Cosmological Hydrogen Recombination: The effect of extremely high-n states

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Daniel Grin

in collaboration with Christopher M. Hirata

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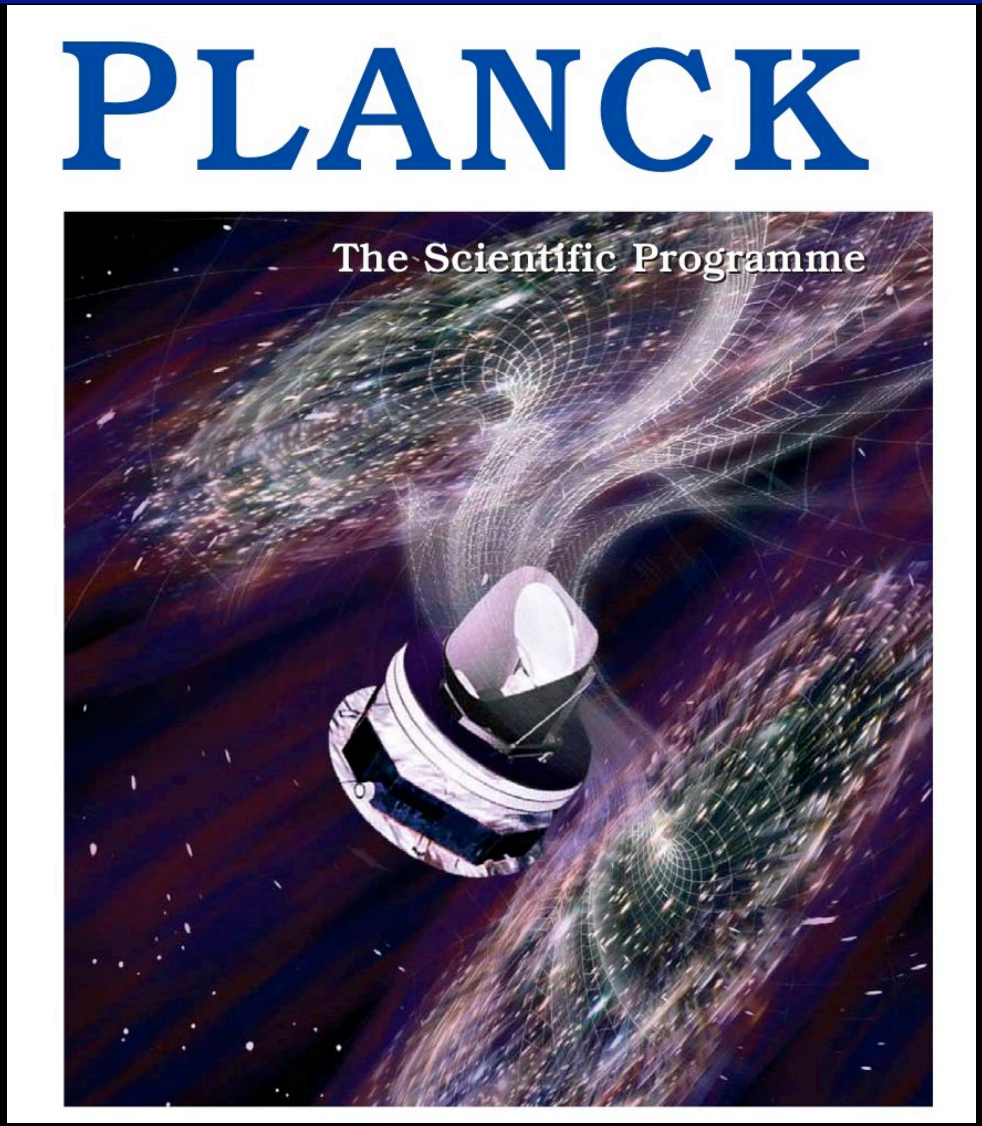
CLONE WARS



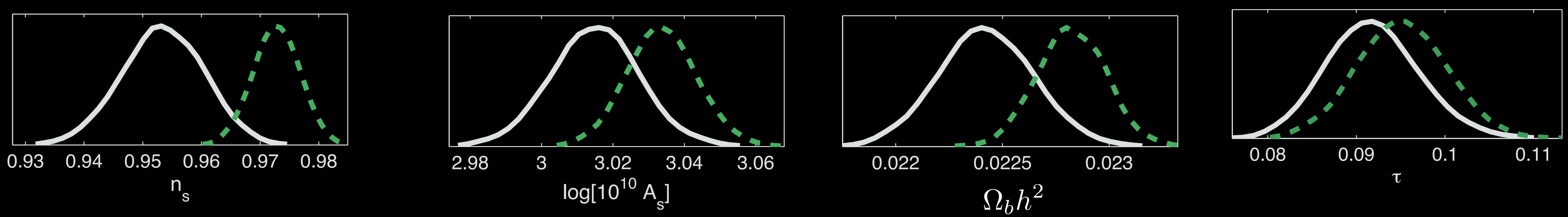
- Planck (launched May 2009) will make cosmic-variance limited CMB anisotropy measurements up to l~2500 (T), and l~1500 (E)

- Wong 2007 and Lewis 2006 show that $x_e(z)$ needs to be predicted to 0.1% accuracy for Planck data analysis

RECOMBINATION, INFLATION, AND REIONIZATION

$$P(k) = A_s \left(k\eta_0\right)^{n_s}$$

- Planck uncertainty forecasts using MCMC (Wong/Moss/Scott 2007)



- Leverage on new physics comes from high l. Here the details of recombination matter!

