COSMOLOGICAL HYDROGEN RECOMBINATION: THE EFFECT OF HIGH-N STATES

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NUPAC Talk

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OUTLINE

- Cosmological Recombination in a nutshell
- Breaking the naive model
- Why should you care? Effects on CMB, inferences about primordial physics
- Our tools
- Preliminary results!

SAHA EQUILIBRIUM IS INADEQUATE

$$\boxed{p + e^- \leftrightarrow H^{(n)} + \gamma^{(nc)}}$$

- Chemical equilibrium does reasonably well predicting "moment of recombination"

$$\frac{x_e^2}{1 - x_e} = \left(\frac{13.6}{T_{\rm eV}}\right)^{3/2} e^{35.9 - 13.6/T_{\rm eV}}$$

$$\boxed{x_e = 0.5 \text{ when } T = T_{\rm rec} \simeq 0.3 \text{ eV}} \quad z_{\rm rec} \simeq 1300$$

- Further evolution falls prey to reaction freeze-out

$$\Gamma = 6 \times 10^{-22} \text{ eV } x_e(T) \, (13.6/T_{\rm eV})^{-5/2} \ln(13.6/T_{\rm eV})$$

$$H = 1.1 \times 10^{-26} \text{ eV } T_{\rm eV}^{3/2}$$

$$\boxed{\Gamma < H \text{ when } T < T_{\rm F} \simeq 0.25 \text{ eV}}$$

BOTTLENECKS AND ESCAPE ROUTES

- BOTTLENECKS
  - Ground state recombinations are ineffective

$$\boxed{\tau^{-1}_{c\to 1s} = 10^{-1}\ \mathrm{s}^{-1} \gg H \simeq 10^{-12}\ \mathrm{s}^{-1}}$$

  - Resonance photons are re-captured, e.g. Lyman α

$$\boxed{\tau^{-1}_{2p\to 1s} = 10^{-2}\ \mathrm{s}^{-1} \gg H \simeq 10^{-12}\ \mathrm{s}^{-1}}$$

- ESCAPE ROUTES (e.g. n=2)
  - Two-photon processes

$$H^{2s} \to H^{1s} + \gamma + \gamma \qquad \boxed{\mathbf{\Lambda_{2s\to 1s} = 8.22}\ \mathrm{s}^{-1}}$$

  - Redshifting off resonance $\boxed{R \sim \left(n_{\mathrm{H}}\lambda_\alpha^3\right)^{-1}\left(\frac{\dot a}{a}\right)}$

EQUILIBRIUM ASSUMPTIONS

- Radiative eq. between different n-states

$$\mathcal{N}_n = \mathcal{N}_2 e^{-(E_n - E_2)/T}$$

- Radiative/collisional eq. between different l

$$\mathcal{N}_{nl} = \mathcal{N}_n \frac{(2l+1)}{n^2}$$

- Matter in eq. with radiation due to Thompson scattering

$$T_m = T_\gamma \text{ since } \frac{\sigma_{\mathrm{T}} a T_\gamma^4 c}{m_e c^2} < H(T)$$

THE PEEBLES PUNCHLINE

- Only n=2 bottlenecks are treated

$$\Gamma_{\mathrm{net,H}} = \Lambda_{2s\to 1s}\left[n_{2s} - n_{1s}e^{-(B_1-B_2)/kT}\right] + \frac{8\pi}{\lambda_\alpha^3}\frac{\dot{a}}{a}\times\left(f_\alpha - e^{-h\nu_\alpha/kT}\right)$$

- Net Rate is suppressed by bottleneck vs. escape factor

$$-\frac{dx_e}{dt} = \sum_{n,l}\alpha_{nl}(T)\left\{n x_e^2 + (2l+1)\,e^{-(B_1-B_n)/kT}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2}\right\}\mathcal{C}$$

$$\mathcal{C} = \frac{\frac{8\pi}{\lambda_\alpha^3 n_{1s}}\frac{\dot{a}}{a} + \Lambda_{2s\to 1s}}{\frac{8\pi}{\lambda_\alpha^3 n_{1s}}\frac{\dot{a}}{a} + (\Lambda_{2s\to 1s} + \beta_c)}$$

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Redshifting term

THE PEEBLES PUNCHLINE

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2γ term

THE PEEBLES PUNCHLINE

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$$\mathcal{C} = \frac{\frac{8\pi}{\lambda_\alpha^3 n_{1s}}\frac{\dot{a}}{a} + \Lambda_{2s\to 1s}}{\frac{8\pi}{\lambda_\alpha^3 n_{1s}}\frac{\dot{a}}{a} + (\Lambda_{2s\to 1s} + \boxed{\beta_c})}$$

Ionization Term

The Peebles Punchline

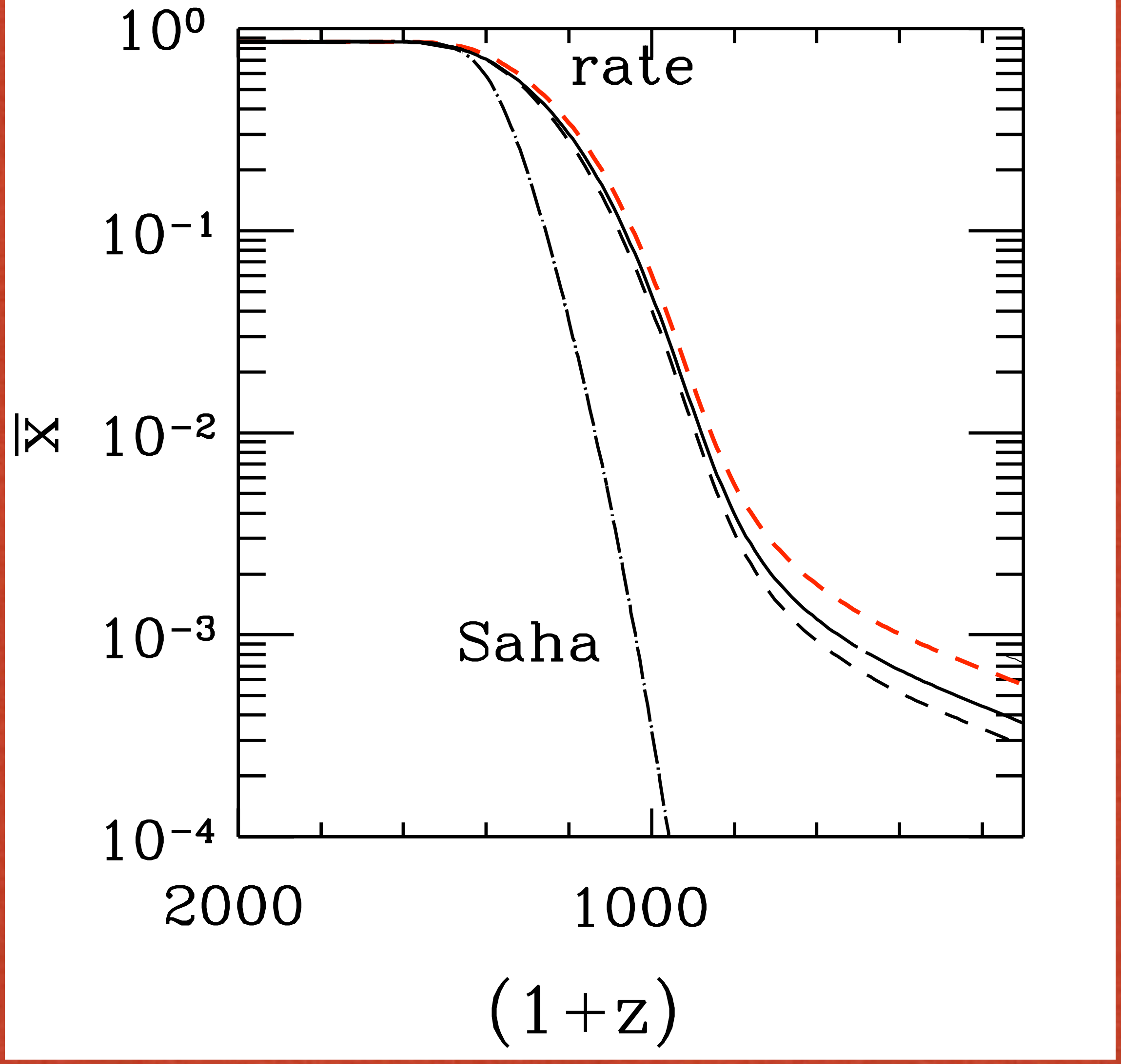
- Net Rate is suppressed by bottleneck vs. escape factor

$$\mathcal{C} = \frac{\frac{8\pi}{\lambda_\alpha^3 n_{1s}}\frac{\dot{a}}{a} + \Lambda_{2s\to 1s}}{\frac{8\pi}{\lambda_\alpha^3 n_{1s}}\frac{\dot{a}}{a} + \left(\Lambda_{2s\to 1s} + \beta_c\right)}$$

$$\frac{\text{redshift term}}{2\gamma\text{ term}} \simeq 0.02\frac{\boldsymbol{\Omega}_m^{1/2}}{\left(1 - x_e\left[z\right]\right)\left(\frac{1+z}{1100}\right)^{3/2}}$$

2γ process dominates until late times $(z \lesssim 850)$

PEEBLES MODEL ASSUMPTIONS/RESULTS



$\Omega_m h^2 \uparrow$
$\Omega_b h^2 \uparrow$

- State of the Art for 30 years!

