Cosmological Hydrogen Recombination: The effect of high-n states

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Caltech

Friday Cosmology Meeting
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OUTLINE

- Cosmological Recombination in a nutshell
- Breaking the naive model
- Why should you care? Effects on CMB, inferences about primordial physics
- Our tools
- Preliminary results!

A BRIEF MESSAGE FROM YOUR PROVIDER

- I will not talk about QCD’s Dyson-Schwinger equations. Sorry!

- Take-home message for the narcoleptic:
  To bring theoretical uncertainties well within the error budget of Planck and other next generation CMB experiments, very high excitation (n>100) states of the hydrogen atom must be included in recombination calculations, with different angular momentum (l) substates separately resolved.

Saha equilibrium is inadequate

$$p + e^- \leftrightarrow H^{(n)} + \gamma^{(nc)}$$

- Chemical equilibrium does reasonably well predicting "moment of recombination"

$$\frac{x_e^2}{1-x_e} = \left(\frac{13.6}{T_{\rm eV}}\right)^{3/2} e^{35.9-13.6/T_{\rm eV}}$$

$$x_e = 0.5 \text{ when } T = T_{\rm rec} \simeq 0.3 \text{ eV} \qquad z_{\rm rec} \simeq 1300$$

- Further evolution falls prey to reaction freeze-out

$$\Gamma = 6 \times 10^{-22} \text{ eV } x_e(T) \left(13.6/T_{\rm eV}\right)^{-5/2} \ln\left(13.6/T_{\rm eV}\right)$$

$$H = 1.1 \times 10^{-26} \text{ eV } T_{\rm eV}^{3/2}$$

$$\Gamma < H \text{ when } T < T_{\rm F} \simeq 0.25 \text{ eV}$$

BOTTLENECKS AND ESCAPE ROUTES

- BOTTLENECKS
  - Ground state recombinations are ineffective

$$\tau_{c\to 1s}^{-1} = 10^{-1}\ \mathrm{s}^{-1} \gg H \simeq 10^{-12}\ \mathrm{s}^{-1}$$

  - Resonance photons are re-captured, e.g. Lyman α

$$\tau_{2p\to 1s}^{-1} = 10^{-2}\ \mathrm{s}^{-1} \gg H \simeq 10^{-12}\ \mathrm{s}^{-1}$$

- ESCAPE ROUTES (e.g. n=2)
  - Two-photon processes

$$H^{2s} \to H^{1s} + \gamma + \gamma \qquad \boldsymbol{\Lambda}_{\mathrm{2s}\to\mathrm{1s}} = \mathbf{8.22}\ \mathrm{s}^{-1}$$

  - Redshifting off resonance

$$R \sim \left(n_{\mathrm{H}}\lambda_\alpha^3\right)^{-1}\left(\frac{\dot{a}}{a}\right)$$

EQUILIBRIUM ASSUMPTIONS

- Radiative eq. between different n-states

$$\mathcal{N}_n = \mathcal{N}_2 e^{-(E_n - E_2)/T}$$

- Radiative/collisional eq. between different l

$$\mathcal{N}_{nl} = \mathcal{N}_n \frac{(2l+1)}{n^2}$$

- Matter in eq. with radiation due to Thompson scattering

$$T_m = T_\gamma \text{ since } \frac{\sigma_{\rm T} a T_\gamma^4 c}{m_e c^2} < H(T)$$

The Peebles Punchline

- Only n=2 bottlenecks are treated

$$\Gamma_{\rm net,H} = \Lambda_{2s\to 1s}\left[n_{2s} - n_{1s}e^{-(B_1-B_2)/kT}\right] + \frac{8\pi}{\lambda_\alpha^3}\frac{\dot a}{a}\times\left(f_\alpha - e^{-h\nu_\alpha/kT}\right)$$

- Net Rate is suppressed by bottleneck vs. escape factor

$$-\frac{dx_e}{dt} = \sum_{n,l>1s}\alpha_{nl}(T)\left\{nx_e^2 - x_{1s}e^{\frac{-B_1}{kT}}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2}\right\}\mathcal{C}$$

$$\mathcal{C} = \frac{\frac{8\pi}{\lambda_\alpha^3 n_{1s}}\frac{\dot a}{a} + \Lambda_{2s\to 1s}}{\frac{8\pi}{\lambda_\alpha^3 n_{1s}}\frac{\dot a}{a} + (\Lambda_{2s\to 1s} + \beta_c)}$$

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Redshifting term

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2γ term

The Peebles Punchline

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Ionization Term

The peebles punchline

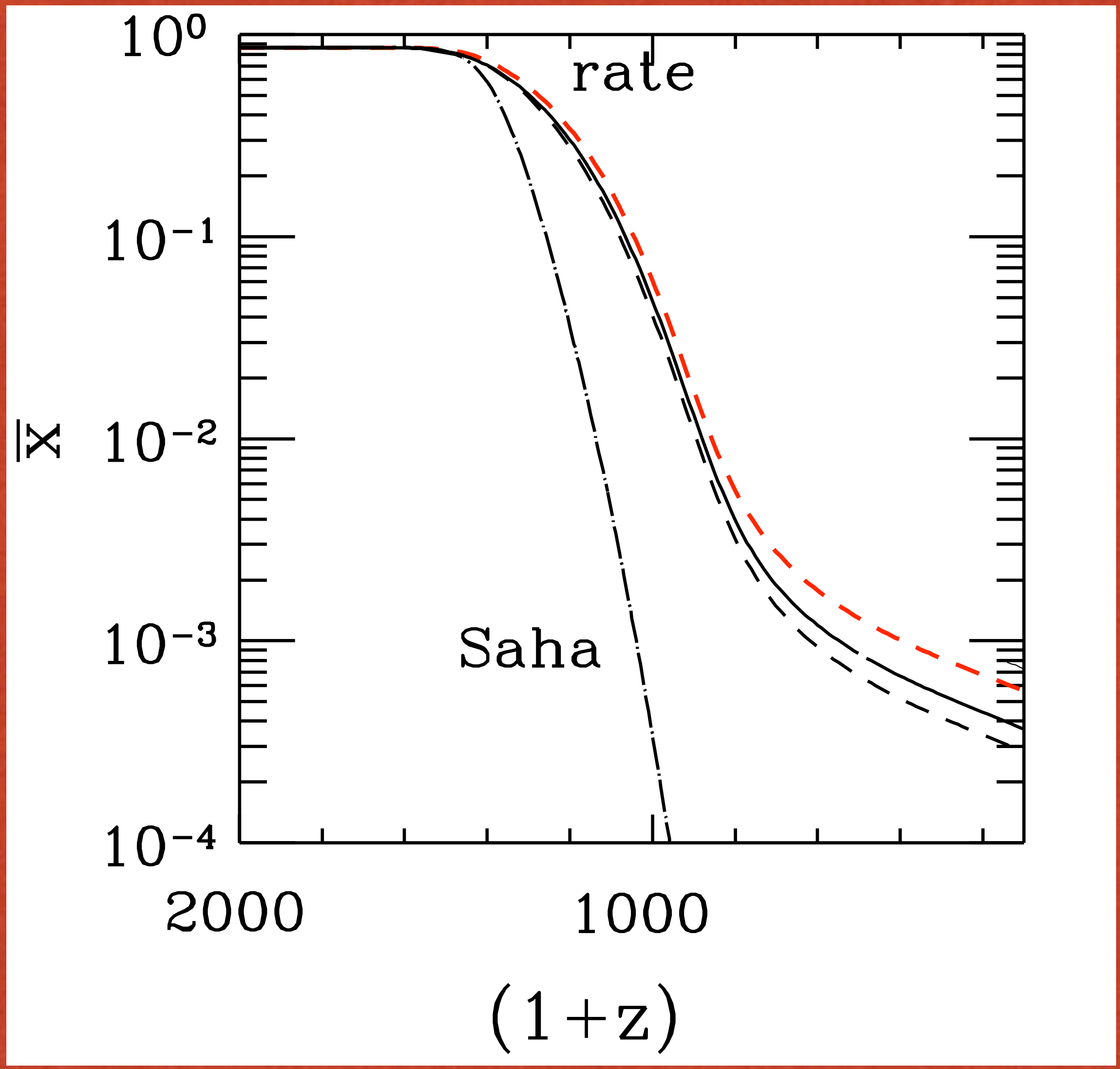
- Net Rate is suppressed by bottleneck vs. escape factor

$$\mathcal{C} = \frac{\frac{8\pi}{\lambda_\alpha^3 n_{1s}}\frac{\dot{a}}{a} + \Lambda_{2s\to 1s}}{\frac{8\pi}{\lambda_\alpha^3 n_{1s}}\frac{\dot{a}}{a} + \left(\Lambda_{2s\to 1s} + \beta_c\right)}$$

$$\frac{\text{redshift term}}{2\gamma\ \text{term}} \simeq 0.02\frac{\Omega_m^{1/2}}{\left(1 - x_e\left[z\right]\right)\left(\frac{1+z}{1100}\right)^{3/2}}$$

2γ process dominates until late times $(z \lesssim 850)$

PEEBLES MODEL ASSUMPTIONS/RESULTS



$\Omega_m h^2 \uparrow$

$\Omega_b h^2 \uparrow$

- State of the Art for 30 years!

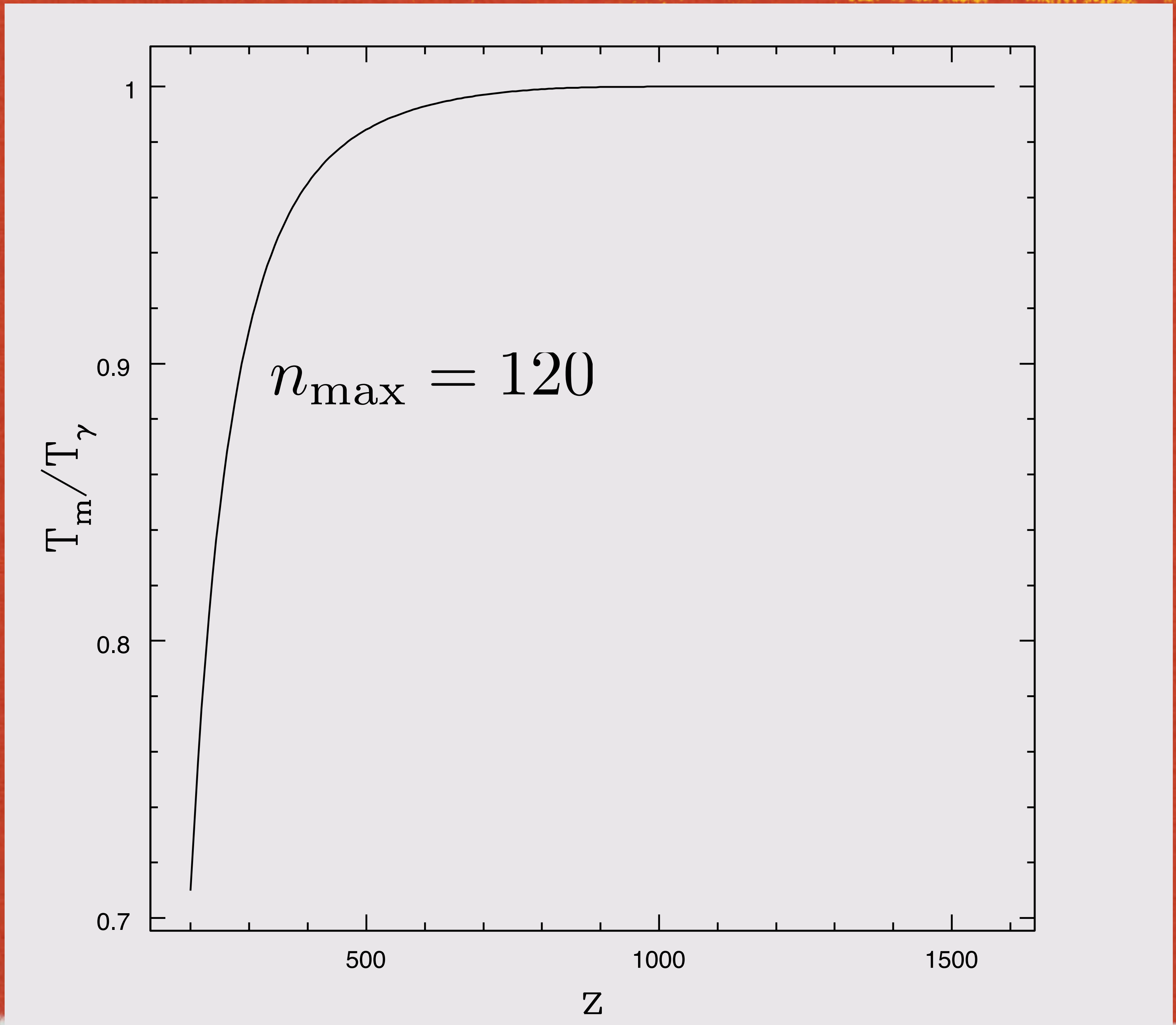
BREAKING THE NAIVE MODEL

- Radiation field is cool: ~~Boltzmann eq. of higher n~~
- Treated by Seager et al. (2000) $n_{\max} = 300$ RecFAST!!!
- ~~Equilibrium~~ between *l states*
- Treated by Chluba et al. (2005) for $n_{\max} = 100$
- Radiation and matter field fall out of eq.