- Inferences about inflation will be wrong if recombination is improperly modeled

$$\mathrm{n}_s = 1 - 4\epsilon + 2\eta \qquad \epsilon = \frac{m_{\rm pl}^2}{16\pi}\left[\frac{V'(\phi)}{V(\phi)}\right]^2 \qquad \mathrm{A}_s^2 = \frac{32}{75}\frac{V}{m_{\rm pl}^4\epsilon}\Bigg|_{k_{\rm pivot}=aH}$$

CAVEAT EMPTOR: $3 \lesssim \ ? \ \lesssim 16$

Need to do eV physics right to infer anything about $10^?$ GeV physics! 3

EQUILIBRIUM ASSUMPTIONS

✳Radiative/collisional eq. between different l

$$\mathcal{N}_{nl} = \mathcal{N}_n \frac{(2l+1)}{n^2}$$

✳ Radiative eq. between different n-states

$$\mathcal{N}_n = \sum_l \mathcal{N}_{nl} = \mathcal{N}_2 e^{-(E_n - E_2)/T}$$

Non-eq rate equations

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Seager/Scott/Sasselov 2000/RECFAST!

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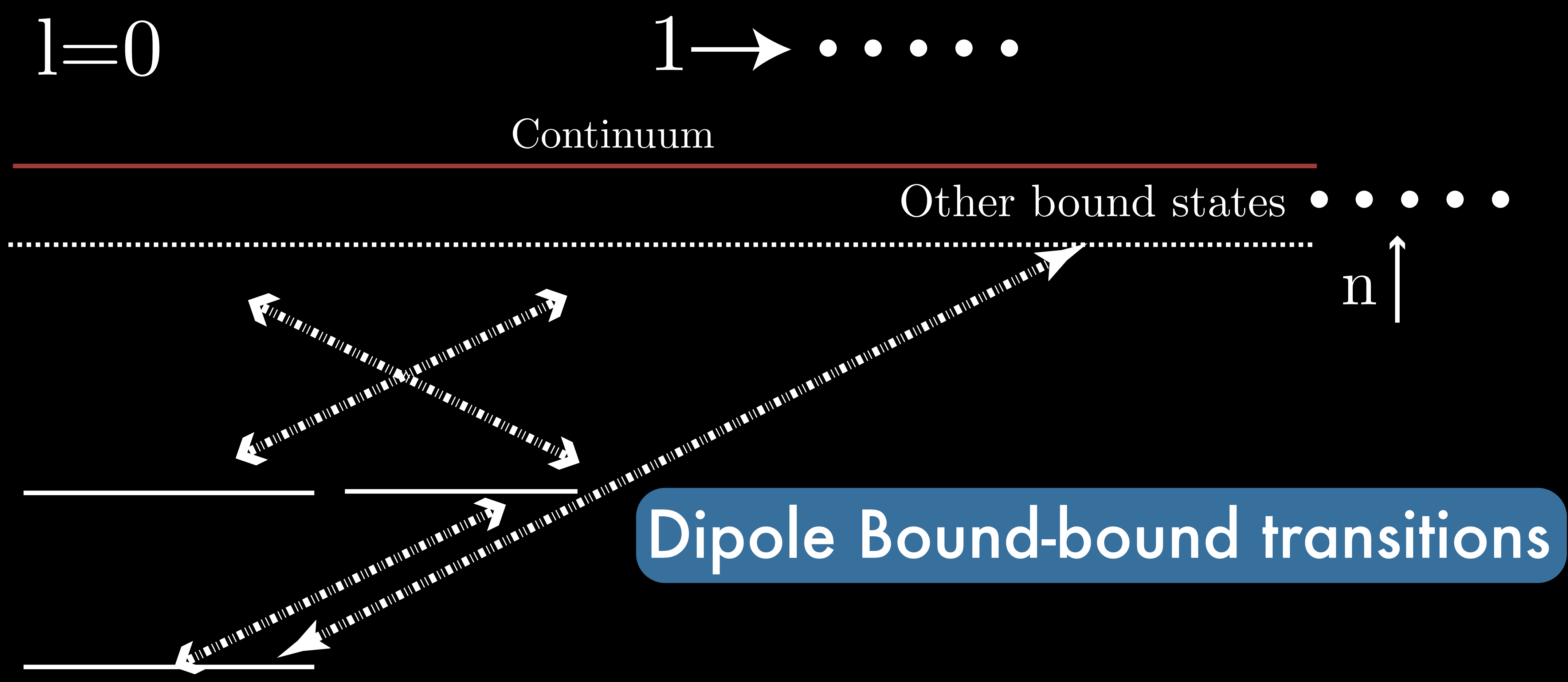
BREAKING EQUILIBRIUM

- Chluba et al. (2005,6) follow l, n separately, get to $n_{\rm max} = 100$
- 0.1 %-level corrections to CMB anisotropies at $n_{\rm max} = 100$
- ~~Equilibrium~~ between *l states:* $\Delta l = \pm 1$ bottleneck
- Beyond this, testing convergence with $n_{\rm max}$ is hard!