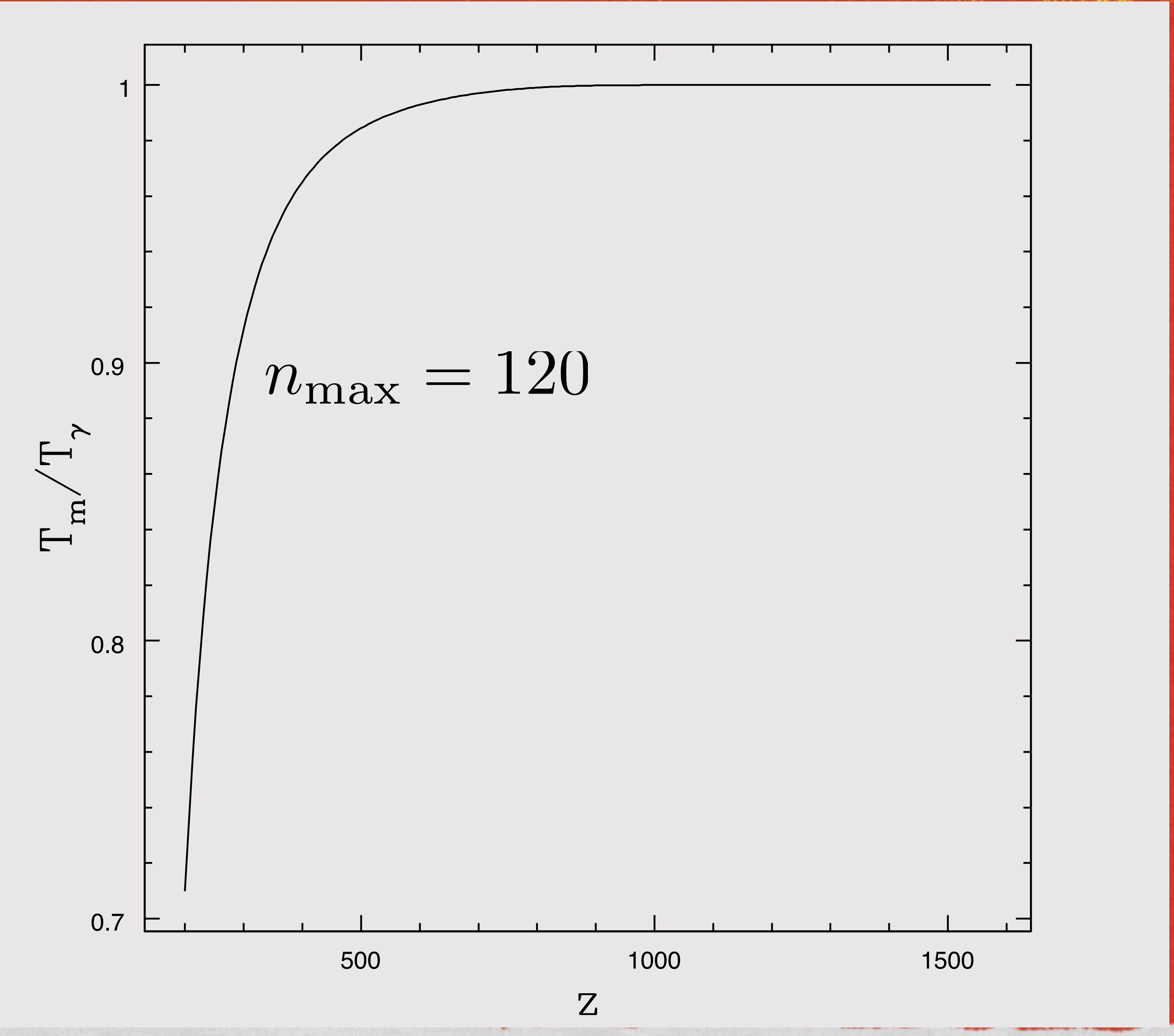
BREAKING THE NAIVE MODEL

- Radiation field is cool: ~~Boltzmann eq. of higher n~~
- $\boxed{\text{Treated by Seager et al. (2000) } n_{\max} = 300}$ RecFAST!!!
- ~~Equilibrium~~ between *l states*
- Treated by $\boxed{\text{Chluba et al. (2005) for } n_{\max} = 100}$
- Radiation and matter field fall out of eq.

$$\dot{T_M} + 2HT_m = \frac{8x_e\sigma_{\rm T}aT_\gamma^4}{3m_ec\,(1 + f_{\rm He} + x_e)}\,(T_M - T_\gamma)$$

- Higher-order 2γ transitions, (Hirata, Ali-Haimoud, in progress)

Decoupling of matter and radiation



BREAKING THE NAIVE MODEL

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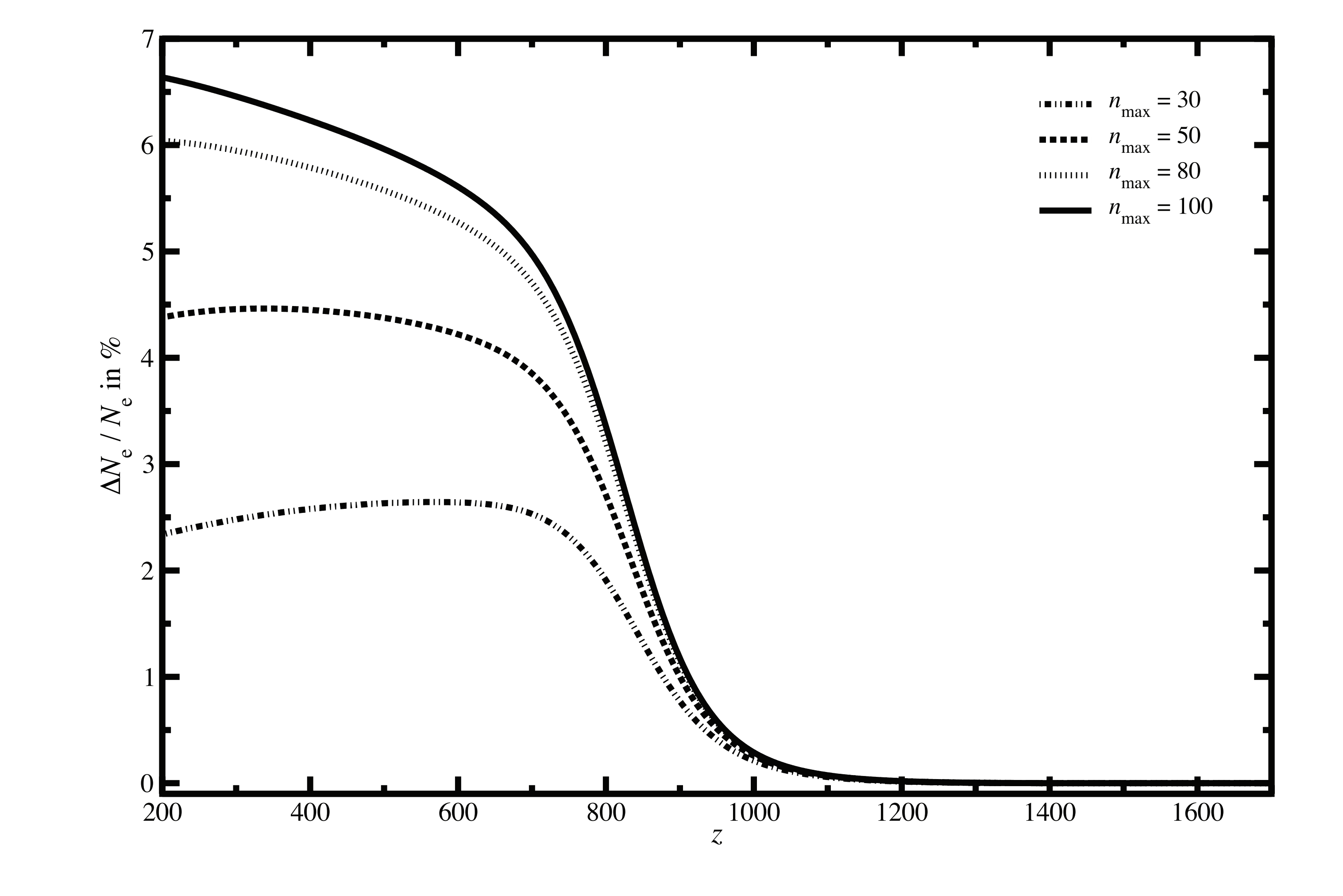
- ~~Equilibrium~~ between *l states*
- Treated by Chluba et al. (2005) for $n_{\rm max} = 100$
- Beyond this, testing convergence with $n_{\rm max}$ is hard!

$$t_{\rm compute} \sim \mathcal{O}\,(\text{weeks})$$

How to proceed if we want 0.1% accuracy in $x_e(z)$?