$$\dot{T_M} + 2HT_m = \frac{8x_e\sigma_{\rm T}aT_\gamma^4}{3m_ec\,(1+f_{\rm He}+x_e)}\,(T_M - T_\gamma)$$

- Higher-order 2γ transitions, (Hirata, Ali-Haimoud, in progress)

Decoupling of matter and radiation



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BREAKING THE NAIVE MODEL

- Radiation field is cool: ~~Boltzmann eq. of higher n~~
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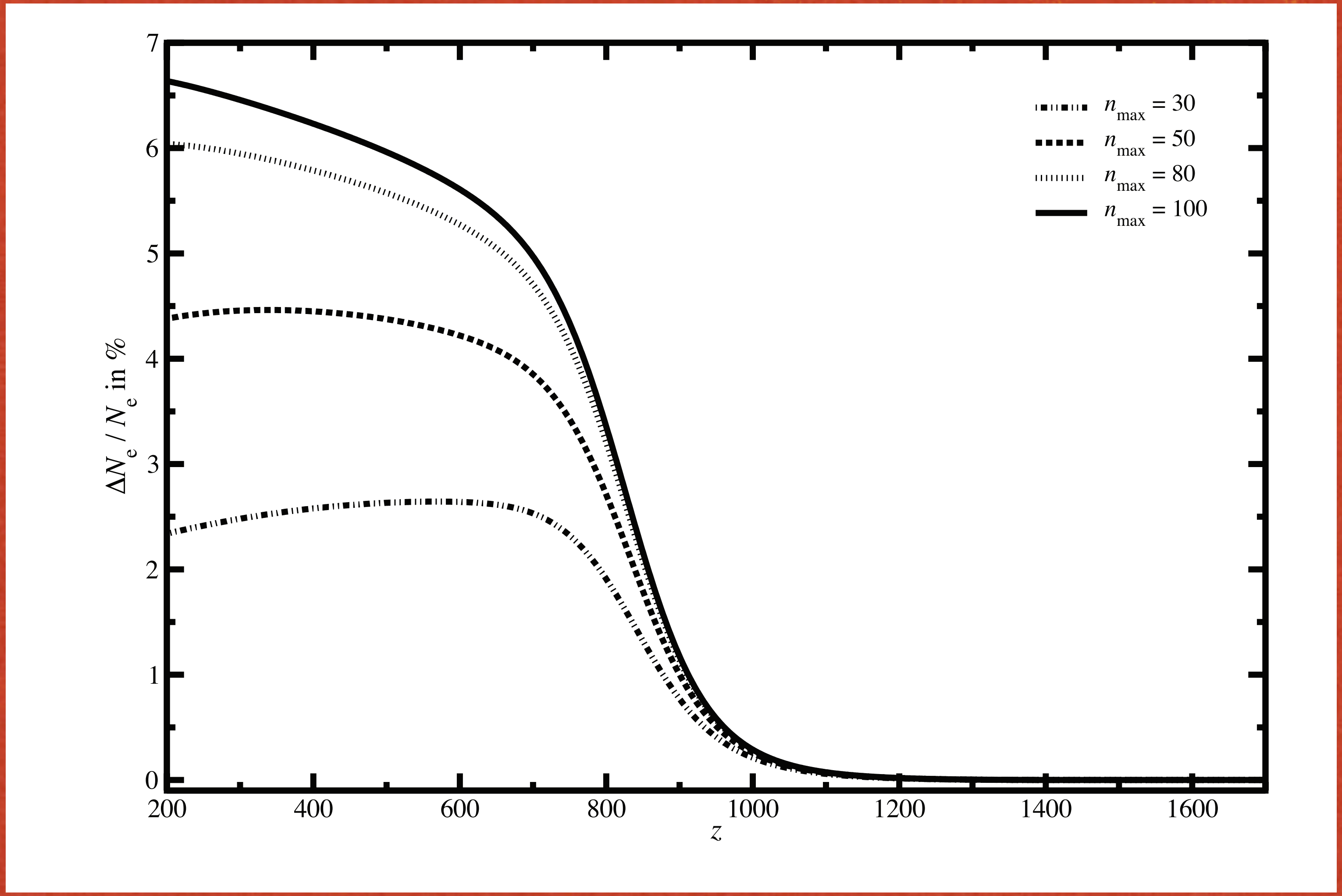
- ~~Equilibrium~~ between *l states*
- Treated by Chluba et al. (2005) for $n_{\mathrm{max}} = 100$
- Beyond this, testing convergence with n_{max} is hard!

$$t_{\mathrm{compute}} \sim \mathcal{O}\left(\mathrm{weeks}\right)$$

How to proceed if we want 0.1% accuracy in $x_e(z)$?

The effect of resolving l- substates



- Putting free-electrons in 'bottlenecked' l-substates slows down the decay to 1s: Recombination is slower

BREAKING THE NAIVE MODEL

- Radiation field is cool: ~~Boltzmann eq. of higher n~~
- Treated by Seager et al. (2000) $n_{\rm max} = 300$ RecFAST!!!
- ~~Eq.~~ between *l states*: dipole selection bottleneck: $\Delta l = \pm 1$
- Treated by Chluba et al. (2005) for $n_{\rm max} = 100$
- Beyond this, testing convergence with $n_{\rm max}$ is hard!

$$t_{\rm compute} \sim \mathcal{O}\,(\text{weeks})$$

WHY PROCEED?

Who Cares?
I. Smearing and moving the surface of last scattering (SLSS)

- Photons kin. decouple when Thompson scattering freezes out

$$\gamma + e^- \Leftrightarrow \gamma + e^-$$

$$\Gamma = n_e \sigma_{\rm T} c = 2.2 \times 10^{-19}\ {\rm s}^{-1} \frac{x_e \Omega_b h^2}{a^3} =$$

$$H = H_0 \Omega_m^{1/2} a^{-3/2} \left[1 + \frac{a_{\rm eq}}{a}\right]^{1/2}$$

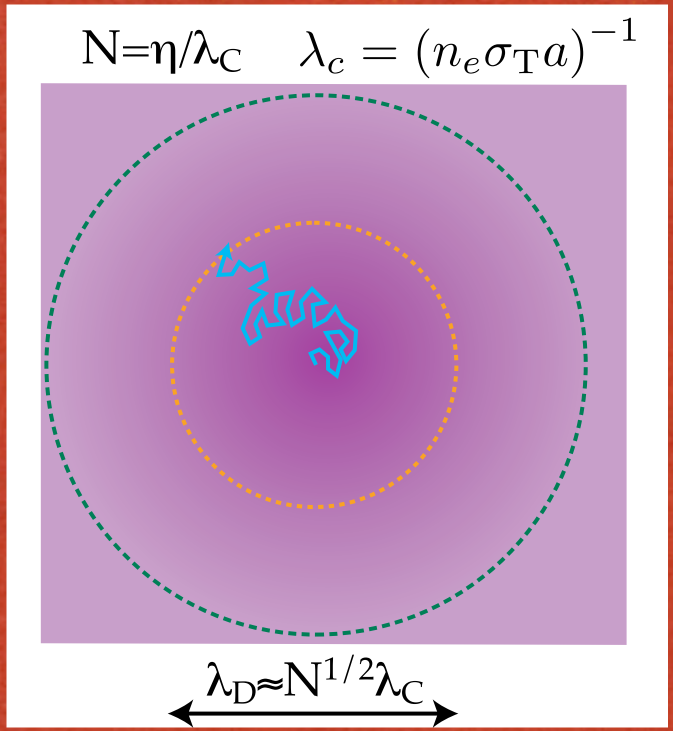
- $z_{\rm dec} \simeq 1100$:Decoupling occurs during recombination

$$C_l \to C_l e^{-2\tau} \qquad \text{if } l > \frac{\eta_0}{\eta_{\rm rec}}.$$

$$\tau = \int_0^{\eta_{\rm dec}} d\eta n_e \left[\eta\right] \sigma_{\rm T} a \left(\eta\right)$$

Who Cares?
II. The Silk Damping Tail

- From Wayne Hu's website



$$l_{\text{damp}} \sim 1000$$

- Inhomogeneities are damped for $\lambda < \lambda_D$

$$k_D^{-2}(\eta) \simeq \int_0^\eta \frac{d\eta'}{6(1+R)n_e[\eta']\sigma_T a[\eta']}\left[\frac{R^2}{1+R}+\frac{8}{9}\right]$$

$$R = \frac{3\rho_b^0}{4\rho^\gamma}$$

$$|\Theta_l(\eta_0)| \simeq \int_0^{\eta_0} d\eta\, \dot{\tau} e^{-\tau(\eta)} e^{ik\int d\eta c_s} e^{-k^2/k_D^2(\eta)} \tilde{\delta}(k) j_l(k(\eta-\eta_0))dk$$

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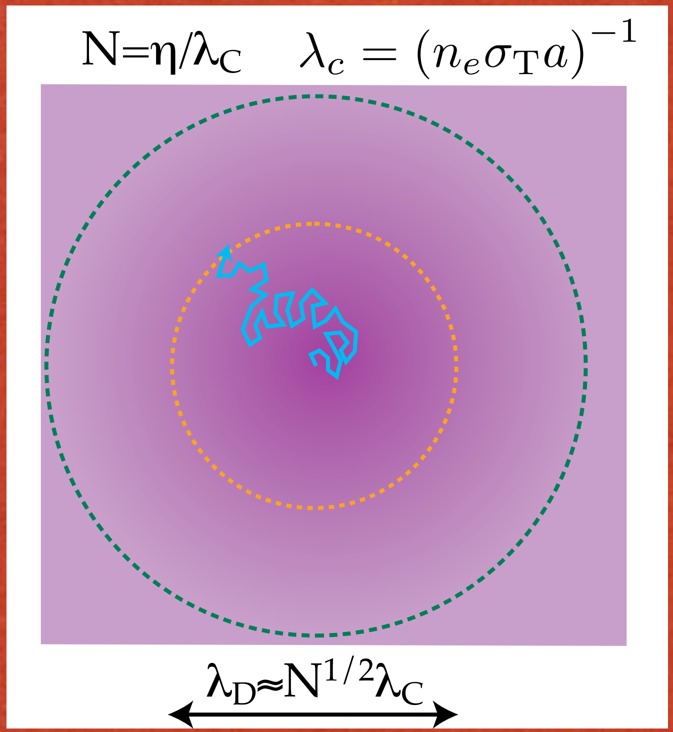

