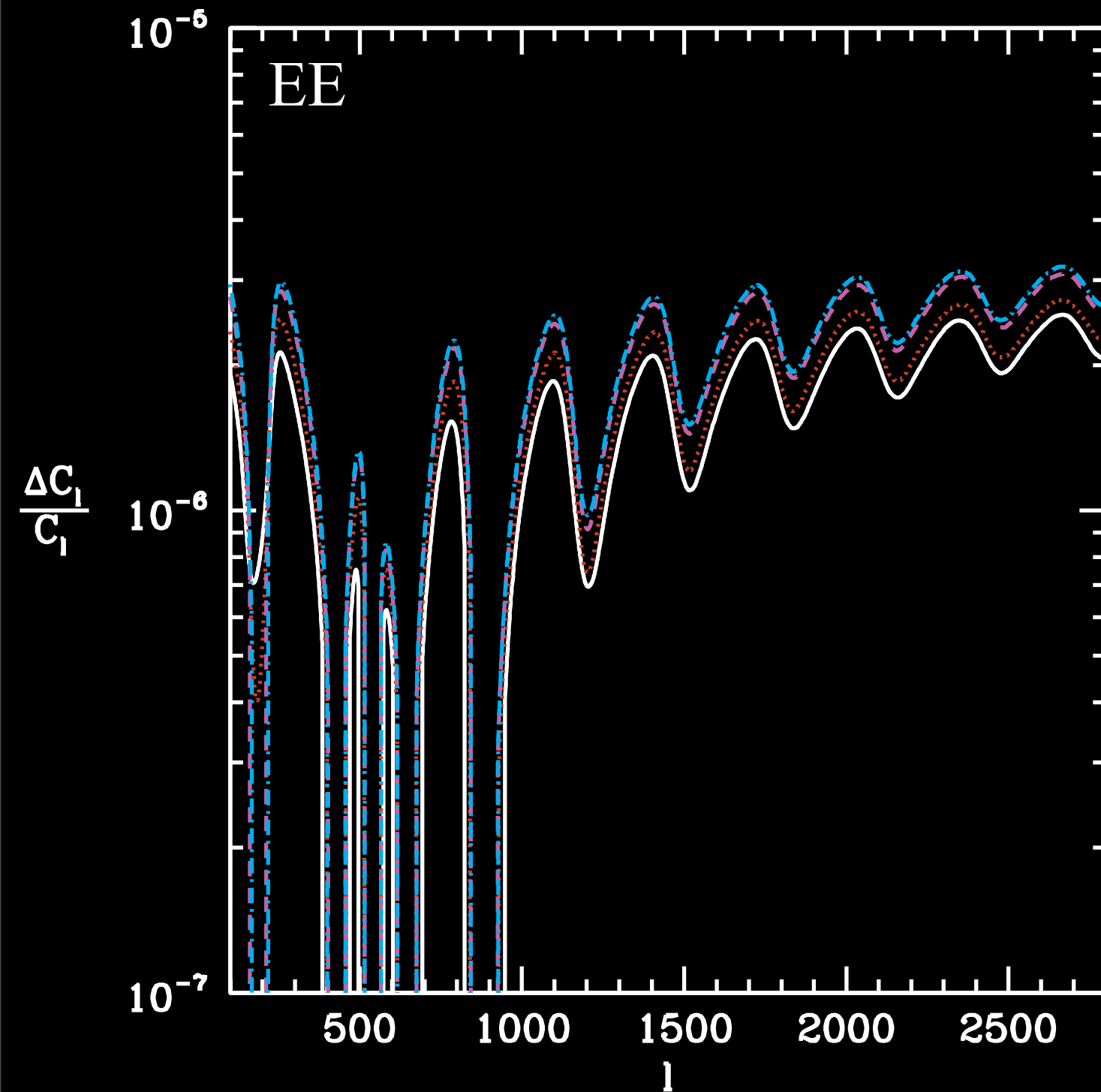
RESULTS: POLARIZATION (EE) C_l s WITH HYDROGEN QUADRUPOLES



$$\Delta C_l \equiv C_l|_{\text{with } E2 \text{ transitions}} - x_e|_{\text{no } E2 \text{ transitions}}.$$