The effect of resolving l- substates



- Putting free-electrons in 'bottlenecked' l-substates slows down the decay to 1s: Recombination is slower

BREAKING THE NAIVE MODEL

- Radiation field is cool: ~~Boltzmann eq. of higher n~~
- $\boxed{\text{Treated by Seager et al. (2000) } n_{\max} = 300}$ RecFAST!!!
- ~~Eq.~~ between *l states*: dipole selection bottleneck: $\Delta l = \pm 1$
- Treated by $\boxed{\text{Chluba et al. (2005) for } n_{\max} = 100}$
- Beyond this, testing convergence with $n_{\max}$ is hard!

$$t_{\text{compute}} \sim \mathcal{O}\,(\text{weeks})$$

$\boxed{\text{WHY PROCEED?}}$

Who Cares?

I. Smearing and moving the surface of last scattering (SLSS)

- Photons kin. decouple when Thompson scattering freezes out

$$\gamma + e^{-} \Leftrightarrow \gamma + e^{-}$$

$$\Gamma = n_e \sigma_{\mathrm{T}} c = 2.2 \times 10^{-19}\ \mathrm{s}^{-1} \frac{x_e \Omega_b h^2}{a^3} =$$

$$H = H_0 \Omega_m^{1/2} a^{-3/2} \left[1 + \frac{a_{\mathrm{eq}}}{a}\right]^{1/2}$$

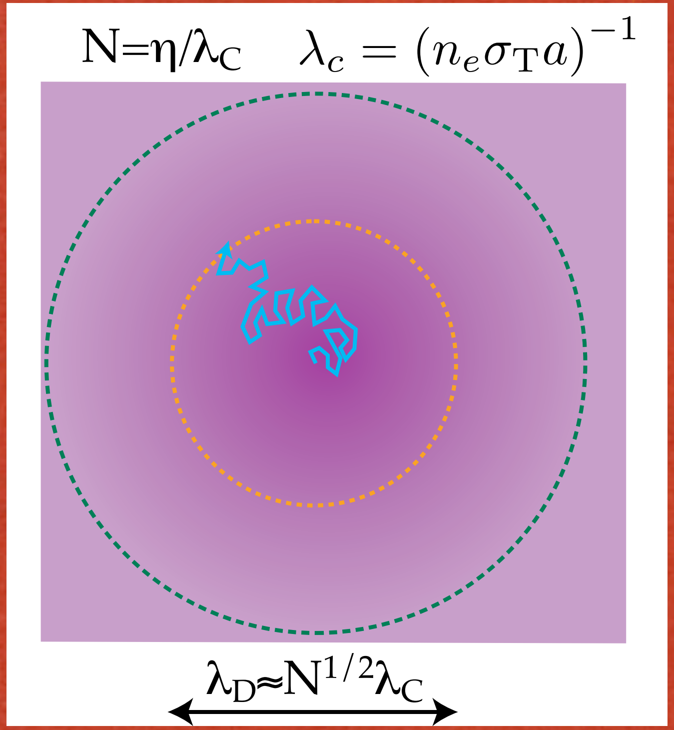
- $z_{\mathrm{dec}} \simeq 1100$:Decoupling occurs during recombination

$$C_l \to C_l e^{-2\tau} \quad \text{if } l > \frac{\eta_{\mathrm{rec}}}{\eta_{\mathrm{hor}}}$$

$$\boxed{\tau = \int_0^{\eta_{\mathrm{dec}}} d\eta n_e [\eta] \sigma_{\mathrm{T}} a(\eta)}$$

Who Cares?
II. The Silk Damping Tail

- From Wayne Hu's website



$$l_{\rm damp} \sim 1000$$

- Inhomogeneities are damped for $\lambda < \lambda_D$

$$k_D^{-2}(\eta) \simeq \int_0^\eta \frac{d\eta'}{6(1+R)n_e[\eta']\sigma_T a[\eta']}\left[\frac{R^2}{1+R}+\frac{8}{9}\right]$$

$$R = \frac{3\rho_b^0}{4\rho^\gamma}$$

$$|\Theta_l(\eta_0)| \simeq \int_0^{\eta_0} d\eta\, \dot{\tau} e^{-\tau(\eta)} e^{ik\int d\eta c_s} e^{-k^2/k_D^2(\eta)} \tilde{\delta}(k) j_l(k(\eta-\eta_0))dk$$

Who Cares?

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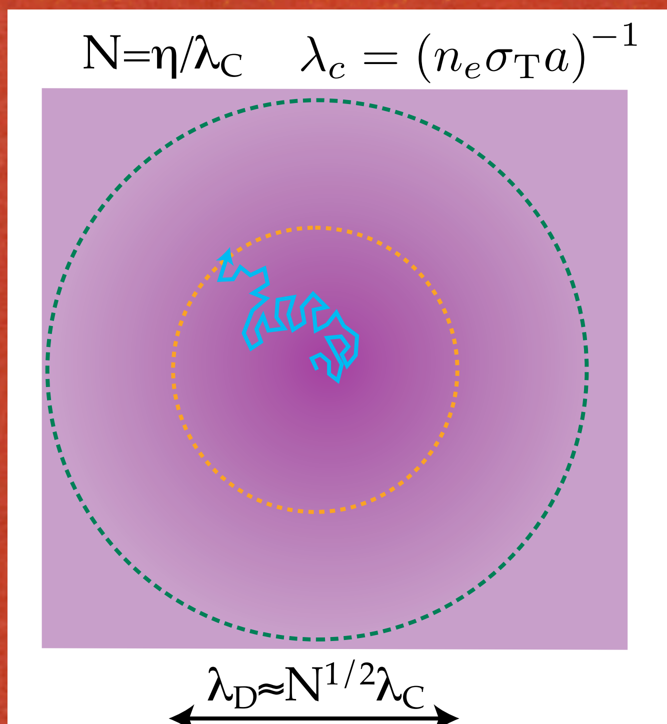

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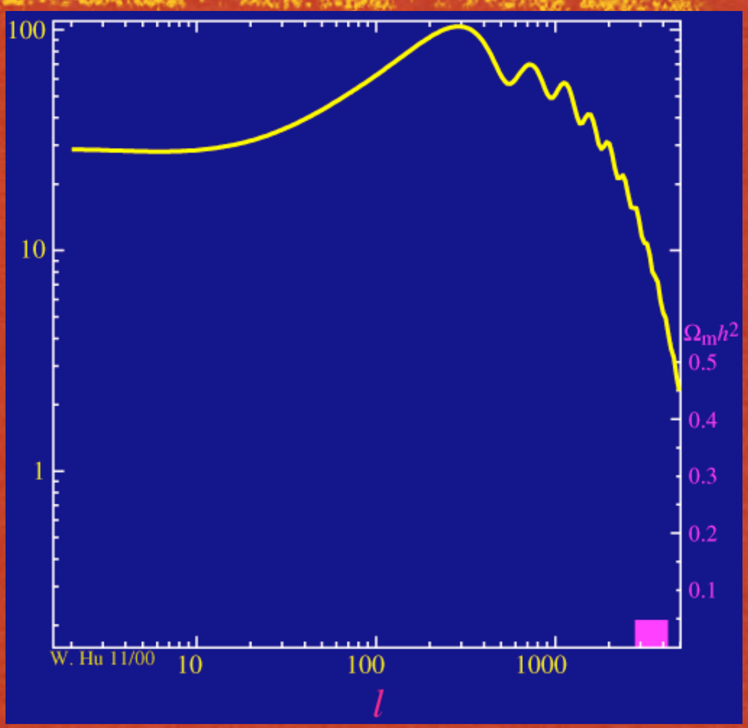

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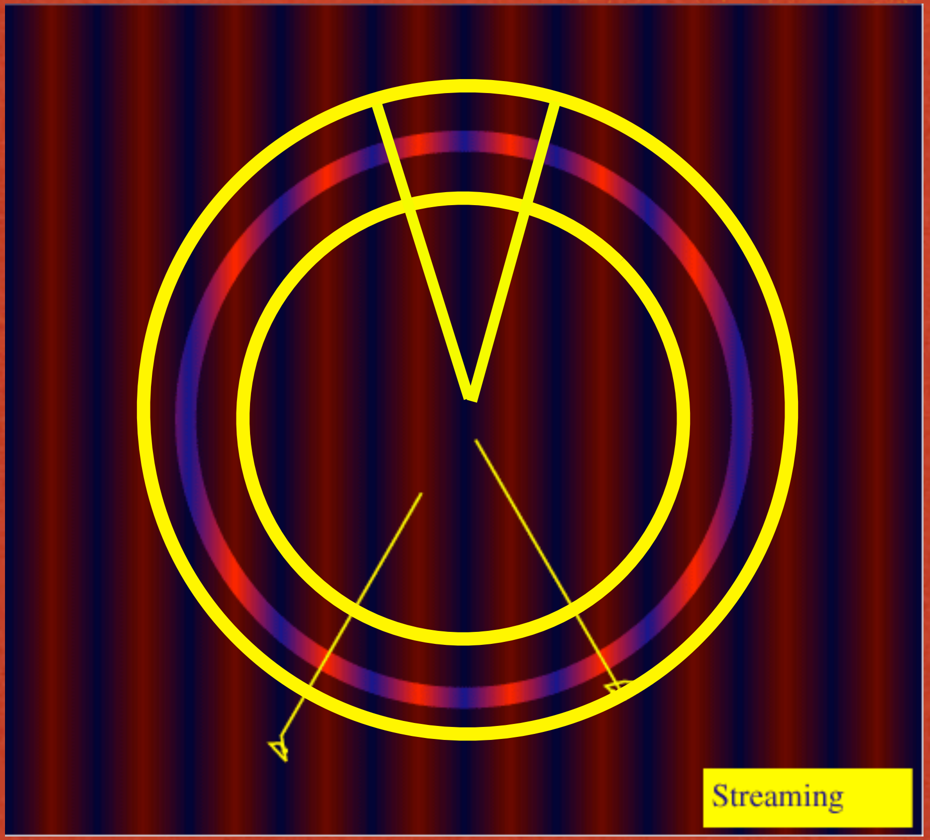
WHO CARES?
III. FINITE THICKNESS OF THE SLSS