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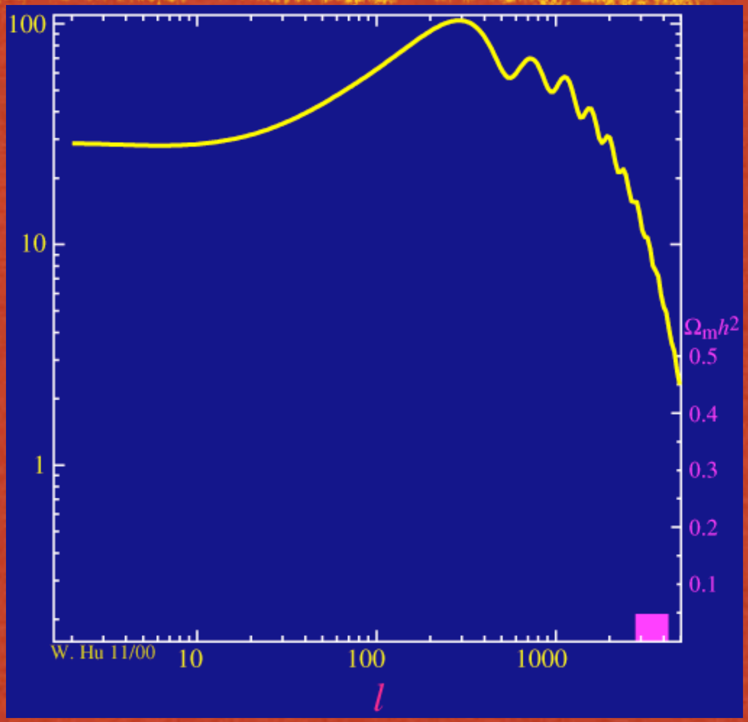

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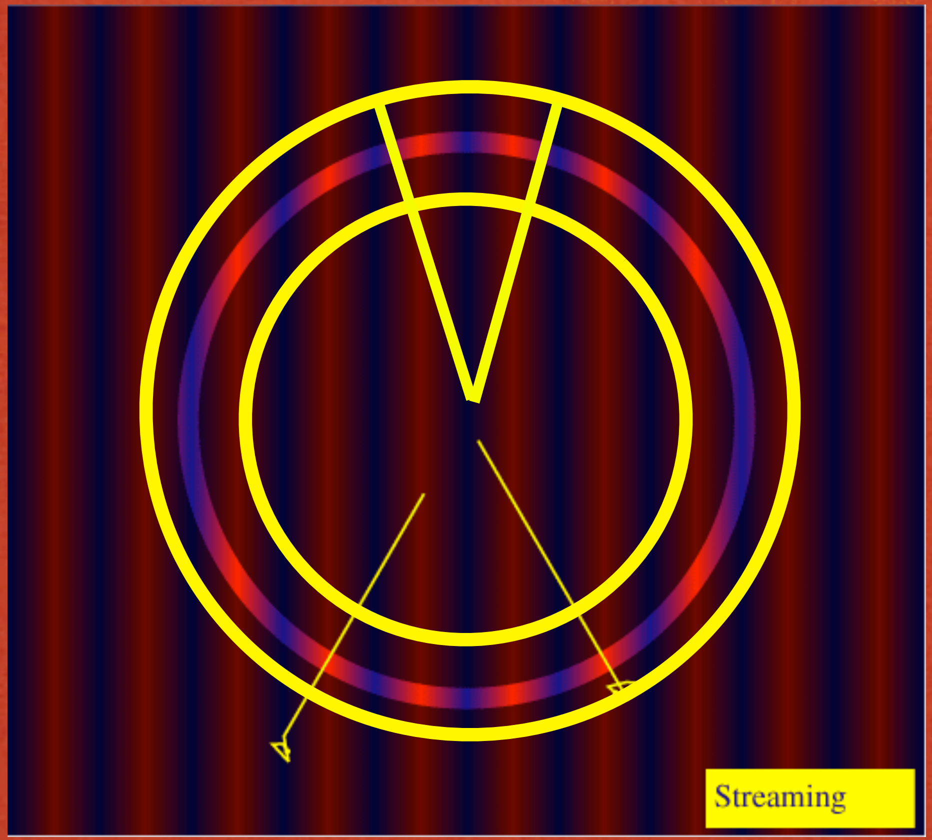
WHO CARES?
III. FINITE THICKNESS OF THE SLSS



- Additional damping of form

$$|\Theta_l\left(\eta_0,k\right)| \rightarrow |\,\Theta_l\left(\eta_0,k\right)|\,e^{-\sigma^2\eta_{\rm rec}^2k^2}$$

Who Cares?
IV. CMB Polarization

- Need to scatter quadrapole to polarize CMB

$$\Theta_l^P(k) = \int d\eta \dot{\tau} e^{-\tau(\eta)} \Theta_{T,2}(k,\eta) \frac{l^2}{(k\eta)^2} j_l(k\eta)$$

- Need time to develop a quadrapole

$$\Theta_l(k\eta) \sim \frac{k\eta}{2\tau} \Theta_l(k\eta) \ll \Theta_l(\eta) \text{ if } l \geq 2, \text{ in tight coupling regime}$$

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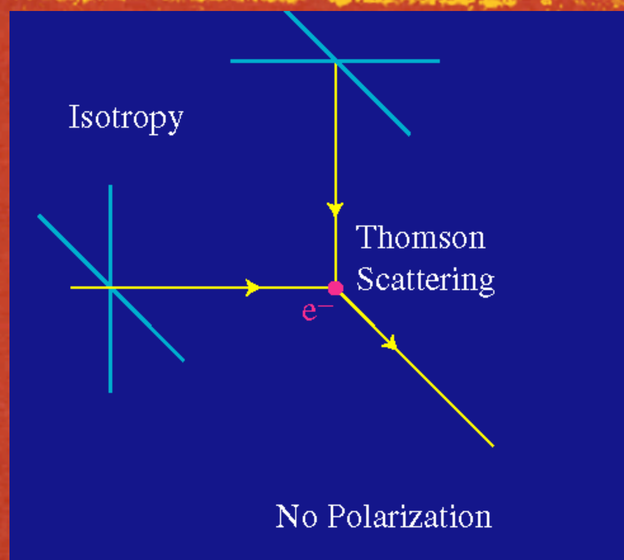

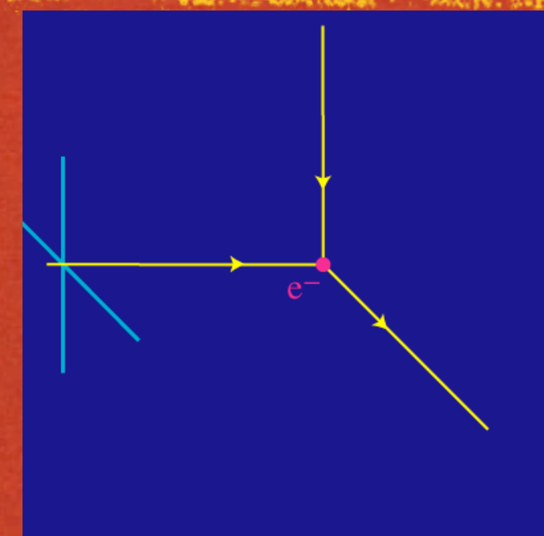

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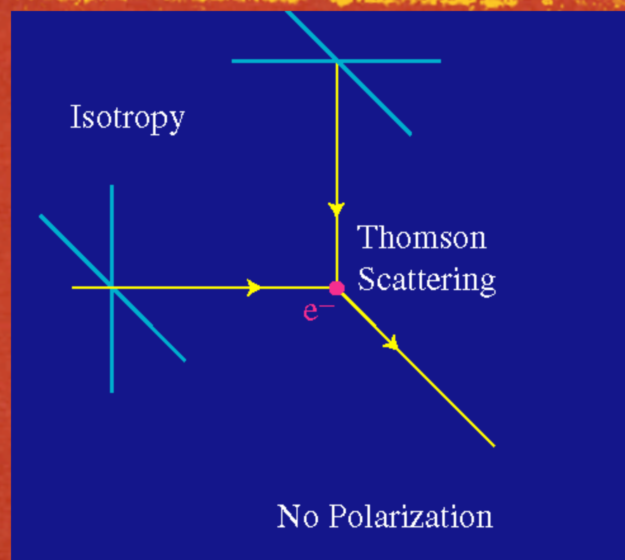

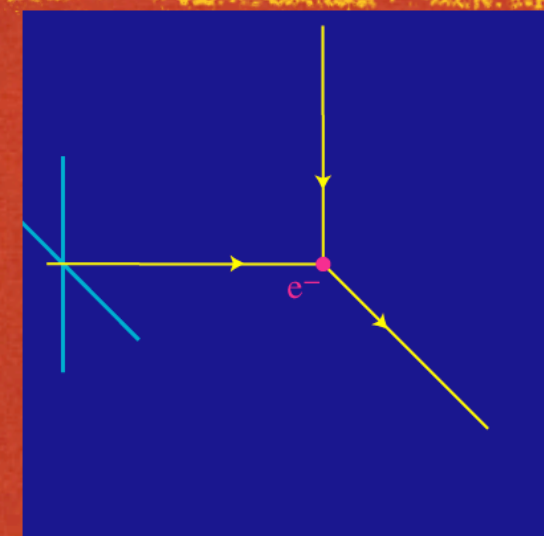

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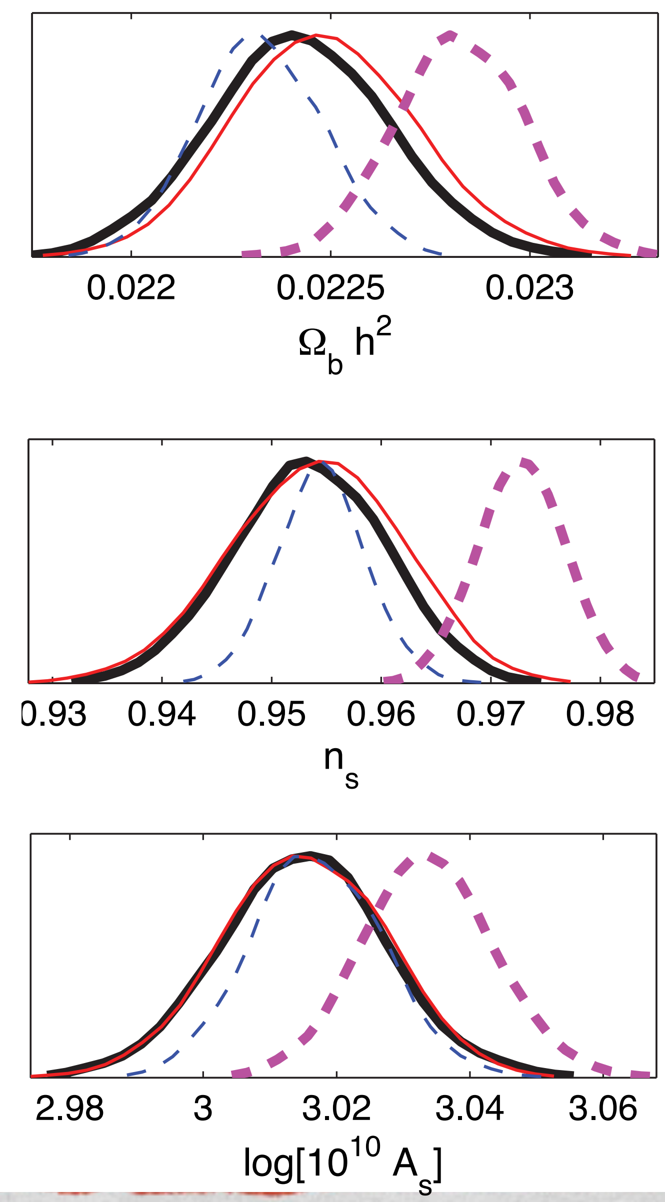
WHO CARES?