$t_{\rm compute} \sim \mathcal{O}\,(\text{years})$ for $\text{n}_{\rm max} = 300$

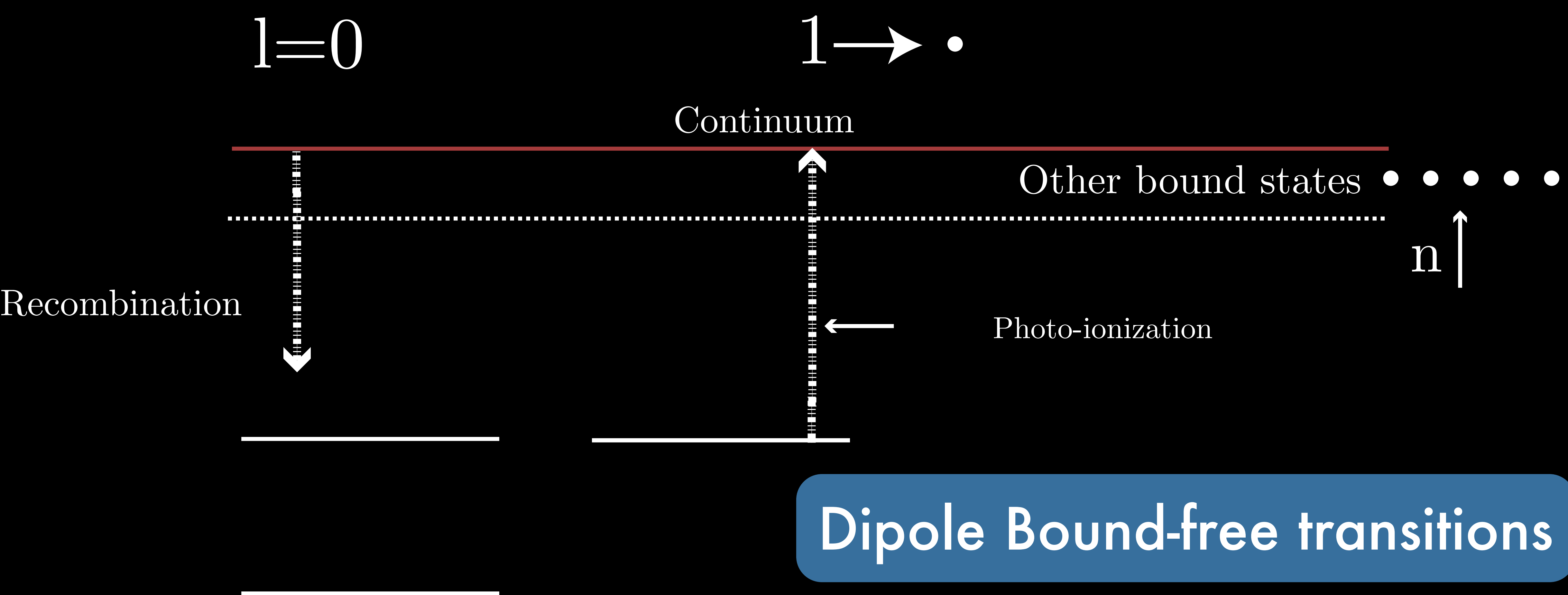
How to proceed if we want 0.01% accuracy in $x_e(z)$?

RecSparse and the multi-level atom



- We implement a multi-level atom computation in a new code, RecSparse!
- Boltzmann eq. solved for T_m (T_γ)
- Spontaneous/stimulated emission/absorption included

RECSPARSE AND THE MULTI-LEVEL ATOM



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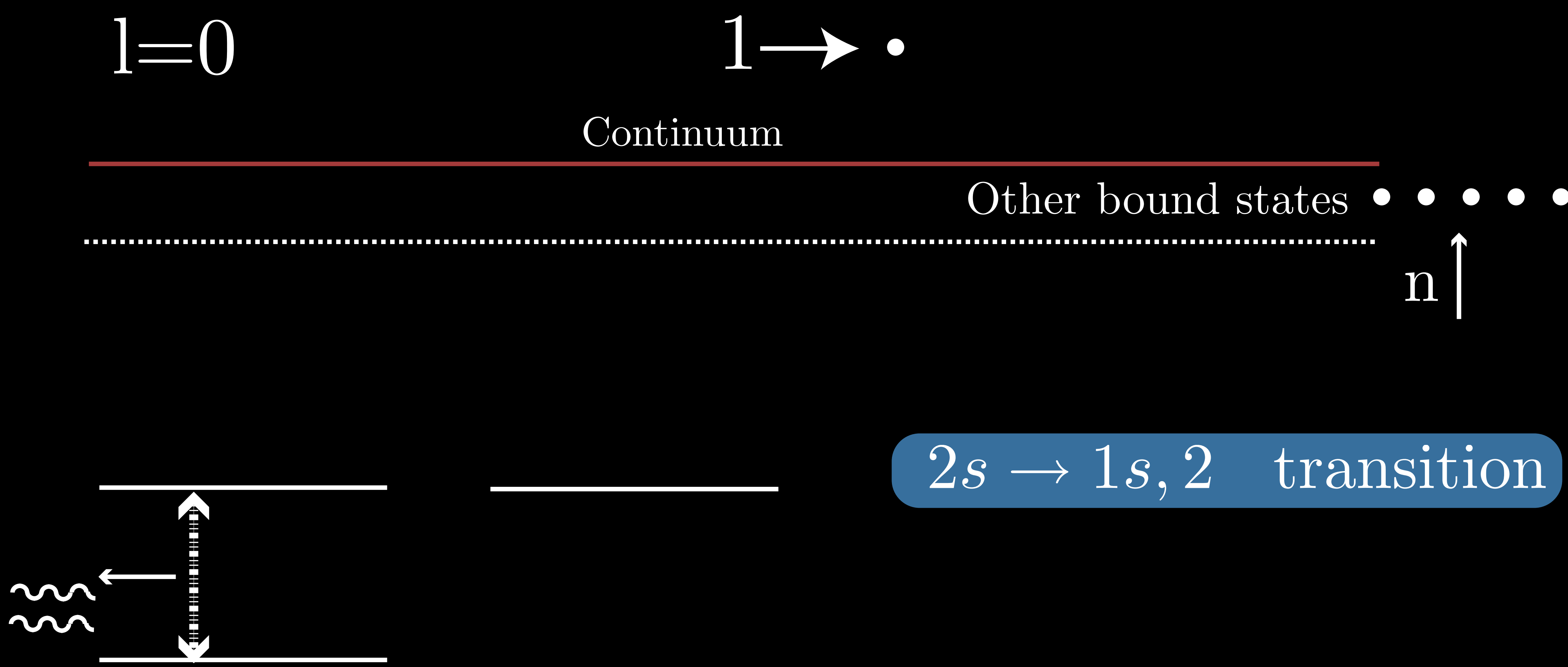