- Additional damping of form

$$|\Theta_l\left(\eta_0, k\right)| \rightarrow |\Theta_l\left(\eta_0, k\right)| \, e^{-\sigma^2 \eta_{\rm rec}^2 k^2}$$

Who Cares?
IV. CMB Polarization

- Need to scatter quadrapole to polarize CMB

$$\Theta_l^P(k) = \int d\eta \dot{\tau} e^{-\tau(\eta)} \Theta_{T,2}(k,\eta) \frac{l^2}{(k\eta)^2} j_l(k\eta)$$

- Need time to develop a quadrapole

$$\Theta_l(k\eta) \sim \frac{k\eta}{2\tau} \Theta_l(k\eta) \ll \Theta_l(\eta) \text{ if } l \geq 2, \text{ in tight coupling regime}$$

Who Cares?
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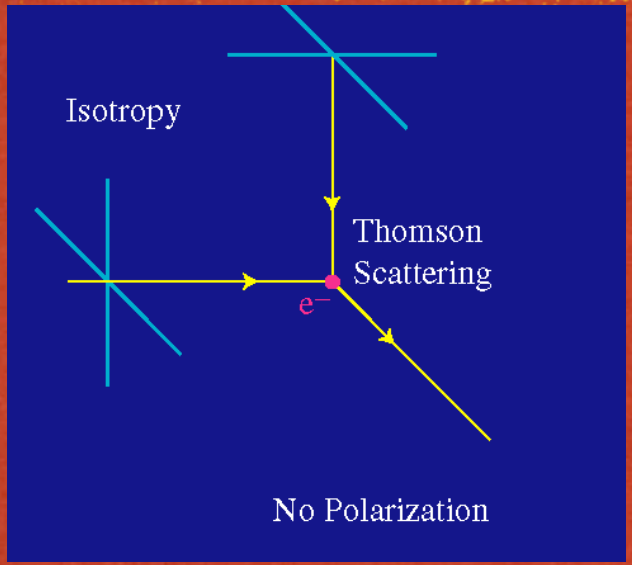

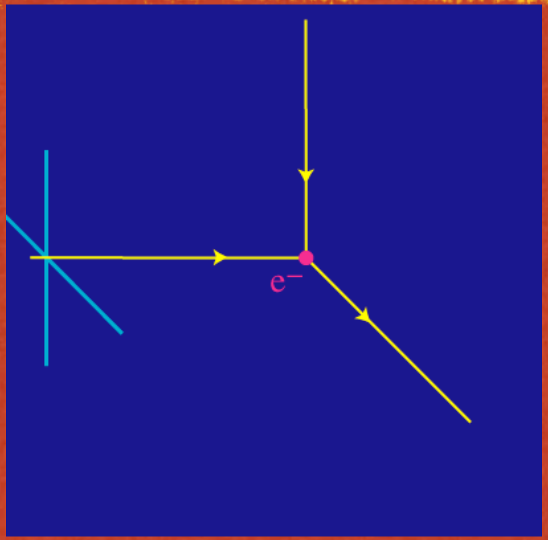

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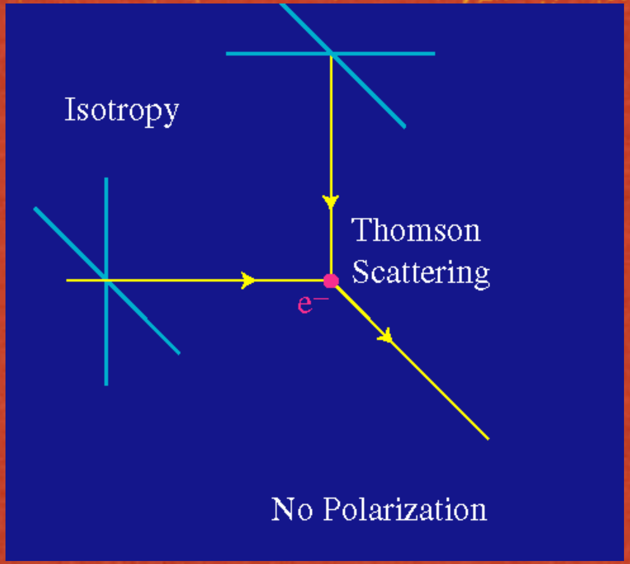

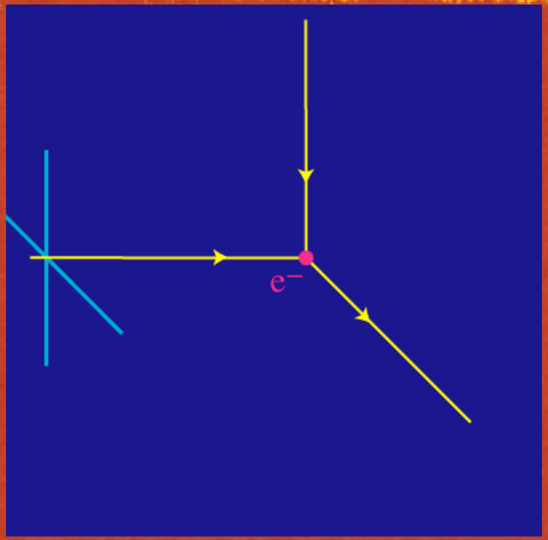

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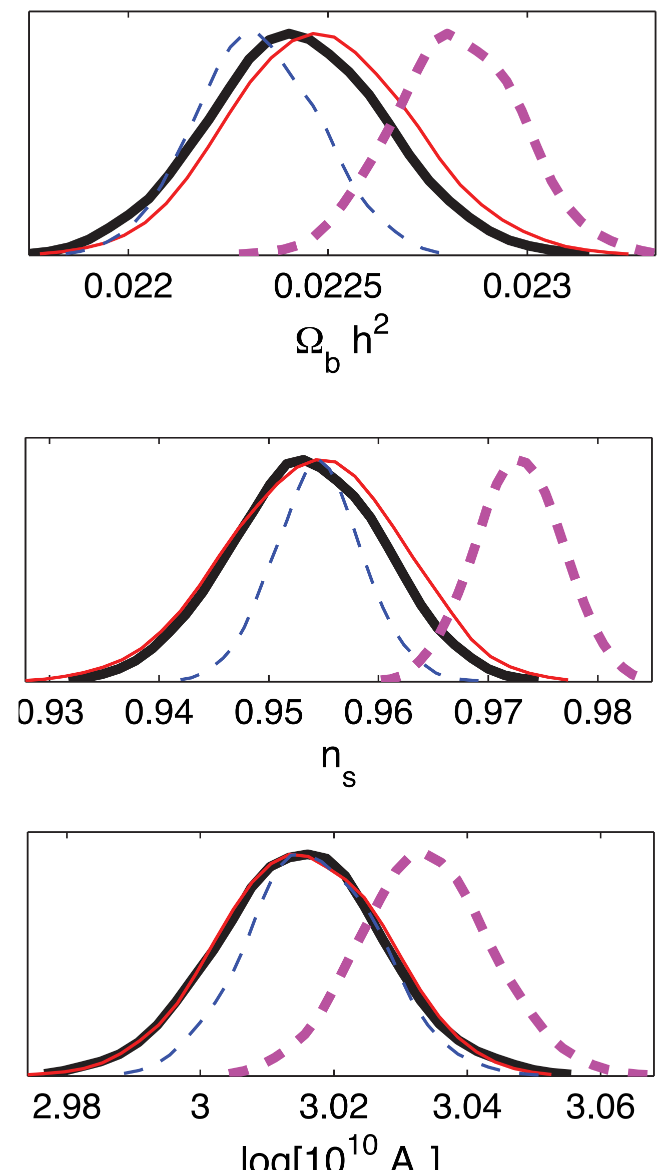
Who Cares?