V. PARAMETER DEGENERACIES

- Planck will be CV limited (T and E) to $l \sim 2500$

- 0.1% accuracy required in $x_e\,(z)$

- Planck uncertainty forecasts using MCMC



THE MULTI-LEVEL ATOM (MLA)

- Bound-free rate equation

$$\dot{x}_{nl}^{bf} = \int dE_{\rm e} P_M(T_m, E_{\rm e}) n_H x_e x_p \left[1 + f(E_e - E_n)\right] \alpha_{nl}(E_{\rm e})$$
$$- \int dE_{\rm e}\ g(E_{\rm E} - E_n)\ x_{nl}\ f(E_{\rm e} - E_{nl}) \alpha_{nl}(E_{\rm E})/g_{nl}$$

- Bound-bound rate equation

$$\dot{x}_{nl}^{bb} = \sum_{n',l'=l\pm1} (A_{nn'}^{ll'} (1 + f_{nn'}) x_{n',l'} - \tfrac{g_{n'l'}}{g_{nl}} f_{nn'} x_{nl}) P_{nn'}^{ll'},$$

THE MULTI-LEVEL ATOM (MLA)

- Bound-free rate equation

Ω_m, Ω_b, h

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- Phase-space density blueward of line
- Escape probability of γ in line

The multi-level atom (MLA)

Stimulated emission/absorption

- Bound-free rate equation

$$\dot{x}_{nl}^{bf} = \int dE_{\mathrm{e}} P_M(T_m, E_{\mathrm{e}}) n_H x_e x_p \left[1 + \boxed{f(E_e - E_n)}\right] \alpha_{nl}(E_{\mathrm{e}})$$
$$- \int dE_{\mathrm{e}}\ g(E_{\mathrm{E}} - E_n)\ x_{nl}\ \boxed{f(E_{\mathrm{e}} - E_{nl})} \alpha_{nl}(E_{\mathrm{E}})/g_{nl}$$

- Bound-bound rate equation

$$\dot{x}_{nl}^{bb} = \sum_{n',l'=l\pm1} (A_{nn'}^{ll'}(1 + \boxed{f_{nn'}}) x_{n',l'} - \frac{g_{n'l'}}{g_{nl}} \boxed{f_{nn'}} x_{nl}) P_{nn'}^{ll'},$$

The multi-level atom (MLA)

Spontaneous Emission

- Bound-free rate equation

$$\dot{x}_{nl}^{bf} = \int dE_{\rm e} P_M(T_m, E_{\rm e}) n_H x_e x_p \left[1 + f(E_e - E_n)\right] \alpha_{nl}(E_{\rm e})$$
$$- \int dE_{\rm e}\; g(E_{\rm E} - E_n)\; x_{nl}\; f(E_{\rm e} - E_{nl}) \alpha_{nl}(E_{\rm E})/g_{nl}$$

- Bound-bound rate equation

$$\dot{x}_{nl}^{bb} = \sum_{n',l'=l\pm1} (A_{nn'}^{ll'} (1 + f_{nn'}) x_{n',l'} - \frac{g_{n'l'}}{g_{nl}} f_{nn'} x_{nl}) P_{nn'}^{ll'},$$

THE MULTI-LEVEL ATOM (MLA)

- Two photon transitions between n=1 and n=2 are included:

$$\dot{x}_{2s\to 1s,2\gamma} = -\dot{x}_{1s\to 2s,2\gamma} = \Lambda_{2s}(-x_{2s} + x_{1s}e^{-E_{2s\to 1s}/T_\gamma})$$

- Net recombination rate:

$$x_e \simeq 1 - x_{1s} \to \dot{x}_e \simeq -\dot{x}_{1s} = -\dot{x}_{1s\to 2s}$$
$$+\sum_{n,l>1s} A_{n1}^{l0} P_{n1}^{l0} \left\{ \frac{g_{nl}}{2} f_{n1}^{+} x_{1s} - (1 + f_{n1}^{+}) x_{nl} \right\}$$

RATE COEFFICIENTS

- Bound-bound rates given by Fermi's golden rule and matrix element

$$\rho(n'l',nl) = \int_0^\infty u_{n'l'}(r)u_{nl}(r)r^3dr = \mathcal{C}\times\left[F_{2,1}\left(-n+l+1,-n'+l,2l,\frac{-4nn'}{(n-n')^2}\right)\right.$$
$$\left.-\left(\frac{n-n'}{n+n'}\right)^2 F_{2,1}\left(-n+l-1,-n'+l,2l,\frac{-4nn'}{(n-n')^2}\right)\right]^2$$

- Power-series destabilizes at high-n, recursion relation used
- Bound-free rates at temperature T given by phase space integral of matrix element $g_{nl} = \int_0^\infty u_{nl}(r) f_{El}(r)\, r^3 dr$
- Rates are tabulated at all n and l of interest, at a variety of energies, and integrated at each time step

RATE COEFFICIENTS

- Rates are tabulated at all n and l of interest, at a variety of energies, and integrated at each time step $\rho(n'l', nl) = a_0 n^2 \int_{-\pi}^{\pi} d\tau e^{i\Omega\tau} \left(1 + cos\eta\right)$

$$\Omega = \omega_n - \omega_{n'}$$

$$r = r_{\max} \left(1 + \cos\eta\right)/2$$

$$\tau = \eta + \sin\eta$$

Fourier transform of classical orbit!
Application of correspondence principle!

- Similar WKB approximation can be used to check stability of BF matrix elements

Radiation Field: Black Body+

- Escape probability treated in Sobolev approx.

$$P_{n,n'}^{l,l'} = \frac{1-e^{-\tau_s}}{\tau_s}$$

$$\boxed{\tau_s = \frac{c^3 n_{\rm H}}{8\pi H \nu_{nn'}^3} A_{nn'}^{ll'} \left[\frac{g_{n'}^{l'}}{g_n^l} x_n^l - x_{n'}^{l'} \right]}$$

$$\mathcal{R}(\nu,\nu') = \phi(\nu)\phi(\nu') \qquad \frac{v_{\rm th}}{H(z)} \ll \lambda$$

- Excess line photons injected into radiation field

$$\left(\frac{8\pi\nu_{nn'}^3}{c^3 n_H}\right)\left(f_{nn'}^+ - f_{nn'}^-\right) = A_{nn'}^{ll'} P_{nn'}^{ll'} \left[x_n^l \left(1+f_{nn'}^+\right) - \frac{g_n^l}{g_{n'}^{l'}} x_{n'}^{l'} f_{nn'}^+ \right]$$

- Photons are conserved outside of line regions

$$f_{n1}^{+10} = f_{n+1,1}^{-10} \left[\frac{1-(n+1)^{-2}}{1-n^{-2}} (1+z) - 1 \right]$$

Radiation Field: Black Body+

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$$P^{l,l'}_{n,n'} = \frac{1 - e^{-\tau_s}}{\tau_s}$$

$$\boxed{\tau_s = \frac{c^3 n_{\mathrm{H}}}{8\pi H \nu^3_{nn'}} A^{ll'}_{nn'} \left[\frac{g^{l'}_{n'}}{g^l_n} x^l_n - x^{l'}_{n'} \right]}$$

$$\mathcal{R}(\nu, \nu') = \phi(\nu)\phi(\nu')$$

- Forbes, Hirata, and Ali-Haimoud are solving FP eqn. to obtain evolution of $f(\nu)$ more generally, including atomic recoil/diffusion, 2γ decay and full time-dependence of problem, coherent and incoherent scattering, overlap of higher-order Lyman lines

STEADY-STATE APPROXIMATION FOR EXCITED STATES

- Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

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$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

$$\vec{x} = \begin{pmatrix} \vec{x}_0 \\ \vec{x}_1 \\ \ldots \\ \vec{x}_{n_{\max}-1} \end{pmatrix}$$

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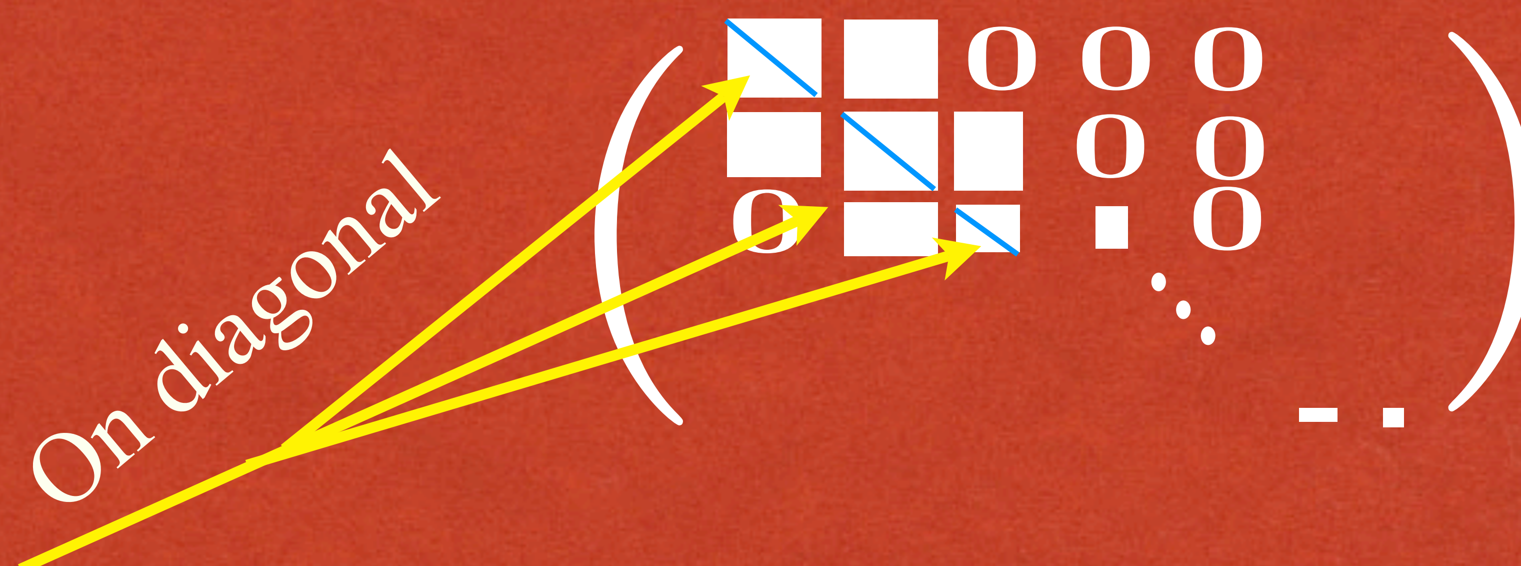

For state l, includes BB transitions out of l to all other l'', photo-ionization, 2γ transitions to ground state