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RECSPARSE AND THE MULTI-LEVEL ATOM

✴ Free electron fraction evolved according to

$$\begin{aligned} \dot{x}_e &= -\dot{x}_{1s} \\ &= -\Lambda_{2s\to 1s}\left(x_{2s} - x_{1s}e^{-E_{2s\to 1s}/T_\gamma}\right) + \sum_{n,l>1s} A_{n1}^{l\,0} P_{n1}^{l0}\left\{g(T,n,l)\right\} \end{aligned}$$

2s-1s decay rate

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RecSparse and the multi-level atom

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Lyman series current to ground state

RecSparse and the multi-level atom

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Einstein coeff.

RecSparse and the multi-level atom

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$$
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\end{aligned}
$$

Escape probability

Other corrections to recombination

- Deviations from steady-state approx (Chluba/Sunyaev 2008)
- Coherent scattering (Forbes and Hirata 2009, Switzer/Hirata 2007)
- Atomic recoil (Forbes and Hirata 2009, Dubrovich and Grachev 2008)
- Feedback from hydrogen/helium (Chluba/Sunyaev 2007)
- Higher-n two-photon processes (Chluba/Sunyaev 2007, Hirata 2008) in hydrogen and Helium (Switzer/Hirata 2007)

Steady-state for excited levels

- Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

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$$\vec{x}_l = \begin{pmatrix} x_{l,l+1} \\ \cdots \\ x_{l,n_{\max}} \end{pmatrix}$$

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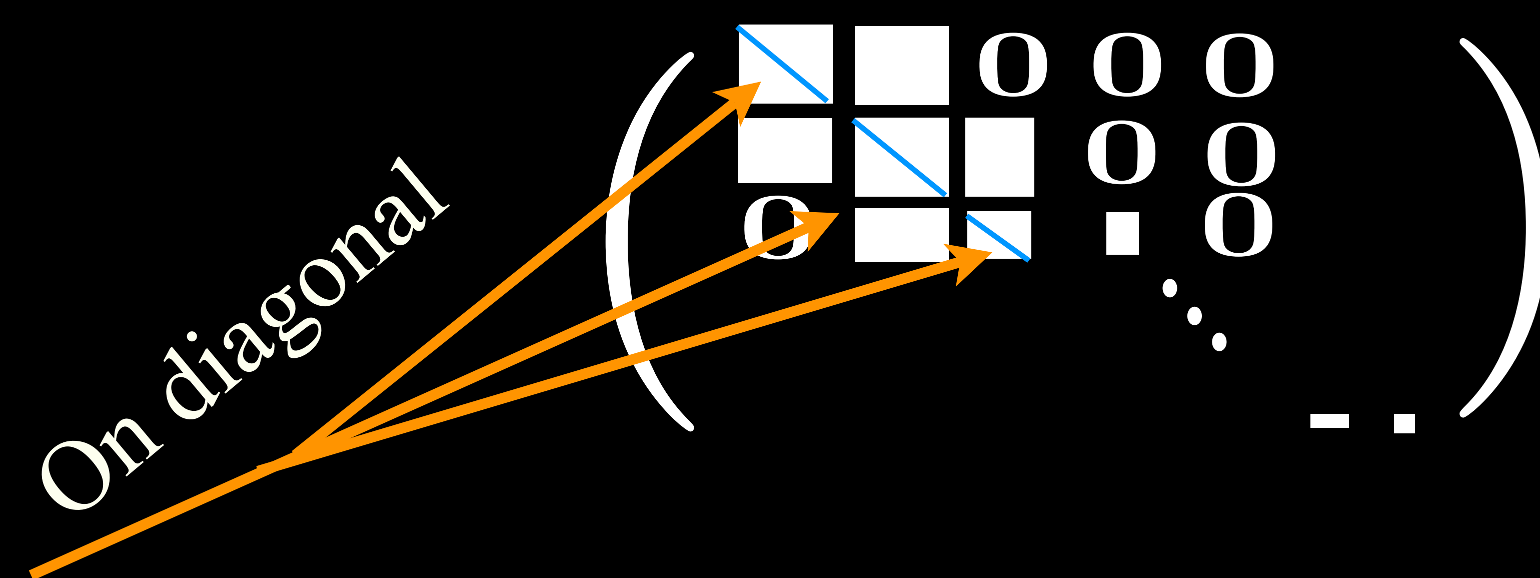

For state l, includes BB transitions out of l to all other l”, photo-ionization, 2γ transitions to ground state