V. PARAMETER DEGENERACIES

- Planck will be CV limited (T and E) to $l \sim 2500$

- 0.1% accuracy required in $x_e\,(z)$

- Planck uncertainty forecasts using MCMC



THE MULTI-LEVEL ATOM (MLA)

- Bound-free rate equation

$$\dot{x}_{nl}^{bf} = \int dE_{\rm e} P_M(T_m, E_{\rm e}) n_H x_e x_p \left[1 + f(E_e - E_n)\right] \alpha_{nl}(E_{\rm e})$$
$$- \int dE_{\rm e}\ g(E_{\rm E} - E_n)\ x_{nl}\ f(E_{\rm e} - E_{nl}) \alpha_{nl}(E_{\rm E})/g_{nl}$$

- Bound-bound rate equation

$$\dot{x}_{nl}^{bb} = \sum_{n',l'=l\pm1} (A_{nn'}^{ll'}(1 + f_{nn'}) x_{n',l'} - \frac{g_{n'l'}}{g_{nl}} f_{nn'} x_{nl}) P_{nn'}^{ll'},$$

THE MULTI-LEVEL ATOM (MLA)

- Bound-free rate equation

Ω_m, Ω_b, h

$$\dot{x}^{bf}_{nl} = \int dE_\mathrm{e} P_M(T_m, E_\mathrm{e}) n_H x_e x_p \left[1 + f(E_e - E_n)\right] \alpha_{nl}(E_\mathrm{e})$$
$$- \int dE_\mathrm{e}\; g(E_\mathrm{E} - E_n)\; x_{nl}\; f(E_\mathrm{e} - E_{nl}) \alpha_{nl}(E_\mathrm{E})/g_{nl}$$

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The multi-level atom (MLA)

- Bound-free rate equation

$$\dot{x}_{nl}^{bf} = \int dE_{\mathrm{e}} P_M(T_m, E_{\mathrm{e}}) n_H x_e x_p \left[1 + f(E_e - E_n)\right] \alpha_{nl}(E_{\mathrm{e}}) - \int dE_{\mathrm{e}}\ g(E_{\mathrm{E}} - E_n)\ x_{nl}\ f(E_{\mathrm{e}} - E_{nl}) \alpha_{nl}(E_{\mathrm{E}})/g_{nl}$$

- Bound-bound rate equation

$$\dot{x}_{nl}^{bb} = \sum_{n',l'=l\pm 1} (A_{nn'}^{ll'}(1 + f_{nn'}) x_{n',l'} - \tfrac{g_{n'l'}}{g_{nl}} f_{nn'} x_{nl}) P_{nn'}^{ll'},$$

- Phase-space density blueward of line
- Escape probability of γ in line

THE MULTI-LEVEL ATOM (MLA)

Stimulated emission/absorption

- Bound-free rate equation

$$\dot{x}_{nl}^{bf} = \int dE_{\rm e} P_M(T_m, E_{\rm e}) n_H x_e x_p \left[1 + \boxed{f(E_e - E_n)}\right] \alpha_{nl}(E_{\rm e})$$

$$- \int dE_{\rm e}\ g(E_{\rm E} - E_n)\ x_{nl}\ \boxed{f(E_{\rm e} - E_{nl})} \alpha_{nl}(E_{\rm E})/g_{nl}$$

- Bound-bound rate equation

$$\dot{x}_{nl}^{bb} = \sum_{n',l'=l\pm1} (A_{nn'}^{ll'}(1 + \boxed{f_{nn'}}) x_{n',l'} - \frac{g_{n'l'}}{g_{nl}} \boxed{f_{nn'}} x_{nl}) P_{nn'}^{ll'},$$

THE MULTI-LEVEL ATOM (MLA)

Spontaneous Emission

- Bound-free rate equation

$$\dot{x}_{nl}^{bf} = \int dE_{\rm e} P_M(T_m, E_{\rm e}) n_H x_e x_p \left[1 + f(E_e - E_n)\right] \alpha_{nl}(E_{\rm e})$$
$$- \int dE_{\rm e}\; g(E_{\rm E} - E_n)\; x_{nl}\; f(E_{\rm e} - E_{nl}) \alpha_{nl}(E_{\rm E})/g_{nl}$$

- Bound-bound rate equation

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RATE COEFFICIENTS

- Bound-bound rates given by Fermi's golden rule and matrix element

$$\rho(n'l',nl) = \int_0^\infty u_{n'l'}(r)u_{nl}(r)r^3dr = \mathcal{C}\times\left[F_{2,1}\left(-n+l+1,-n'+l,2l,\frac{-4nn'}{(n-n')^2}\right)\right.$$
$$\left.-\left(\frac{n-n'}{n+n'}\right)^2 F_{2,1}\left(-n+l-1,-n'+l,2l,\frac{-4nn'}{(n-n')^2}\right)\right]^2$$

- Power-series destabilizes at high-n, recursion relation used
- Bound-free rates at temperature T given by phase space integral of matrix element $g_{nl} = \int_0^\infty u_{nl}(r)\,f_{El}\,(r)\,r^3dr$
- Rates are tabulated at all n and l of interest, at a variety of energies, and integrated at each time step

RATE COEFFICIENTS

- Rates are tabulated at all n and l of interest, at a variety of energies, and integrated at each time step $\rho(n'l', nl) = a_0 n^2 \int_{-\pi}^{\pi} d\tau e^{i\Omega\tau} \left(1 + cos\eta\right)$

$$\Omega = \omega_n - \omega_{n'}$$

$$r = r_{\max} \left(1 + \cos\eta\right)/2$$

$$\tau = \eta + \sin\eta$$

Fourier transform of classical orbit!
Application of correspondence principle!

- Similar WKB approximation can be used to check stability of BF matrix elements

Radiation Field: Black Body+

- Escape probability treated in Sobolev approx.

$$P^{l,l'}_{n,n'} = \frac{1-e^{-\tau_s}}{\tau_s}$$

$$\boxed{\tau_s = \frac{c^3 n_{\rm H}}{8\pi H \nu^3_{nn'}} A^{ll'}_{nn'} \left[\frac{g^{l'}_{n'}}{g^l_n} x^l_n - x^{l'}_{n'}\right]}$$

$$\mathcal{R}(\nu,\nu') = \phi(\nu)\phi(\nu')$$

$$\frac{v_{\rm th}}{H(z)} \ll \lambda$$

- Excess line photons injected into radiation field

$$\left(\frac{8\pi\nu^3_{nn'}}{c^3 n_H}\right)\left(f^+_{nn'} - f^-_{nn'}\right) = A^{ll'}_{nn'} P^{ll'}_{nn'} \left[x^l_n\left(1+f^+_{nn'}\right) - \frac{g^l_n}{g^{l'}_{n'}} x^{l'}_{n'} f^+_{nn'}\right]$$

- Photons are conserved outside of line regions

$$f^{+10}_{n1} = f^{+10}_{n+1,1}\left[\frac{1-(n+1)^{-2}}{1-n^{-2}}(1+z) - 1\right]$$

Radiation Field: Black Body+

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$$P^{l,l'}_{n,n'} = \frac{1-e^{-\tau_s}}{\tau_s}$$

$$\boxed{\tau_s = \frac{c^3 n_{\mathrm{H}}}{8\pi H \nu^3_{nn'}} A^{ll'}_{nn'} \left[\frac{g^{l'}_{n'}}{g^l_n} x^l_n - x^{l'}_{n'}\right]}$$

$$\mathcal{R}(\nu,\nu') = \phi(\nu)\phi(\nu')$$

- Forbes and Hirata are solving FP eqn. to obtain evolution of $\mathcal{R}(\nu,\nu')$ more generally, including atomic recoil/diffusion and full time-dependence of problem

STEADY-STATE APPROXIMATION FOR EXCITED STATES

- Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

STEADY-STATE APPROXIMATION FOR EXCITED STATES

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$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

$$\vec{x} = \begin{pmatrix} \vec{x}_0 \\ \vec{x}_1 \\ \dots \\ \vec{x}_{n_{\mathrm{max}}-1} \end{pmatrix}$$

STEADY-STATE APPROXIMATION FOR EXCITED STATES

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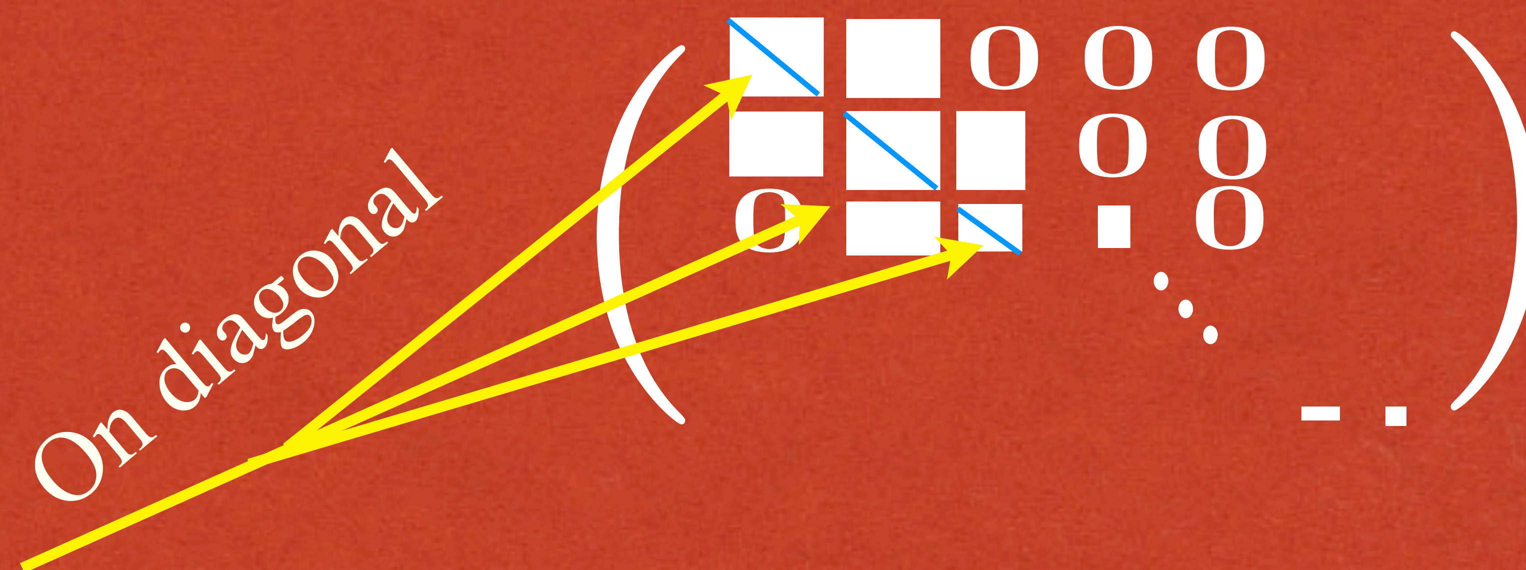