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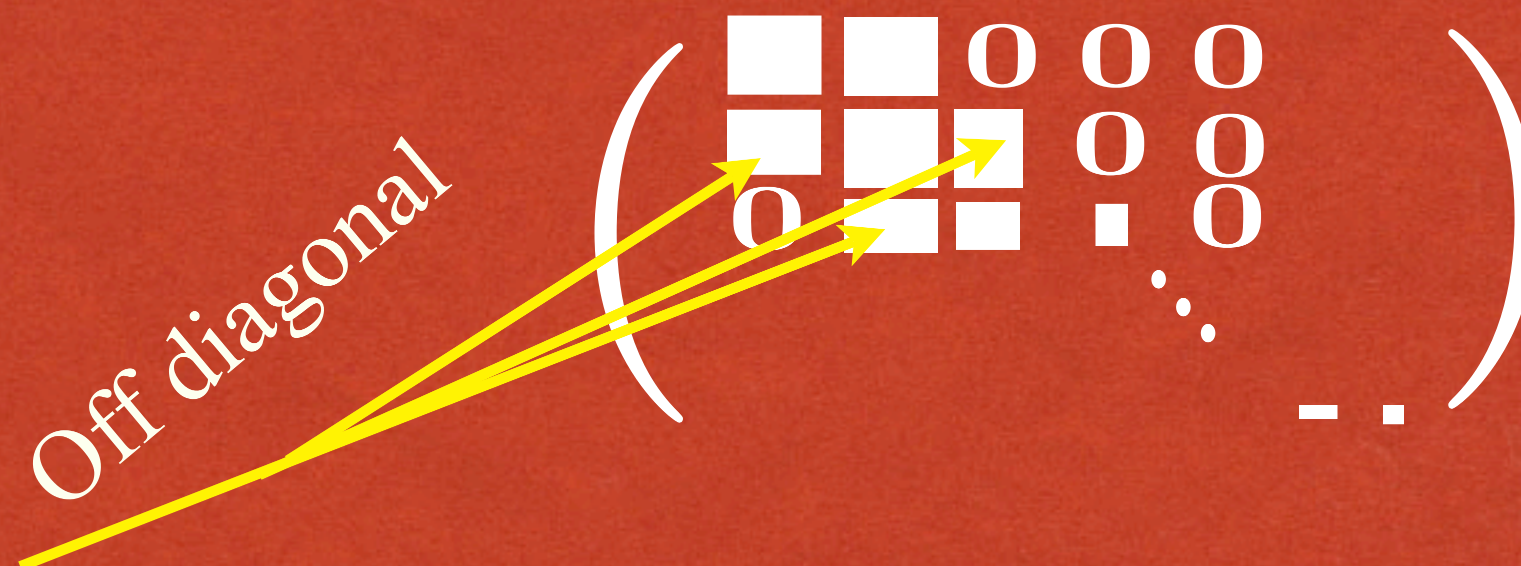

For state l, includes BB transitions into l from all other l’

STEADY-STATE APPROXIMATION FOR EXCITED STATES

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- Includes recombination to l,
1 and 2γ transitions from ground state

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- Evolution equations may be re-written in matrix form

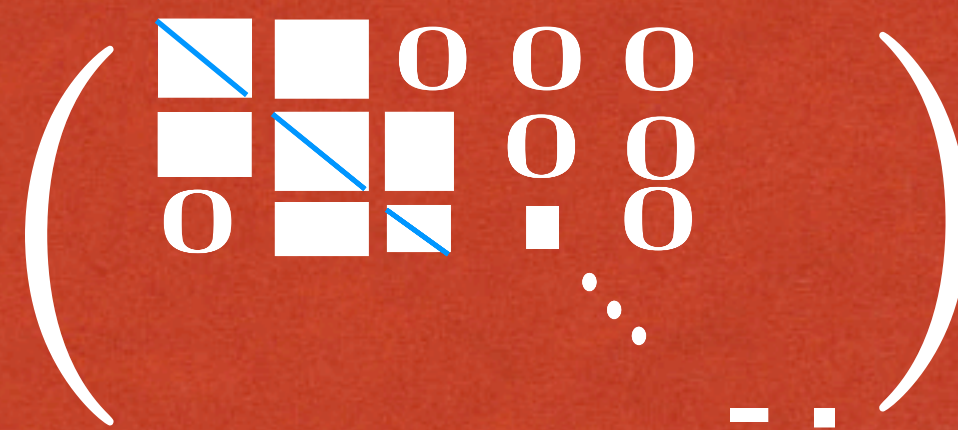
$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

- For n>1, $\boxed{t_{\rm rec}^{-1} \sim 10^{-12} s^{-1} \ll \mathbf{R}\ , \vec{s} \rightarrow \vec{x} \simeq \mathbf{R}^{-1}\vec{s}}$

$\mathbf{R} \lesssim 1\ \mathrm{s}^{-1}$ (e.g. Lyman-α)

RAPID MATRIX INVERSION: SPARSITY TO THE RESCUE

- Matrix is $\sim n_{max}^2 \times n_{max}^2$
- Brute force would require $n_{max}^6 \sim 1000$ s for $n_{max} = 200$ for a single time step
- Sparsity to the rescue $\Delta l = \pm 1$



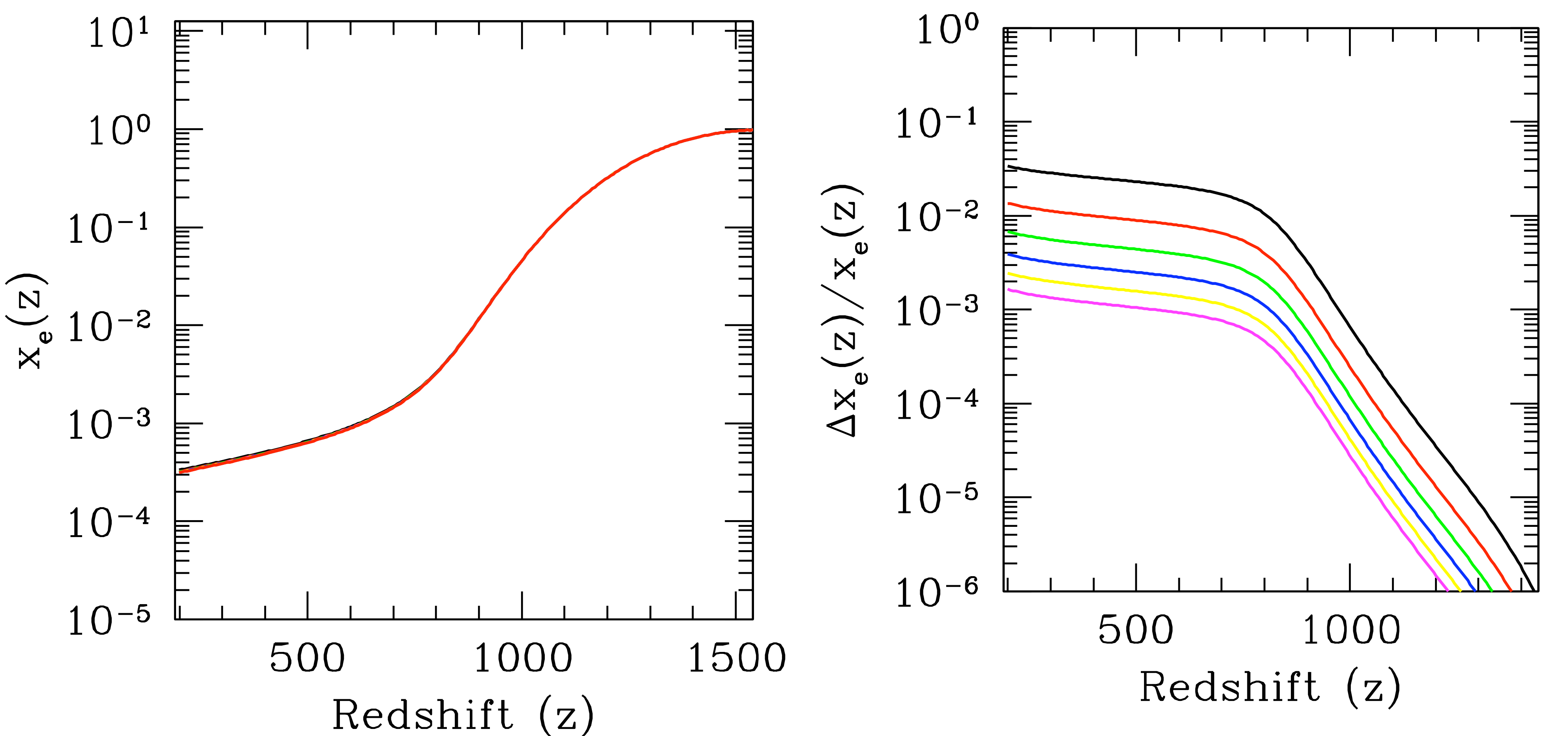
$$\boxed{\mathbf{M}_{l,l-1}\vec{x}_{l-1} + \mathbf{M}_{l,l}\vec{x}_l + \mathbf{M}_{l,l+1}\vec{x}_{l+1} = \vec{s}_l}$$

$$\vec{v}_l = \chi_\mathbf{l}\left[\vec{s}_l - \mathbf{M_{l,l+1}}\vec{v}_l + \Sigma^0_{l'=l-1}\sigma_{l,l'}\vec{s}_{l'}\,(-1)^{l'-l}\right]$$

$$\chi_l = \begin{cases} \mathbf{M}_{00}^{-1} & \text{if } l = 0 \\ \left(\mathbf{M}_{l+1,l+1} - \mathbf{M}_{l+1,l}\chi_l\mathbf{M}_{l,l+1}\right)^{-1} & \text{if } l > 0 \end{cases}$$

$$\sigma_{l,l-1} = \mathbf{M}_{l,l-1}\chi_{l-1}$$
$$\sigma_{l,i} = \sigma_{l,i+1}\mathbf{M}_{i+1,i}\chi_i$$