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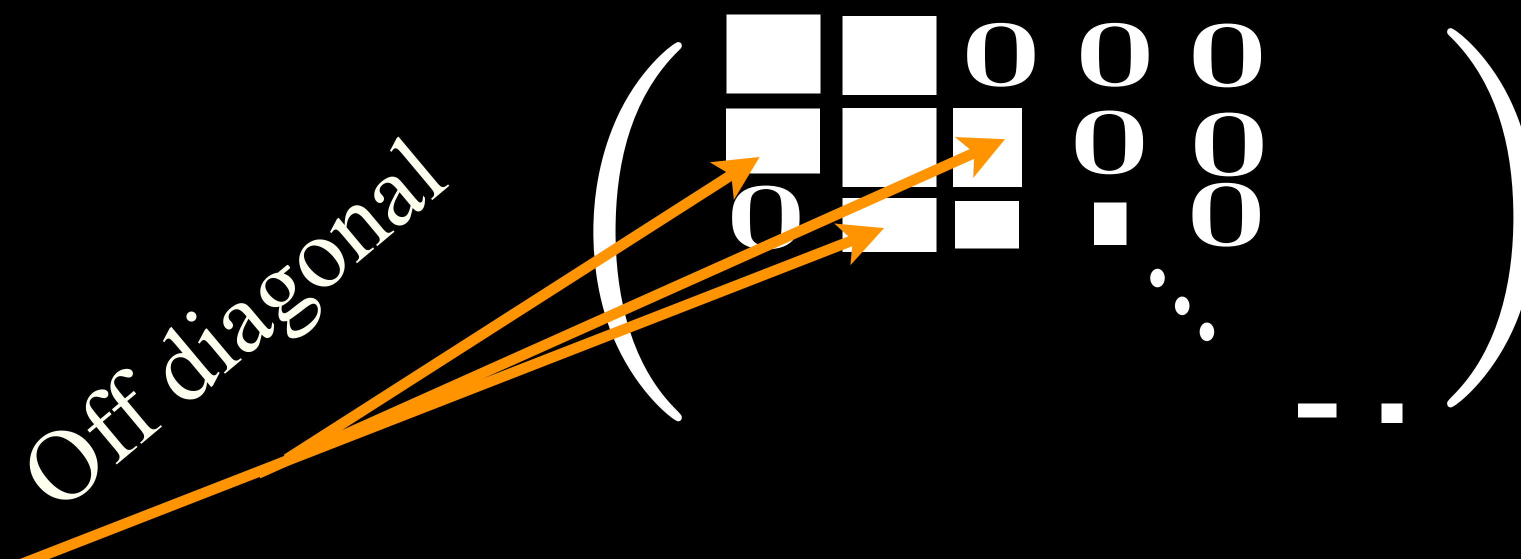

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STEADY-STATE FOR EXCITED LEVELS

- Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

Includes recombination to l,
1 and 2γ transitions from ground state

Steady-state for excited levels

- Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

For n>1, $\mathbf{R}$, $\vec{s} \geq 1\ \mathrm{s}^{-1}$ e.g. Lyman-α

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Steady-state for excited levels

- Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

$t_{\rm rec}^{-1} \sim 10^{-12}\ {\rm s}^{-1}$

For n>1, $\mathbf{R}$, $\vec{s} \geq 1\ {\rm s}^{-1}$ e.g. Lyman-α

STEADY-STATE FOR EXCITED LEVELS

- Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

LHS $\ll$ RHS

$$\vec{x} \simeq -\mathbf{R}^{-1}\vec{s}$$

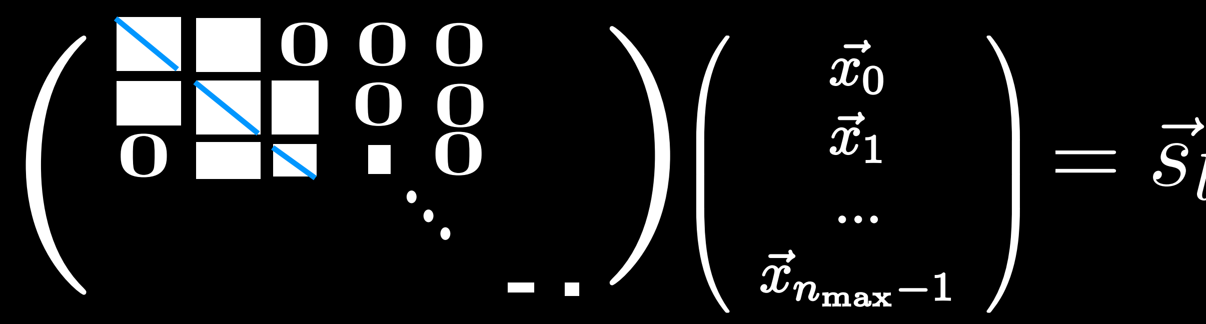
$t_{\rm rec}^{-1} \sim 10^{-12}\ {\rm s}^{-1}$

For n>1, $\mathbf{R}$, $\vec{s} \geq 1\ {\rm s}^{-1}$ e.g. Lyman-α

RAPID MATRIX INVERSION: SPARSITY TO THE RESCUE

- Matrix is $\sim n^2_{max} \times n^2_{max}$
- Dipole selection rules: $\Delta l = \pm 1$

$$\mathrm{M}_{l,l-1}\vec{x}_{l-1} + \mathbf{M}_{l,l}\vec{x}_l + \mathbf{M}_{l,l+1}\vec{x}_{l+1} = \vec{s}_l$$



- RecSparse computation time ~ $n_{max}^{2.5}$: Huge improvement!
- Case of $n_{\rm max} = 100$ runs in less than a day, $n_{\rm max} = 200$ takes ~ 4 days.
- Purely radiative model, collisions would push closer to eq., but theoretical rates are in bad shape!