Bulk of integral from late times, higher $n_{\text{max}} \rightarrow$ lower $x_e \rightarrow$ lower $\tau \rightarrow$ higher $e^{-2\tau} \rightarrow$ higher C_l

RESULTS: POLARIZATION (EE) C_l s WITH HYDROGEN QUADRUPOLES

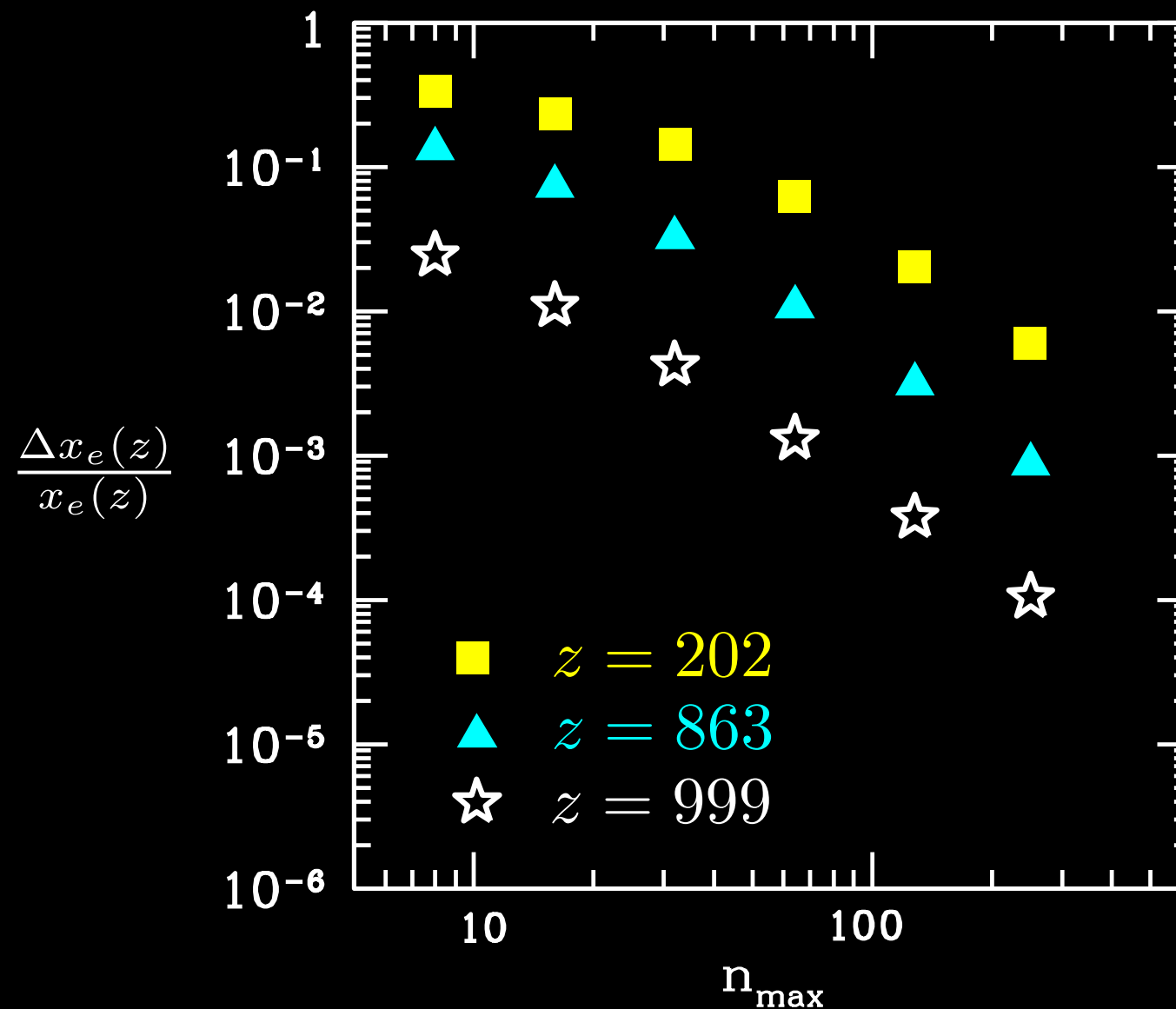


$$\Delta C_l \equiv C_l|_{\text{with } E2 \text{ transitions}} - x_e|_{\text{no } E2 \text{ transitions}}$$

Overall effect is negligible for upcoming CMB experiments!

Bulk of integral from late times, higher $n_{\text{max}} \rightarrow$ lower $x_e \rightarrow$ lower $\tau \rightarrow$ higher $e^{-2\tau} \rightarrow$ higher C_l

CONVERGENCE



- * Relative error well described by power law at high n_{\max}

$$\Delta x_e / x_e \propto n_{\max}^{-1.9}$$

- * Can extrapolate to absolute error

THE UPSHOT FOR COSMOLOGY

- ✦ Can explore effect on overall Planck likelihood analysis

$$Z^2 = \sum_{ll', X, Y} F_{ll'} \Delta C_l^X \Delta C_l^Y$$

$$Z = 1.8 \text{ if } n_{\text{max}} = 64,$$

$$Z = 0.50 \text{ if } n_{\text{max}} = 128,$$

$$Z = 0.14 \text{ if } n_{\text{max}} = 250.$$

CONCLUSIONS

- * RecSparse: a new tool for MLA recombination calculations (*arXiv:0911.1359*)
- * Highly excited levels ($n \sim 64$ and higher) are relevant for Planck CMB data analysis