For state l, includes BB transitions out of l to all other l”, photo-ionization, 2γ transitions to ground state

STEADY-STATE APPROXIMATION FOR EXCITED STATES

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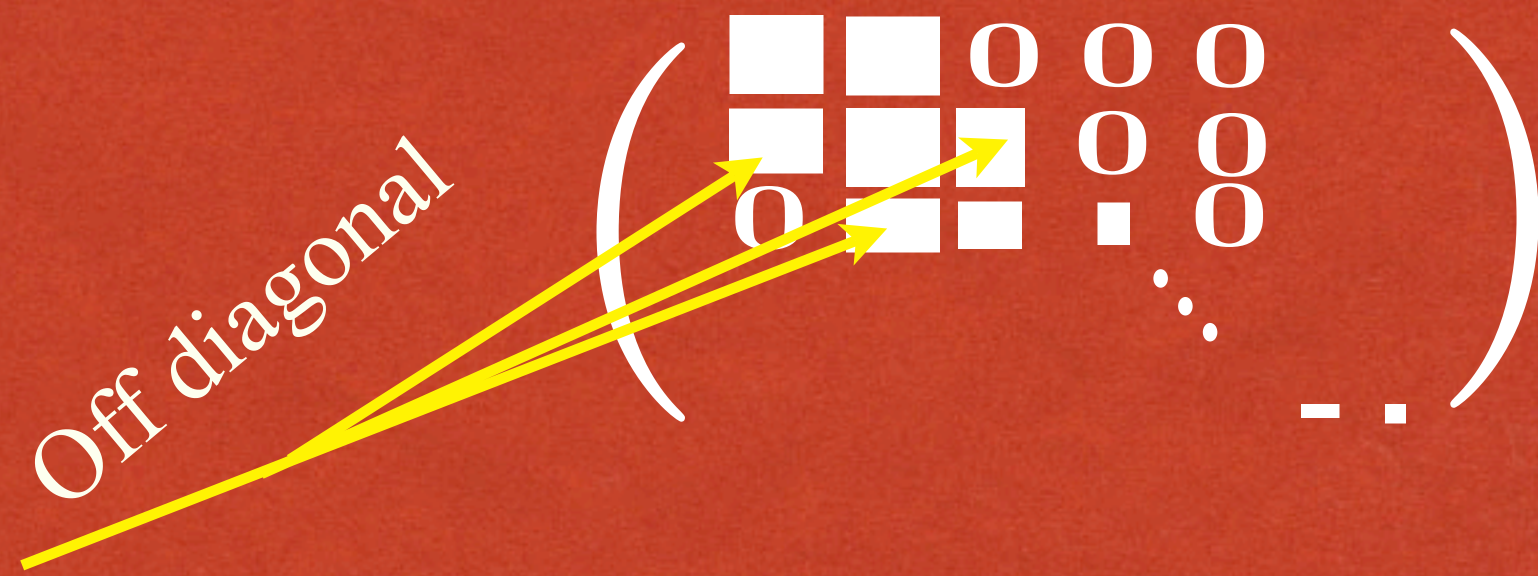


For state l, includes BB transitions into l from all other l’

STEADY-STATE APPROXIMATION FOR EXCITED STATES

- Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \boxed{\vec{s}}$$

- Includes recombination to l,
  1 and 2γ transitions from ground state

STEADY-STATE APPROXIMATION FOR EXCITED STATES

- Evolution equations may be re-written in matrix form

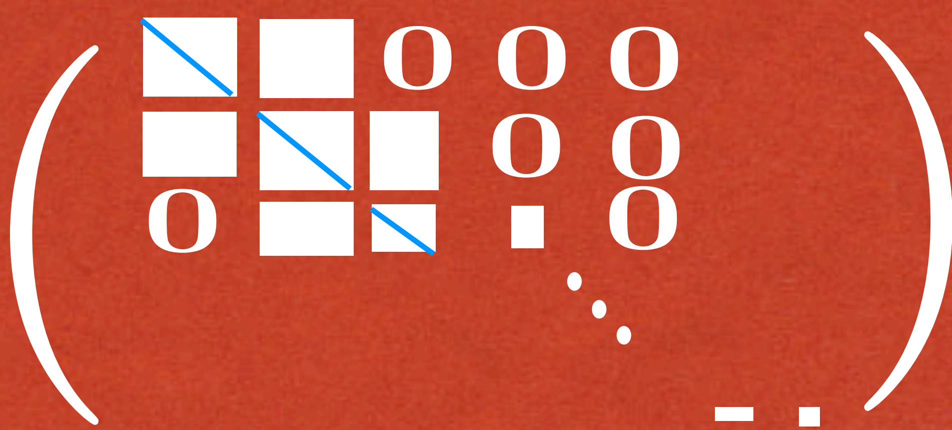
$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

- For n>1, $\boxed{t_{\rm rec}^{-1} \sim 10^{-12} s^{-1} \ll \mathbf{R}\ , \vec{s} \rightarrow \vec{x} \simeq \mathbf{R}^{-1}\vec{s}}$

$$\mathbf{R} \lesssim 1\ \mathrm{s}^{-1}\ \ (\text{e.g. Lyman-}\alpha)$$

RAPID MATRIX INVERSION: SPARSITY TO THE RESCUE

- Matrix is $\sim n_{max}^2 \times n_{max}^2$
- Brute force would require $n_{max}^6 \sim 1000$ s for $n_{max} = 200$ for a single time step
- Sparsity to the rescue $\Delta l = \pm 1$



$$\boxed{\mathbf{M}_{l,l-1}\vec{x}_{l-1} + \mathbf{M}_{l,l}\vec{x}_l + \mathbf{M}_{l,l+1}\vec{x}_{l+1} = \vec{s}_l}$$

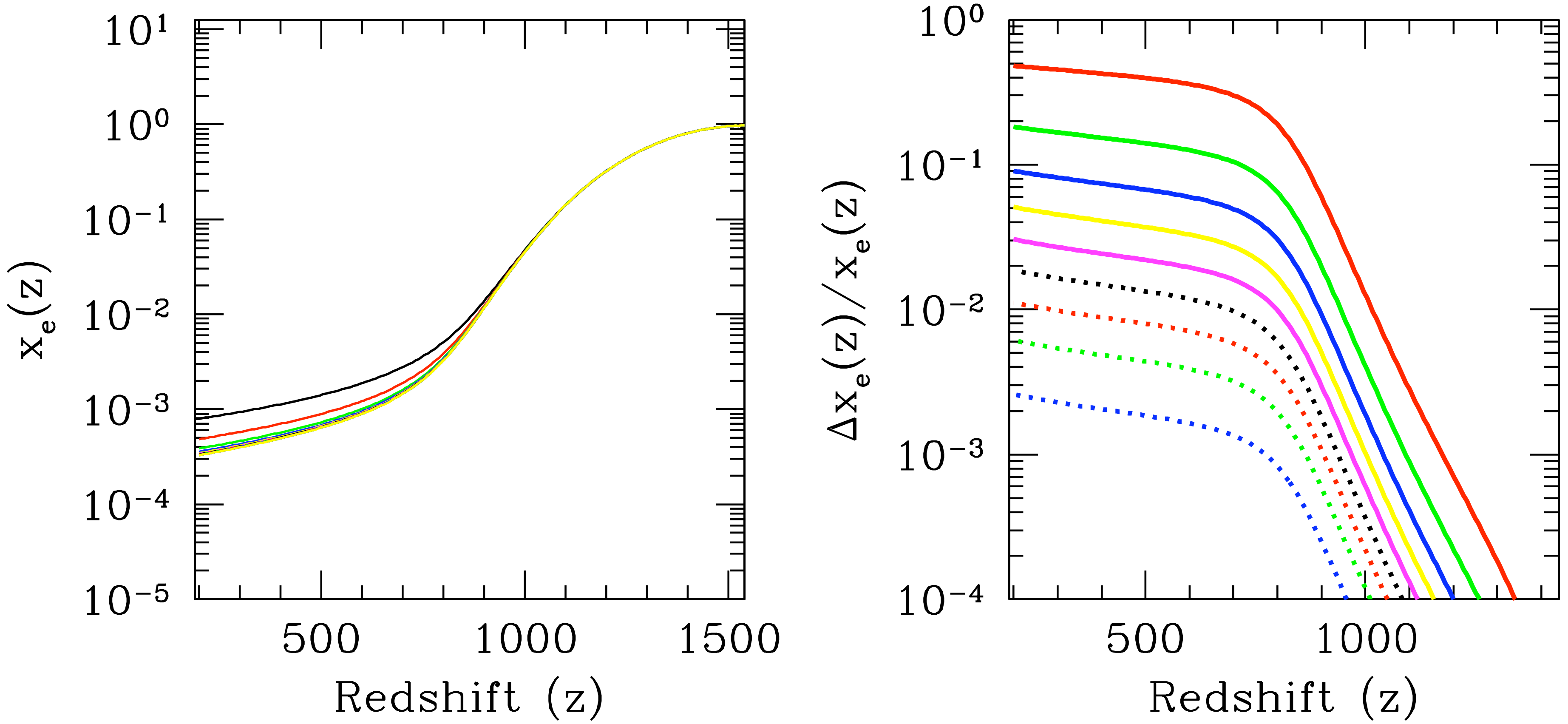
$$\vec{v}_l = \chi_\mathbf{l}\left[\vec{s}_l - \mathbf{M_{l,l+1}}\vec{v}_l + \Sigma_{l'=l-1}^{0}\sigma_{l,l'}\vec{s}_{l'}\left(-1\right)^{l'-l}\right]$$