FORBIDDEN TRANSITIONS AND RECOMBINATION

- Higher-n 2γ transitions in H important at 7-σ for Planck (TT/EE) data analysis (Hirata 2008, Kholupenko 2006)
- Some forbidden transitions are important in Helium recombination (Dubrovich 2005, Lewis 2006).
- ***Unfinished business***: *Are other forbidden transitions in hydrogen important, particularly for Planck data analysis? Maybe quadrupole transitions, since they are optically thick?*

QUADRUPOLE TRANSITIONS AND RECOMBINATION

- Lyman lines are optically thick, so $nd \rightarrow 1s$ immediately followed by $1s \rightarrow np$, so this can be treated as an effective $d \rightarrow p$ process with rate $A_{nd\rightarrow 1s} x_{nd}$.

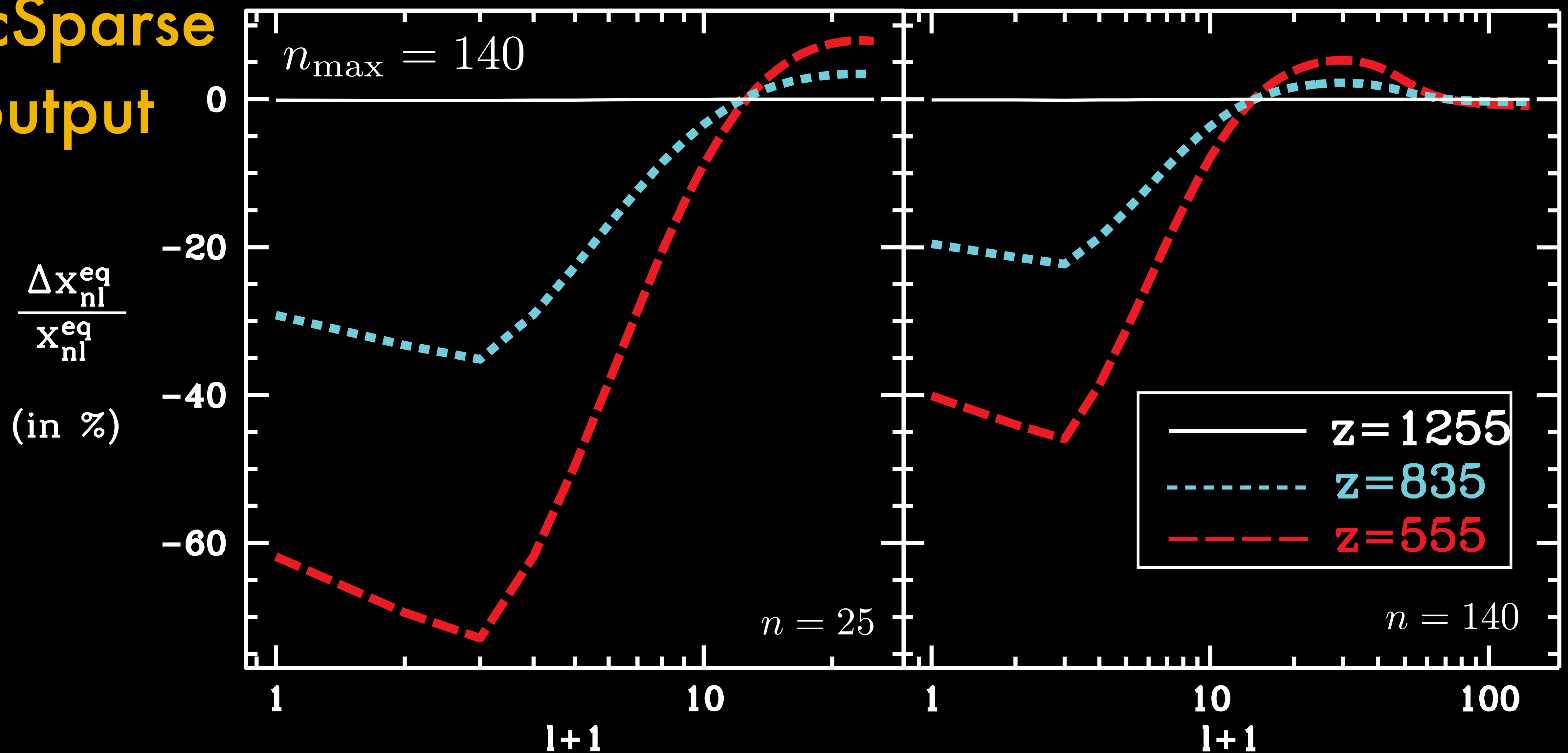
- Same sparsity pattern of rate matrix, similar to l-changing collisions

- Detailed balance yields net rate

$$R^{\mathrm{quad}}_{nd\rightarrow np} = A_{nd\rightarrow 1s}\left(x_{nd} - \frac{5}{3}x_{np}\right)$$