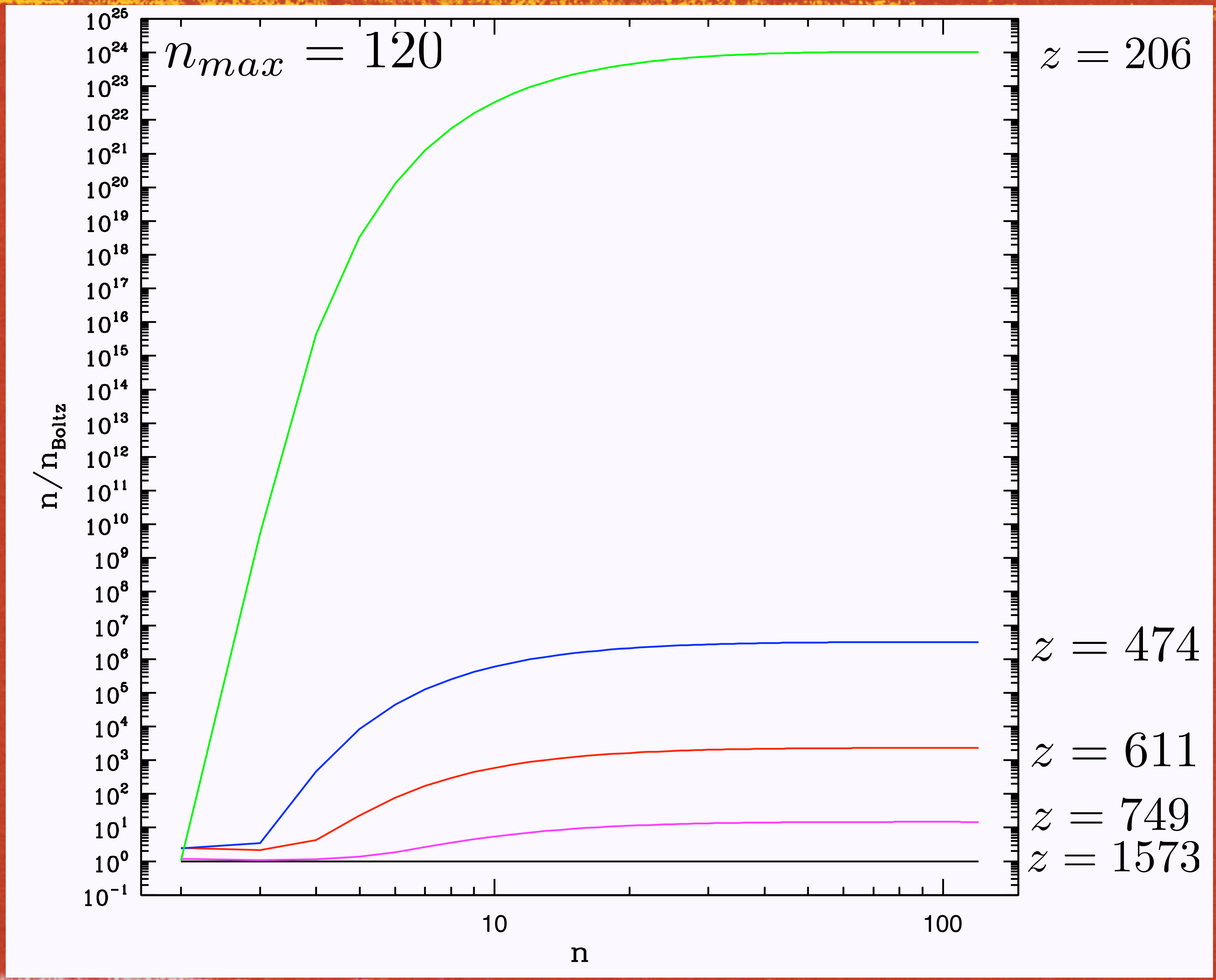
Recombination histories



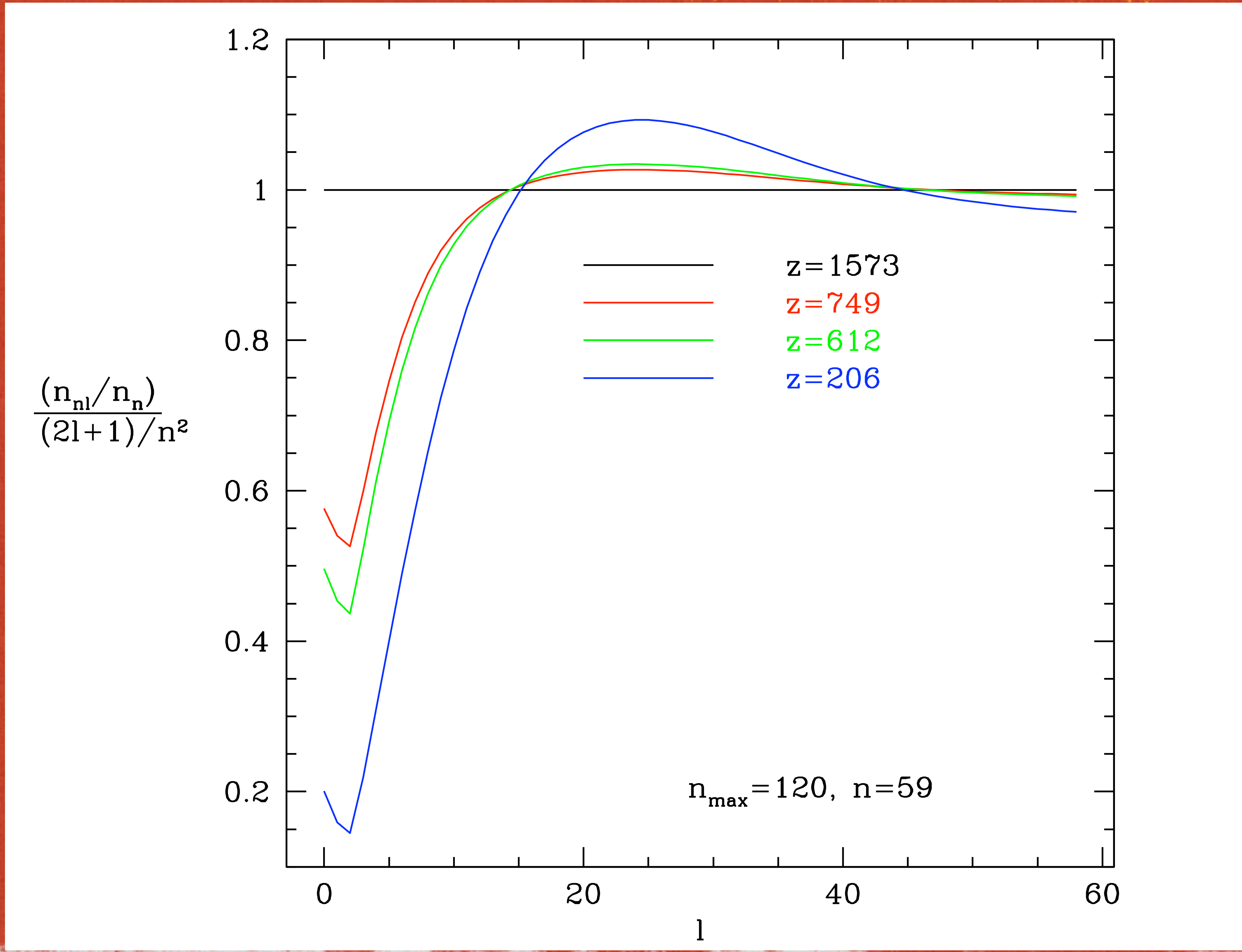
- $x_e(z)$ falls with increasing $n_{\max} = 10 \rightarrow 200$, as expected.
- Rec Rate>downward BB Rate> Ionization, upward BB rate
- For $n_{max} = 100$, code computes in only 2 hours

DEVIATIONS FROM BOLTZMANN EQ: HIGH-N

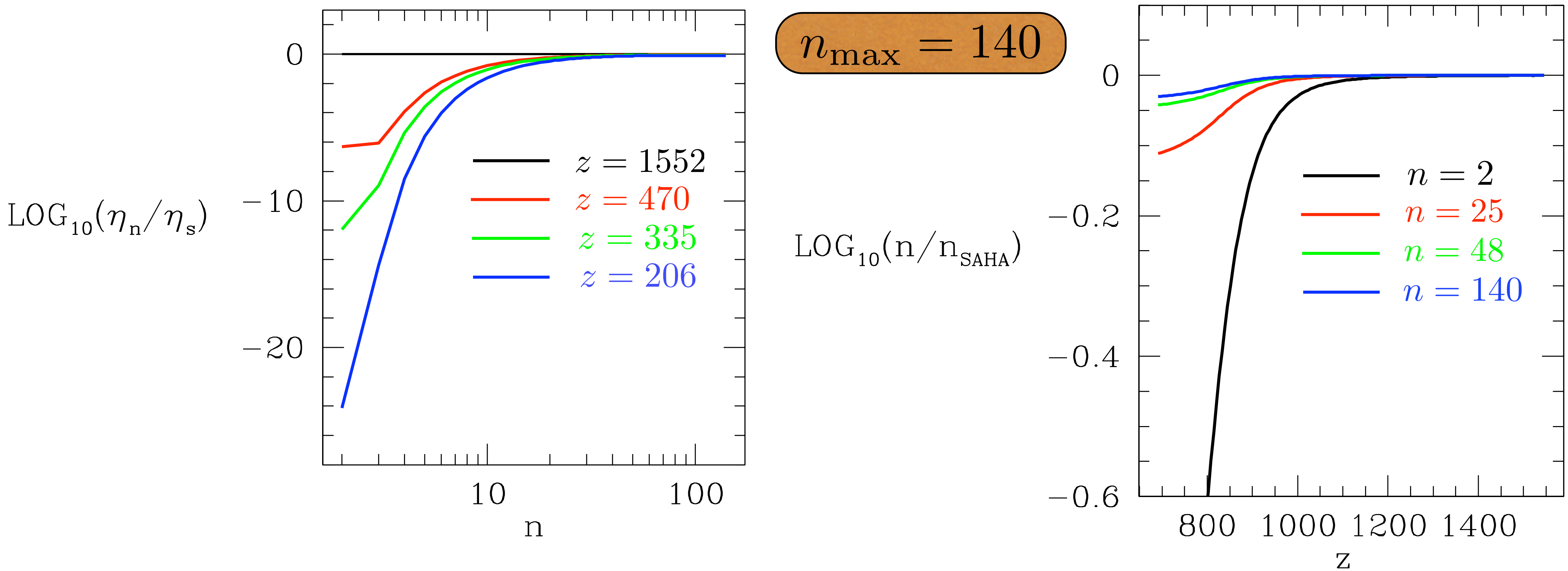
- $\alpha n \gtrsim A_{\mathrm{bb,down}}$.