- * E2 transitions in H are not relevant for Planck CMB data analysis

FUTURE WORK

- * Include line-overlap
- * Develop cutoff method for excluded levels
- * Generalize **RecSparse** to calc. rec. line. spectra
- * Compute and include collisional rates
- * Monte-Carlo analyses
- * Cosmological masers

Bound-free rates

- * Using continuum wave functions, bound-free rates are obtained (Burgess 1957)
- * Bound-free matrix elements satisfy a convenient recursion relation:
- Matrix elements compared with Burgess 1965 (5 digits) and with WKB approximation (5%):
- At each temperature, thermal recombination/ionization rates obtained using 11-point Newton-Cotes formula, agreement with Burgess to 4 published digits

BB Rate coefficients: verification

- WKB estimate of matrix elements $\rho(n'l', nl) = a_0 n^2 \int_{-\pi}^{\pi} d\tau e^{i\Omega\tau} (1 + \cos\eta)$

Fourier transform of classical orbit!
Application of correspondence principle!

$$\Omega = \omega_n - \omega_{n'}$$

$$r = r_{\max} (1 + \cos \eta) / 2$$

$$\tau = \eta + \sin \eta$$

$$\rho^{\text{dipole}}(n, l, n', l') = \frac{n_c^2}{s} \left\{ J_{s-1}(s\epsilon) - \frac{1 \mp \sqrt{1 - \epsilon^2}}{\epsilon} J_s(s\epsilon) \right\}$$