$$\chi_l = \begin{cases} \mathbf{M}_{00}^{-1} & \text{if } l = 0 \\ \left(\mathbf{M}_{l+1,l+1} - \mathbf{M}_{l+1,l}\chi_l\mathbf{M}_{l,l+1}\right)^{-1} & \text{if } l > 0 \end{cases}$$

$$\sigma_{l,l-1} = \mathbf{M}_{l,l-1}\chi_{l-1}$$

$$\sigma_{l,i} = \sigma_{l,i+1}\mathbf{M}_{i+1,i}\chi_i$$

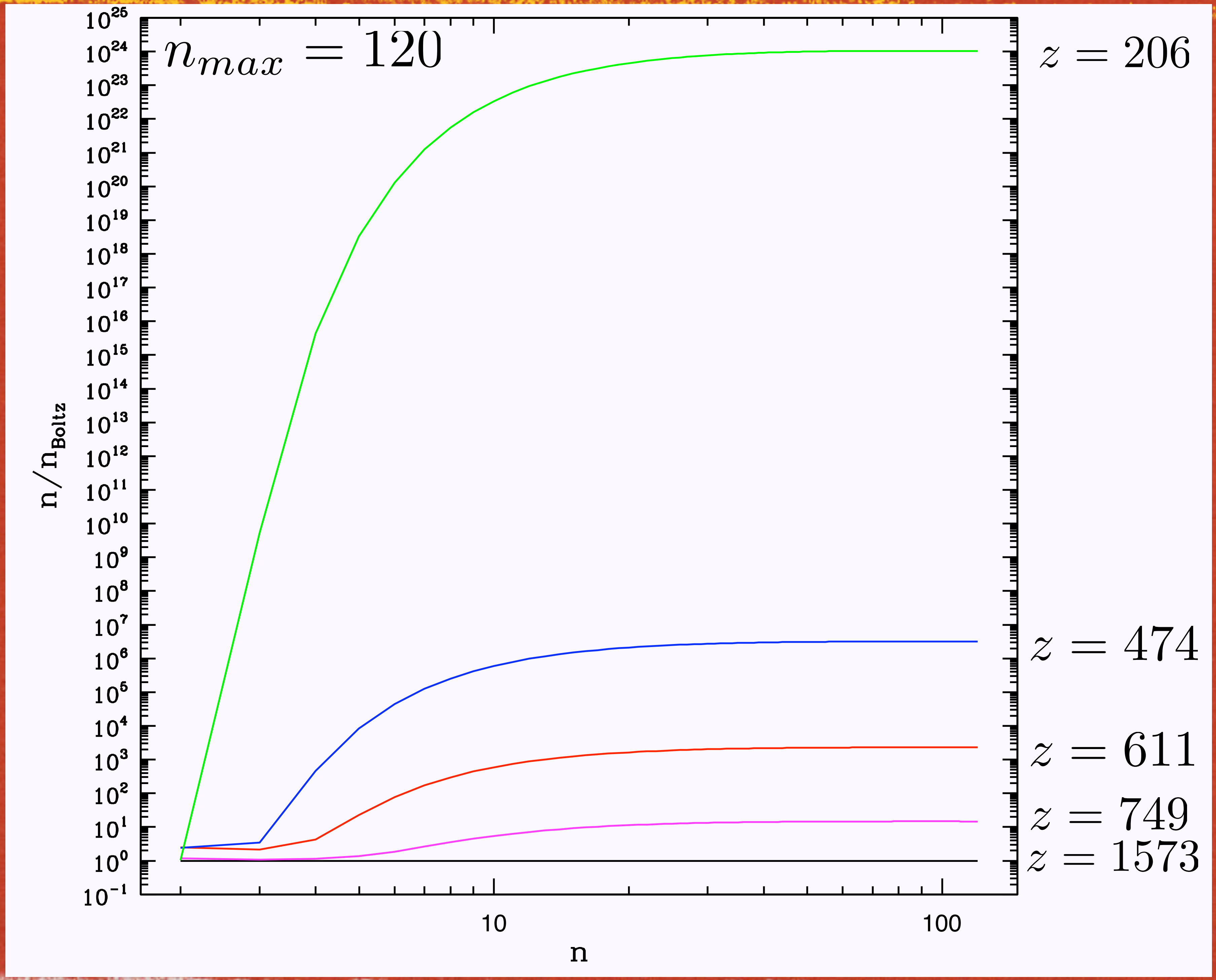
Recombination histories



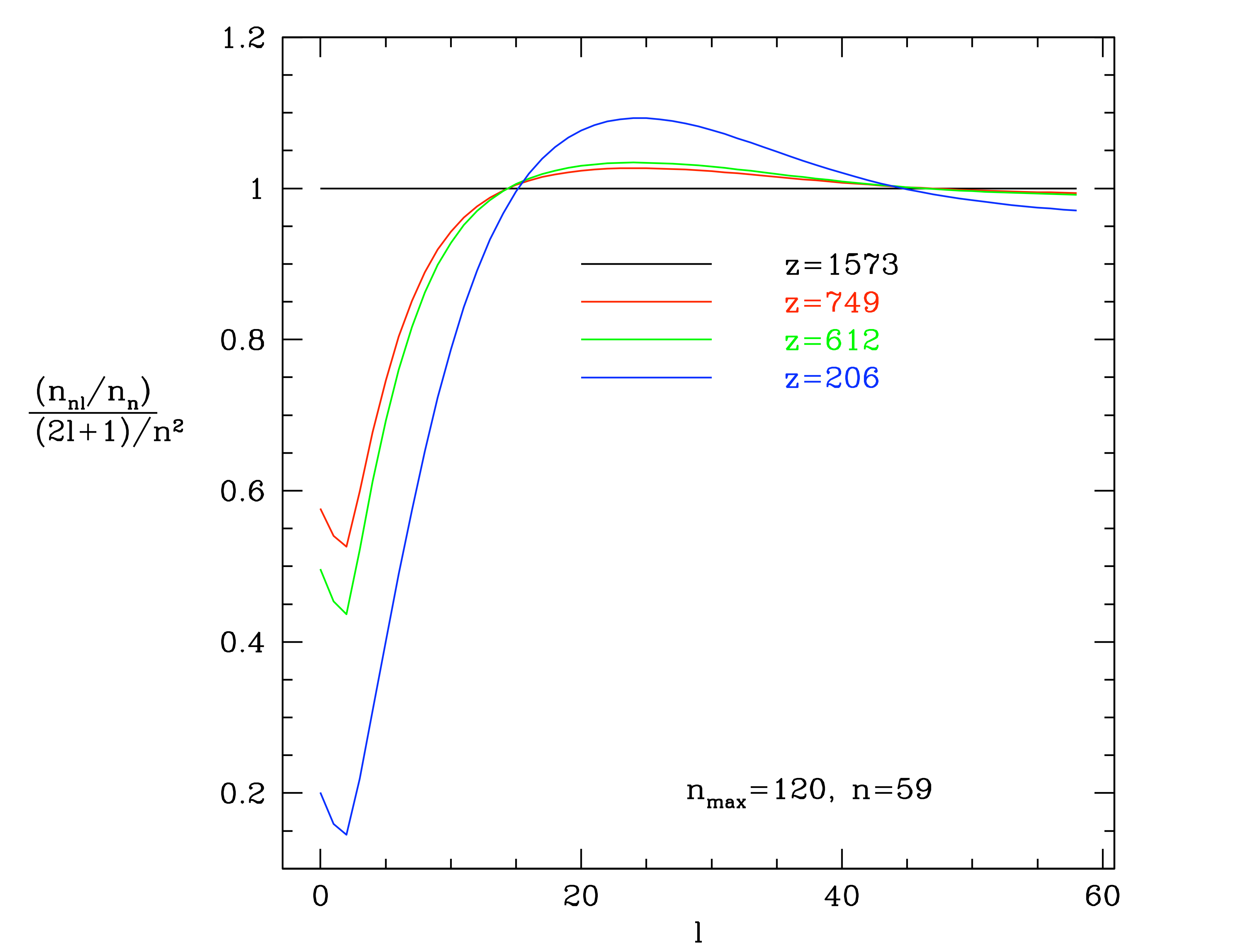
- $x_e(z)$ falls with increasing $n_{max} = 10 \rightarrow 100$, as expected:
- Rec Rate>downward BB Rate> Ionization, upward BB rate
- Even for $n_{max} = 100$, code computes in only 2 hours