DEVIATIONS FROM BOLTZMANN EQ: RESOLVING l

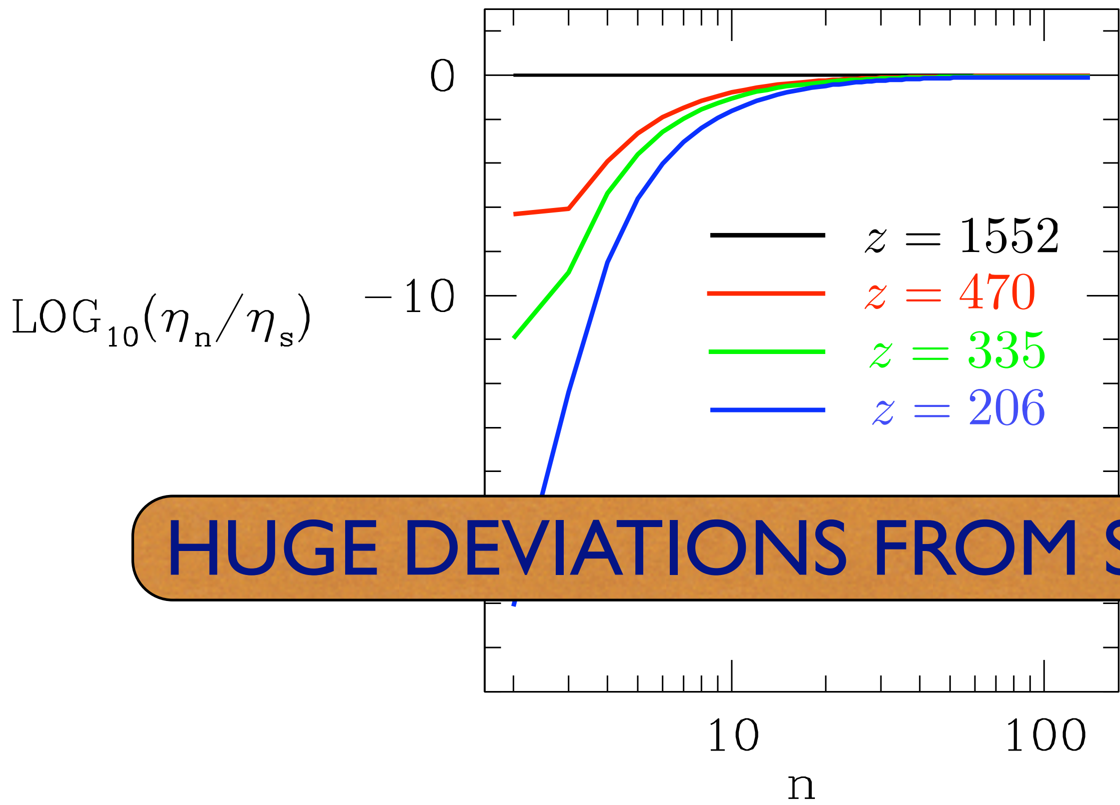


DEVIATIONS FROM SAHA EQUILIBRIUM



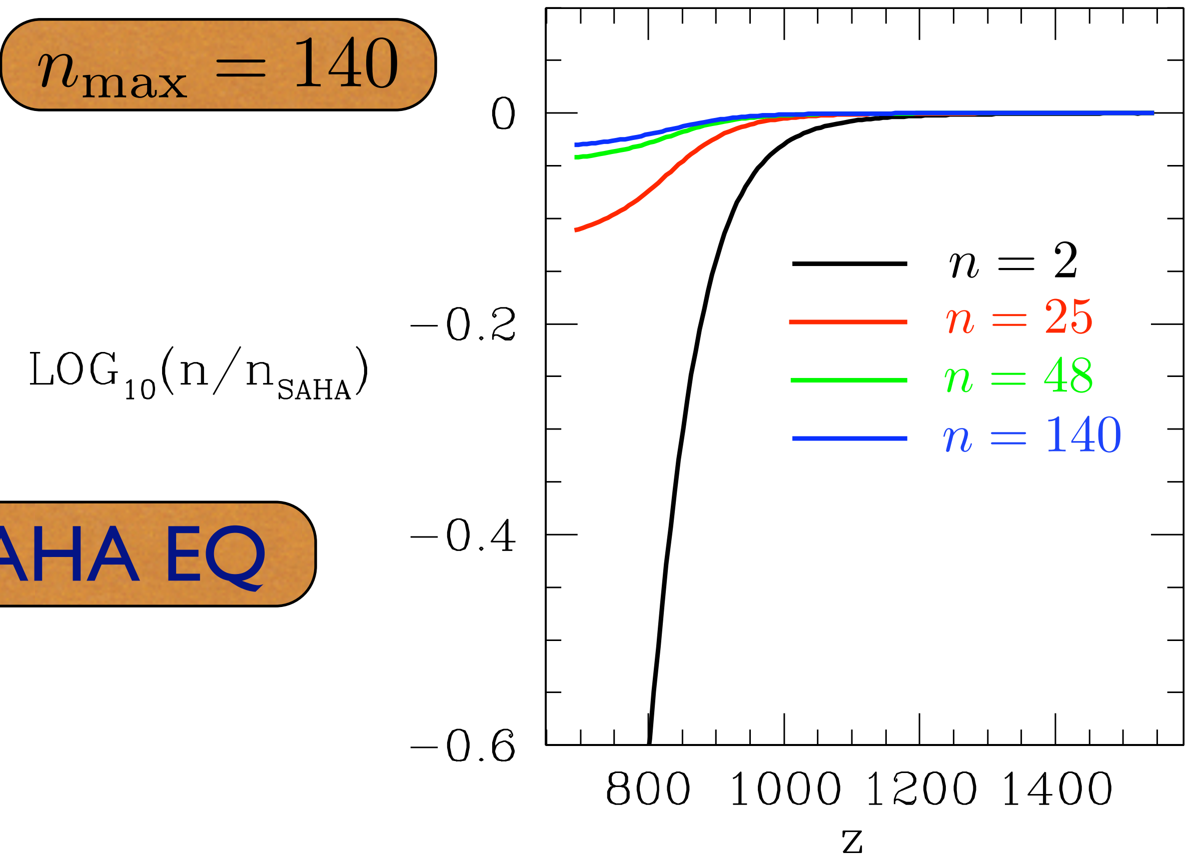
- n=1 suppressed due to freeze-out of x_e
- Remaining levels 'try' to remain in Boltzmann eq. with n=2
- Super-Boltz effects and two-γ transitions (n=1 $\longrightarrow$ n=2) yield less suppression for n>1
- Problem gets worse at late times (low z) as rates fall

DEVIATIONS FROM SAHA EQUILIBRIUM



HUGE DEVIATIONS FROM SAHA EQ

$n_{\max} = 140$



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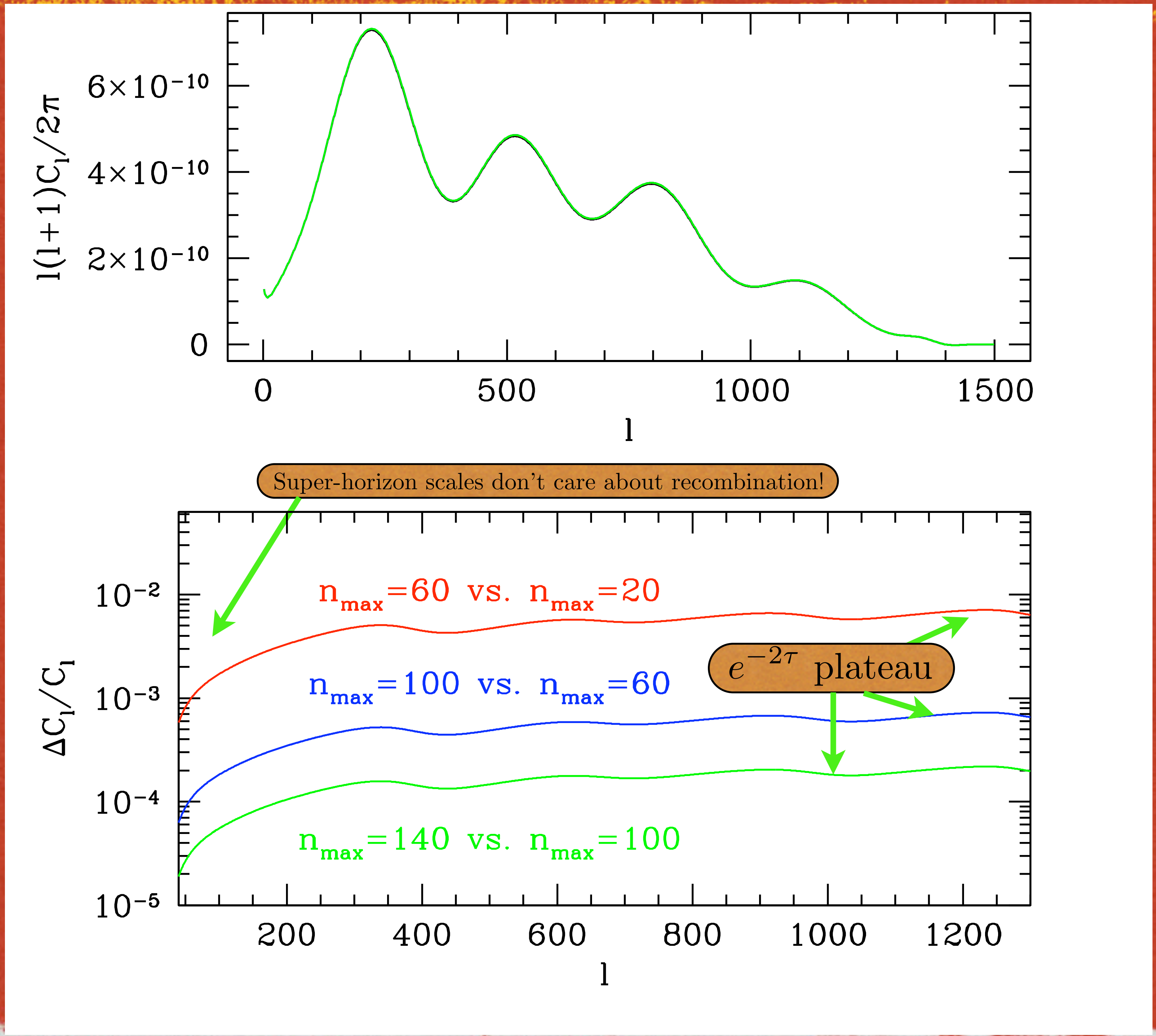
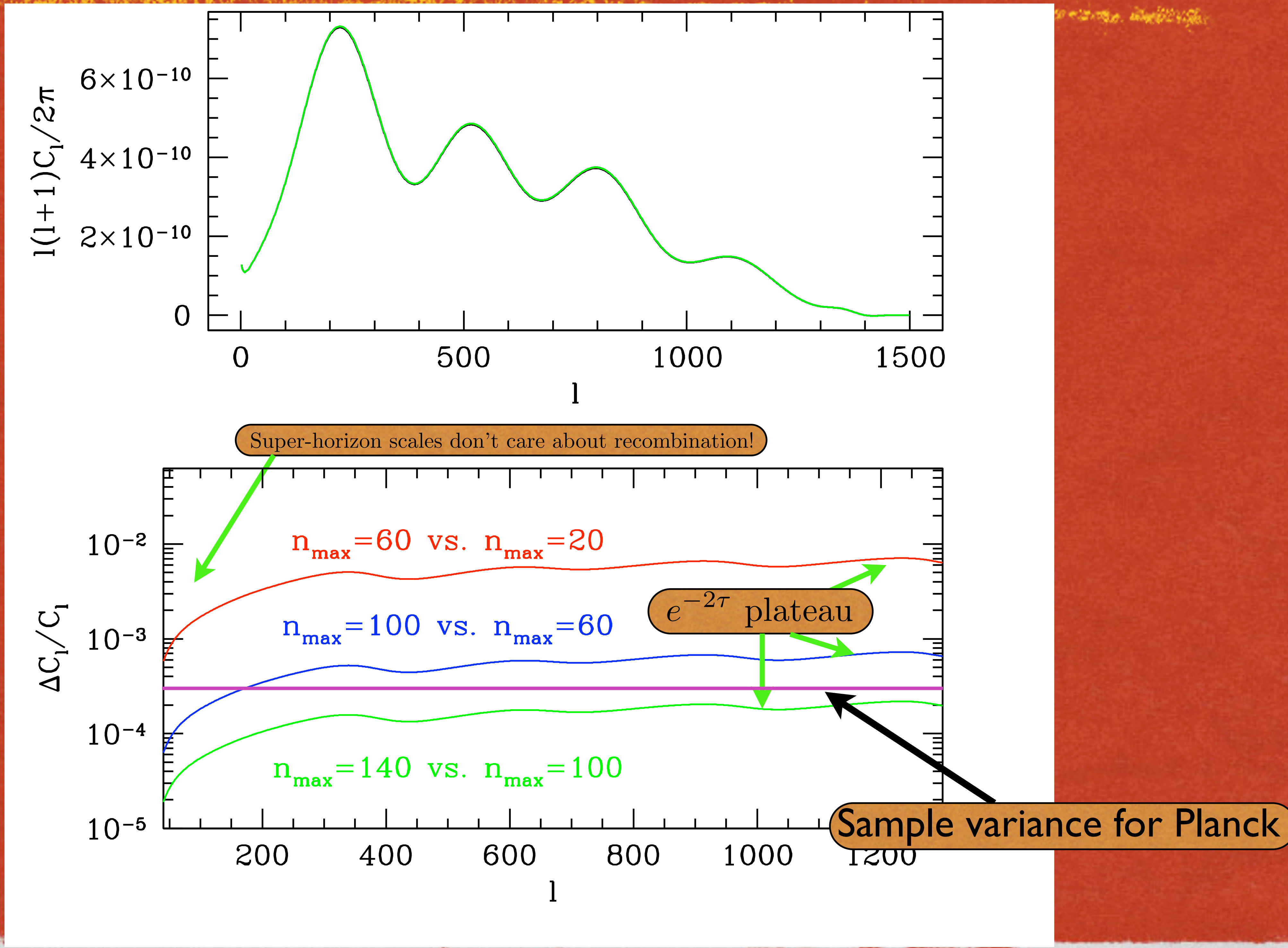
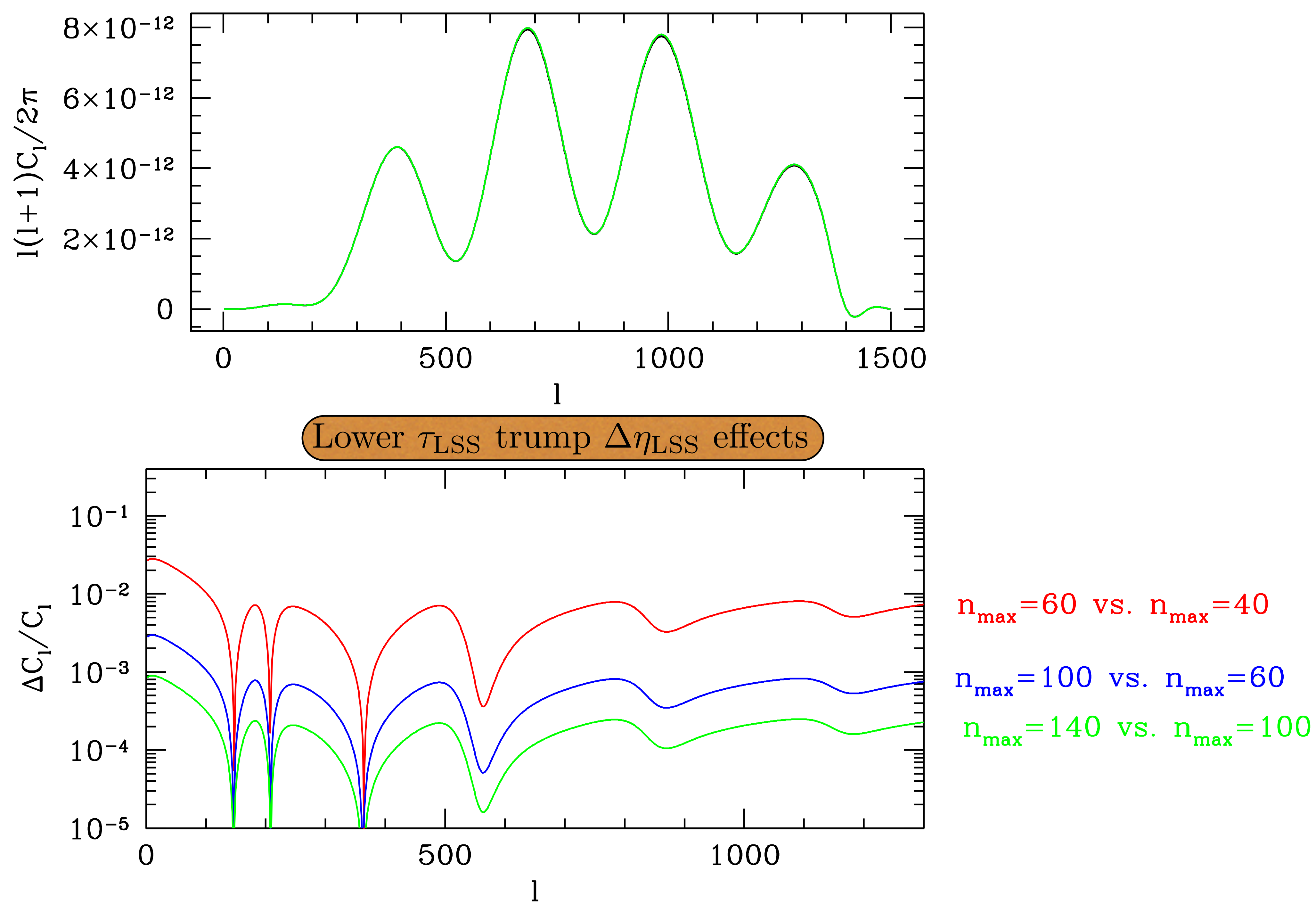
TEMPERATURE $C_l s$