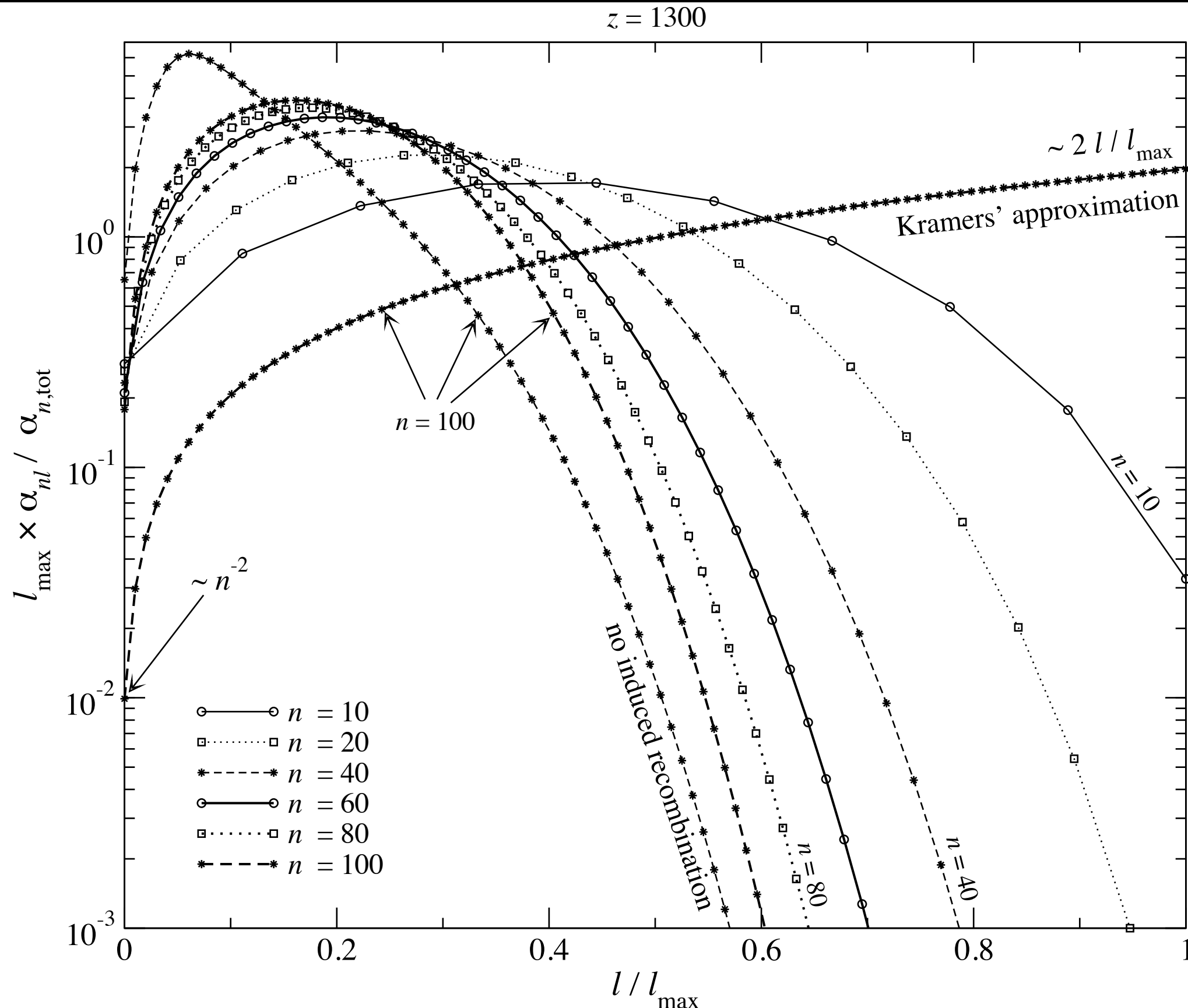
$$\epsilon = \left(1 - \frac{l(l+1)}{n^2} \right)^{1/2}$$

$$s = n - n'$$

- Radial matrix elements checked against WKB (10%), published rates of Brocklehurst (1971), Green, Rush, and Chandler (1967) (agreement to their published 4 digits)