DEVIATIONS FROM BOLTZMANN EQ: L-SUBSTATES

RecSparse output



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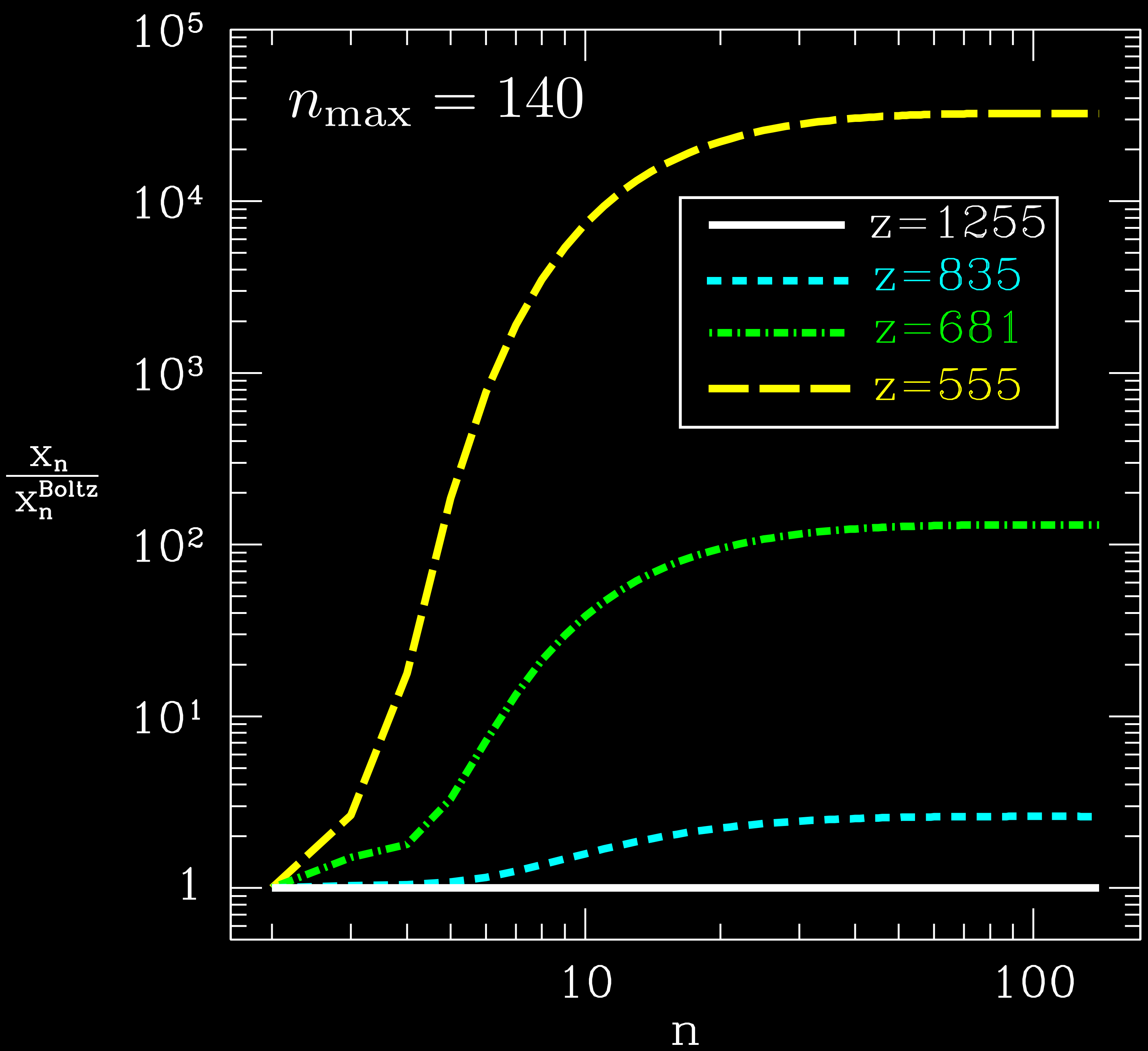
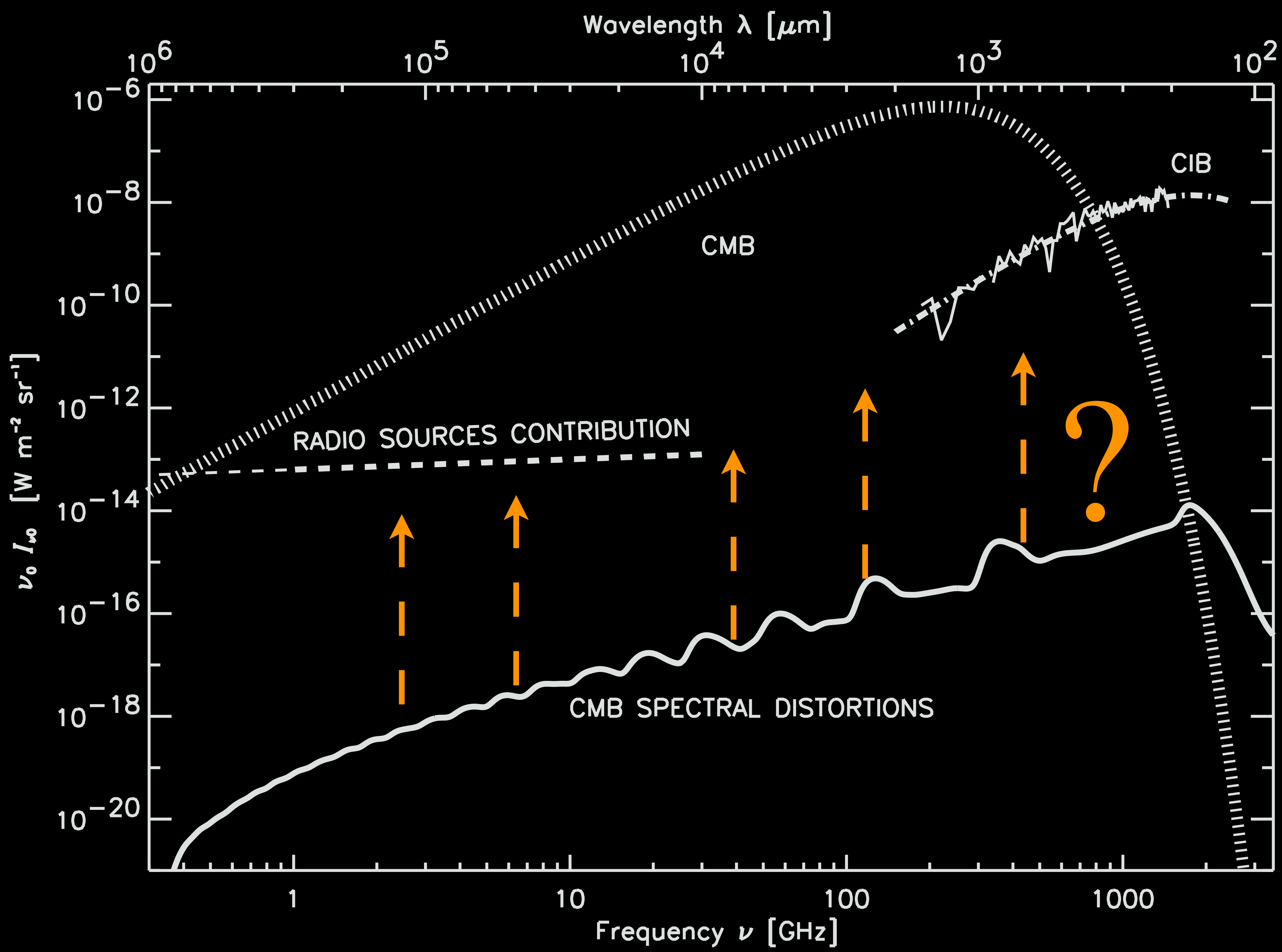
DEVIATIONS FROM BOLTZMANN EQUILIBRIUM: DIFFERENT n-SHELLS