TEMPERATURE $C_l s$



POLARIZATION $C_l s$



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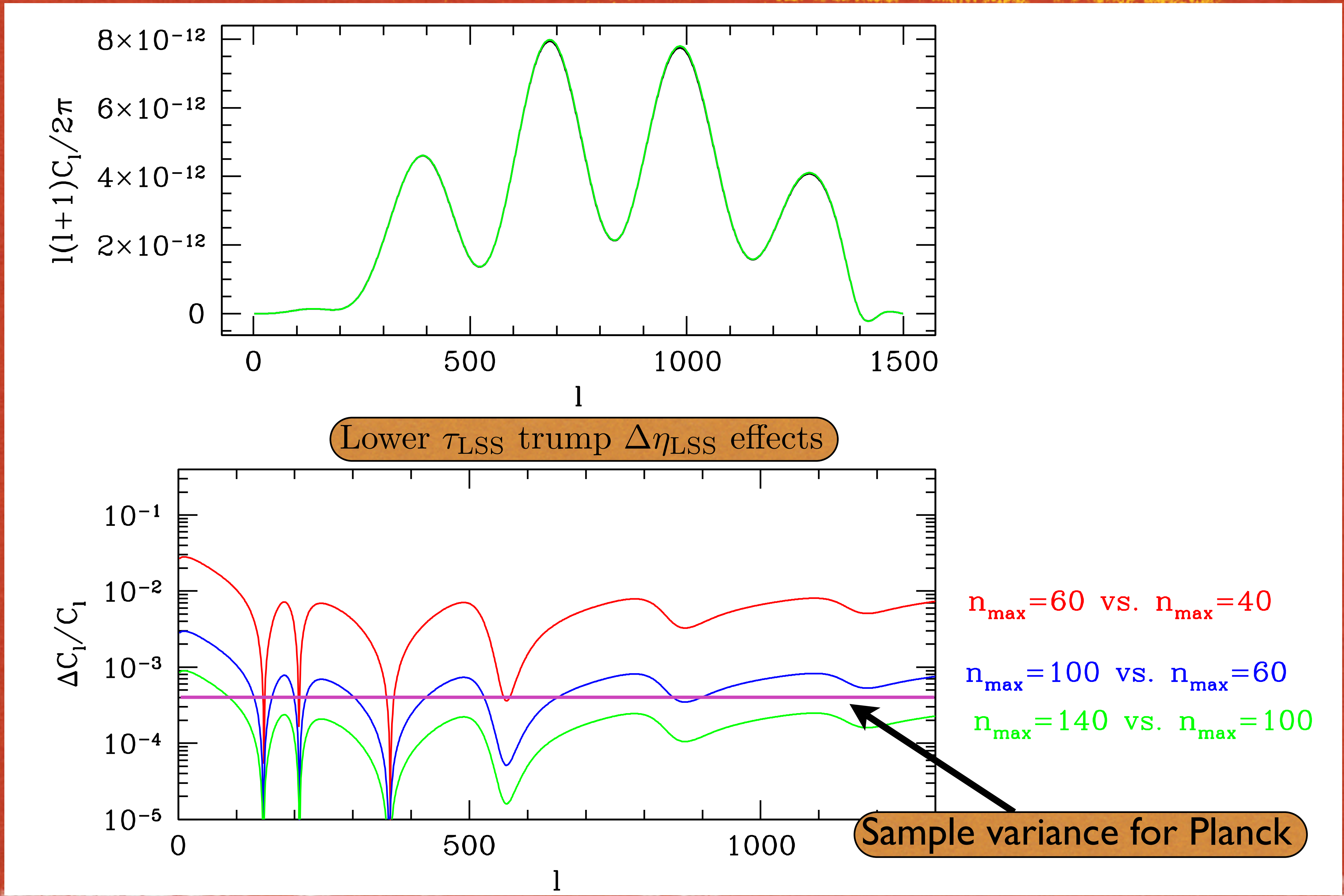
POLARIZATION $C_l s$



ATOMIC COLLISIONS

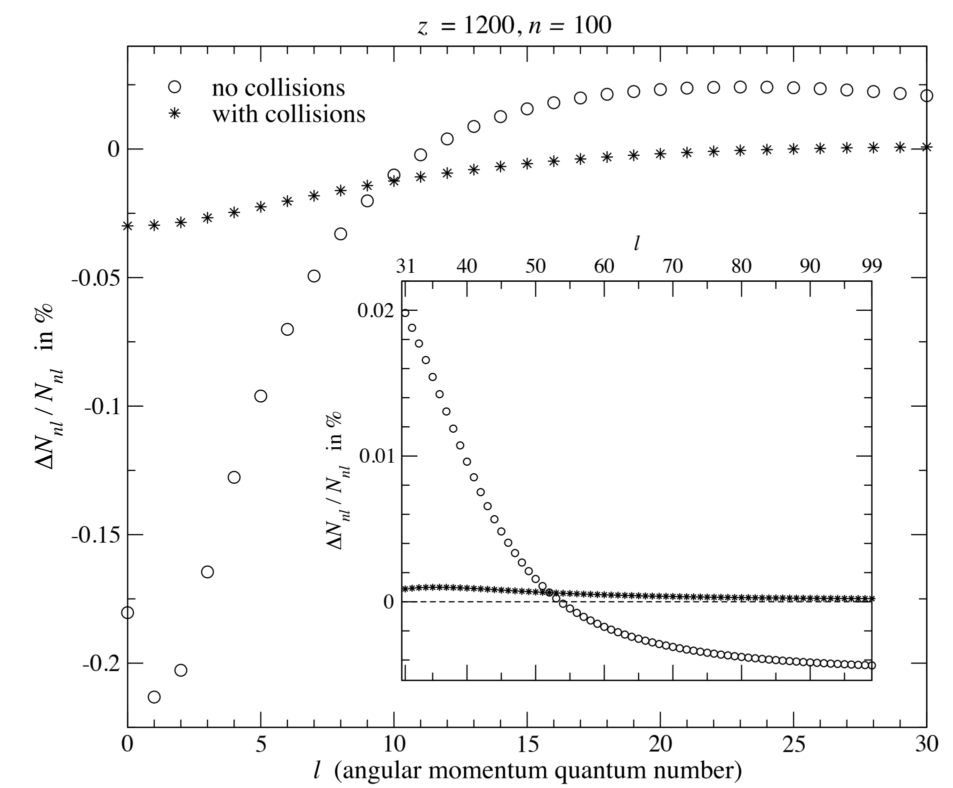
- For fixed n, l-changing collisions bring different-l substates closer to statistical equilibrium (SE)

- Being closer to SE speeds up rec. by mitigating high-l bottleneck

- Theoretical collision rates unknown to factors of 2!
  - $b < a_0 n^2 \rightarrow$ multi-body QM!
  - $t_{\text{pass}} < t_{\text{orbit}} \rightarrow$ Impulse approximation breaks down!

- Order-of-magnitude inclusion under way to determine if better theory needed for rec.

$n_{\text{max}} = 100$



QUADRAPOLE TRANSITIONS

- $\Delta l = \pm 2$ transitions may also play a role
- Rates are given by $A_{nn'}^{ll',m} = C\omega^5 \langle f|r^2 Y_{2m}|i\rangle$
- Moments may be evaluated with radial wf. raising/lowering operators
- Transitions to/from 1s will dominate
- Transitions from nd to 1s will immediately be followed by transitions up to mp, etc...
- Rate can thus be rewritten as an effective $\Delta l = \pm 1$ transition rate, thus respecting our sparsity pattern

WRAPPING UP

- Convergence with $n_{\max}$ of rec. history and CMB observables (for Planck, etc...) is now within reach, thanks to sparse matrix methods
- To do:
  - Line feedback via iterative procedure
  - Collisions
  - Quadrapole transitions
  - Effective source term for omitted higher levels- near Saha eq., should be tractable
  - Full incorporation into CMBFAST/CAMB and analysis of errors/degeneracies with cosmo. parameters, including other heretofore bits of atomic physics