DEVIATIONS FROM BOLTZMANN EQ: L-SUBSTATES

Chluba/Rubino-Martin/Sunyaev 2006



Quadrupole rates: basic formalism

$$\star A_{n_a, l_a \rightarrow n_b, l_b}^{\text{quad}} = \frac{\alpha}{15} \frac{1}{2l_a + 1} \frac{\omega_{ab}^5}{c^4} \left\langle l_a || C^{(2)} || l_b \right\rangle^2 \left({}^2 R_{n_b l_b}^{n_a l_a} \right)^2$$

- Reduced matrix element evaluated using Wigner 3J symbols:

$$\left\langle l_a || C^{(2)} || l_b \right\rangle = (-1)^{l_a} \sqrt{(2l_a + 1)(2l_b + 1)} \begin{pmatrix} l_a & 2 & l_b \\ 0 & 0 & 0 \end{pmatrix}$$

- Radial matrix element evaluated using operator methods

$${}^2 R_{n_b l_b}^{n_a l_a} \equiv \int_0^\infty r^4 R_{n_a l_a}(r) R_{n_b l_b}(r) dr$$

Quadrupole rates: Operator algebra

✱ Radial Schrödinger equation can be factored to yield:

$$^{-}\Omega_{nl} = \frac{1}{lA_{nl}} \left[1 - l \left(\frac{d}{dr} + \frac{l+1}{r} \right) \right] \quad ^{+}\Omega_{nl} = \frac{1}{lA_{nl}} \left[1 + l \left(\frac{d}{dr} - \frac{l-1}{r} \right) \right]$$

$$\begin{aligned} ^{-}\Omega_{nl} R_{nl}(r) &= R_{n \ l-1}(r) \\ ^{+}\Omega_{n \ l-1} R_{nl}(r) &= R_{nl}(r) \end{aligned} \quad A_{nl} = \frac{\sqrt{n^2 - l^2}}{nl}$$

• This algebra can be applied to radial matrix elements:

Quadrupole rates: Operator algebra

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• This algebra can be applied to radial matrix elements:

$$^2R_{n' \ l-1}^{n \ l-1} = \frac{1}{A_{nl}} \left\{ A_{n'l}^2 R_{n'l}^{nl} + 2^{(1)}R_{n' \ l-1}^{nl} \right\} \quad ^{(2)}R_{n' \ n'-1}^{n \ n'-1} = \frac{2nn'}{\sqrt{n^2 - n'^2}} ^{(1)}R_{n \ n'-1}^{nn'}$$

Diagonal!

Quadrupole rates: Operator algebra

✱ Radial Schrödinger equation can be factored to yield:

$$^{-}\Omega_{nl} = \frac{1}{lA_{nl}} \left[1 - l \left(\frac{d}{dr} + \frac{l+1}{r} \right) \right] \quad ^{+}\Omega_{nl} = \frac{1}{lA_{nl}} \left[1 + l \left(\frac{d}{dr} - \frac{l-1}{r} \right) \right]$$

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✱ This algebra can be applied to radial matrix elements:

$$l(2l+3)A_{n'l}^{(2)} R_{n' \ l-1}^{n \ l+1} = (2l+1)(l+2)A_{n \ l+2}^{(2)} R_{n'l}^{n \ l+2} + 2(l+1)A_{n' \ l+1}^{(2)} R_{n' \ l+1}^{n \ l+1} + 2(2l+1)(3l+5)^{(1)} R_{n'l}^{n \ l+1} \quad (1 \leq l \leq n' - 1)$$

$$^{(2)} R_{n' \ n'+1}^{n \ n'-1} = 0$$

$$^{(2)} R_{n' \ n'-1}^{n \ n'+1} = (-1)^{n-n'} 2^{2n'+4} \left[\frac{(n+n'+1)!}{(n-n'-2)!(2n'-1)!} \right]^{1/2} n' (nn')^{n'+3} \frac{(n-n')^{n-n'-3}}{(n+n')^{n+n'+3}}$$

Off-diagonal!