DEVIATIONS FROM BOLTZMANN EQ: HIGH-N

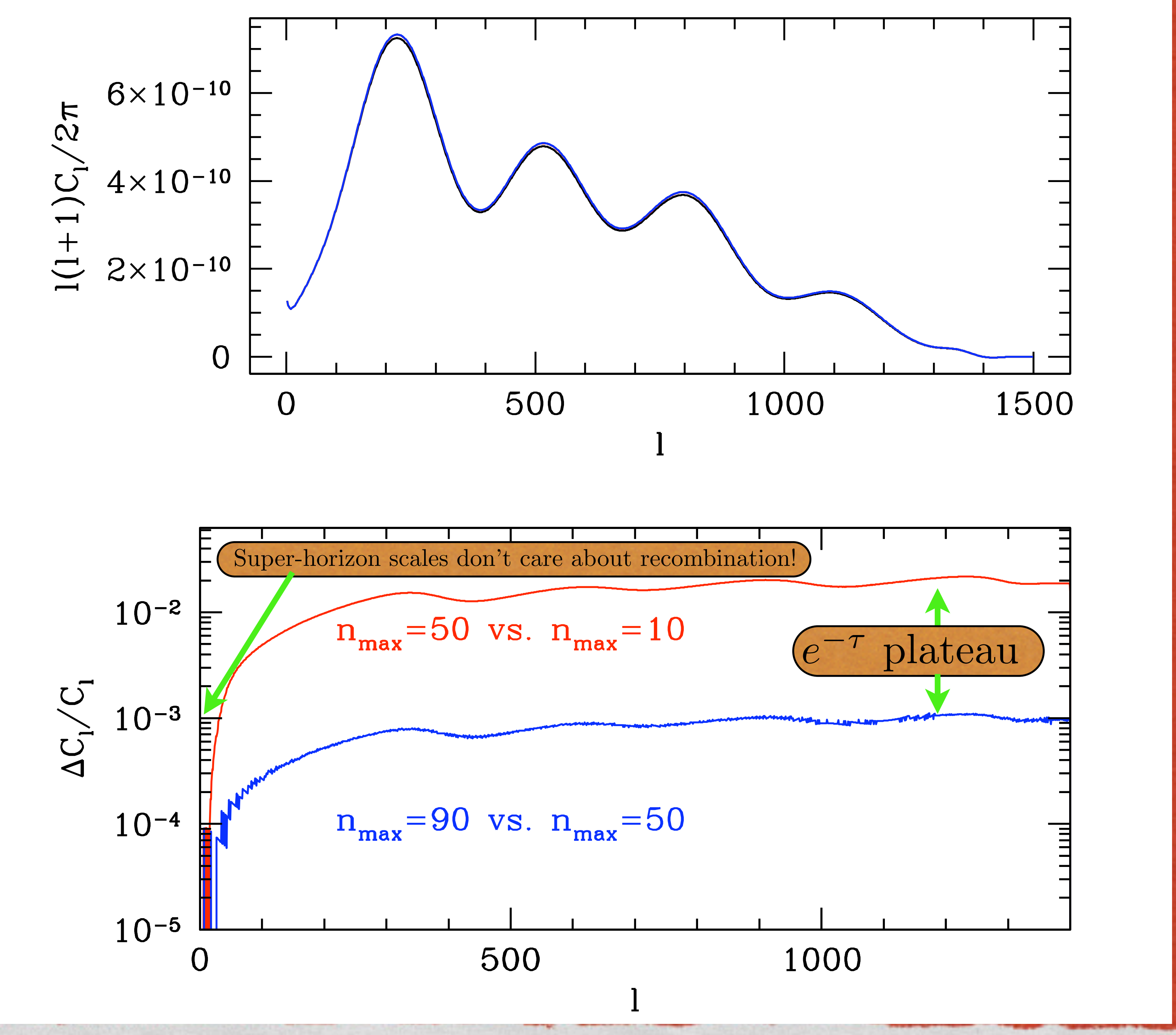
- $\alpha n \gtrsim A_{\mathrm{bb,down}}$.



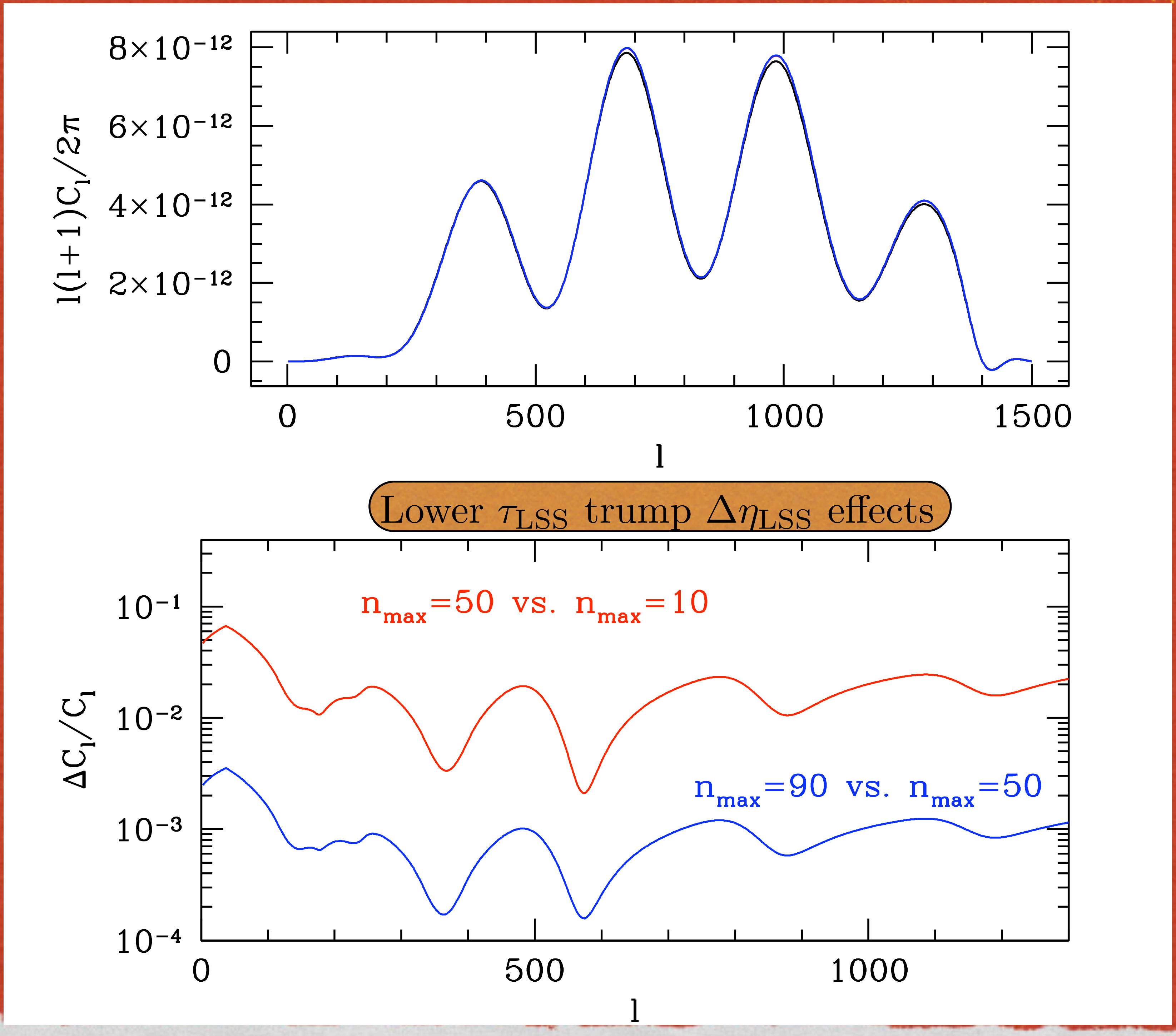
Deviations from Boltzmann eq: resolving l



TEMPERATURE $C_l s$



POLARIZATION $C_l s$



WRAPPING UP

- To do:
  - Sobolev iteration and higher lines
  - Collisions!
  - Effective source term for higher levels
  - Full incorporation into CMBFAST/CAMB and analysis of errors/degeneracies with cosmo. parameters