$$\alpha_n n_e > \sum_{n'l}^{n'<n} A_{nn'}^{ll\pm1}$$

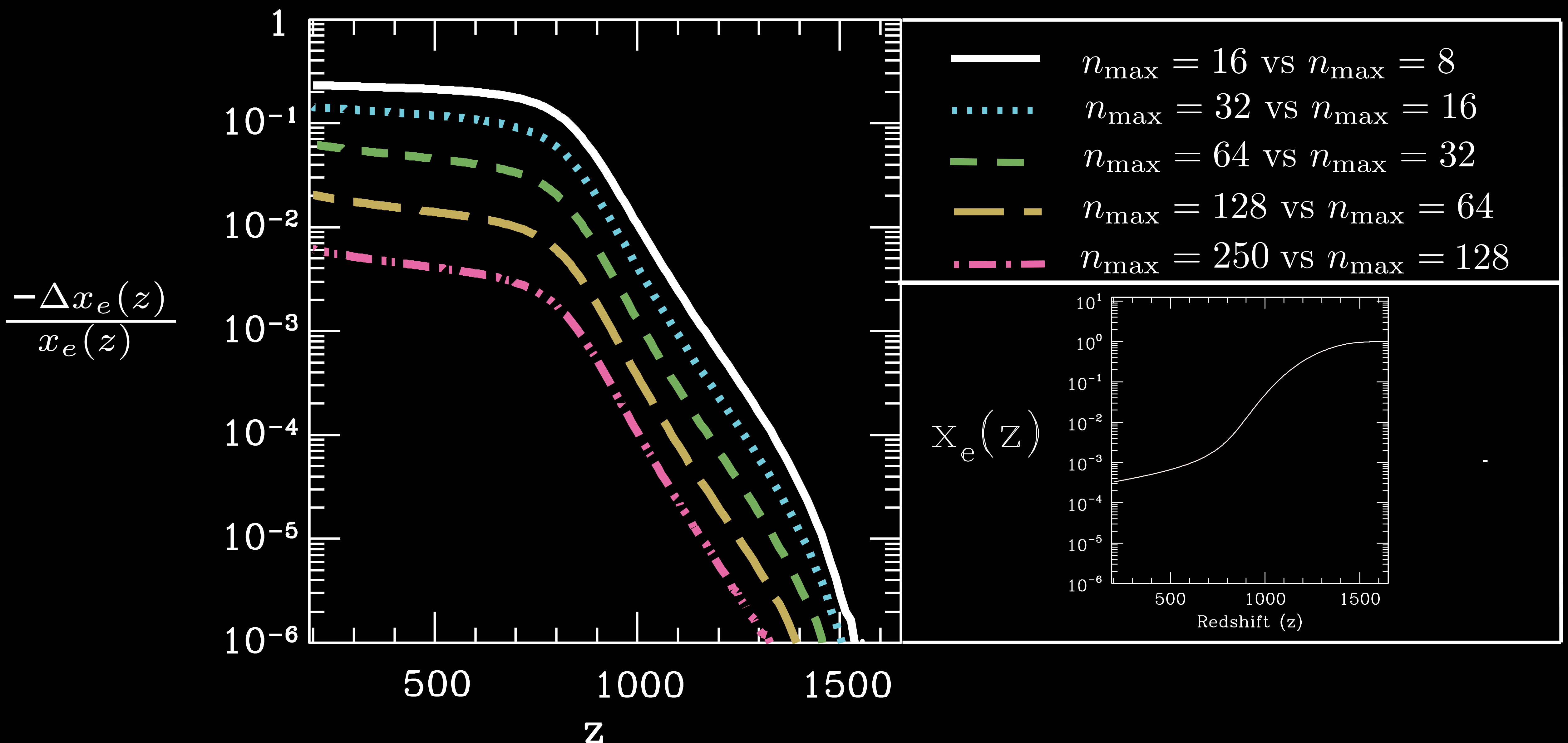
- No inversion relative to n=2 (just over-population)
- Population inversion seen between some excited states: Does radiation stay coherent? Does recombination mase?

DEVIATIONS FROM BOLTZMANN EQUILIBRIUM: DIFFERENT n-SHELLS



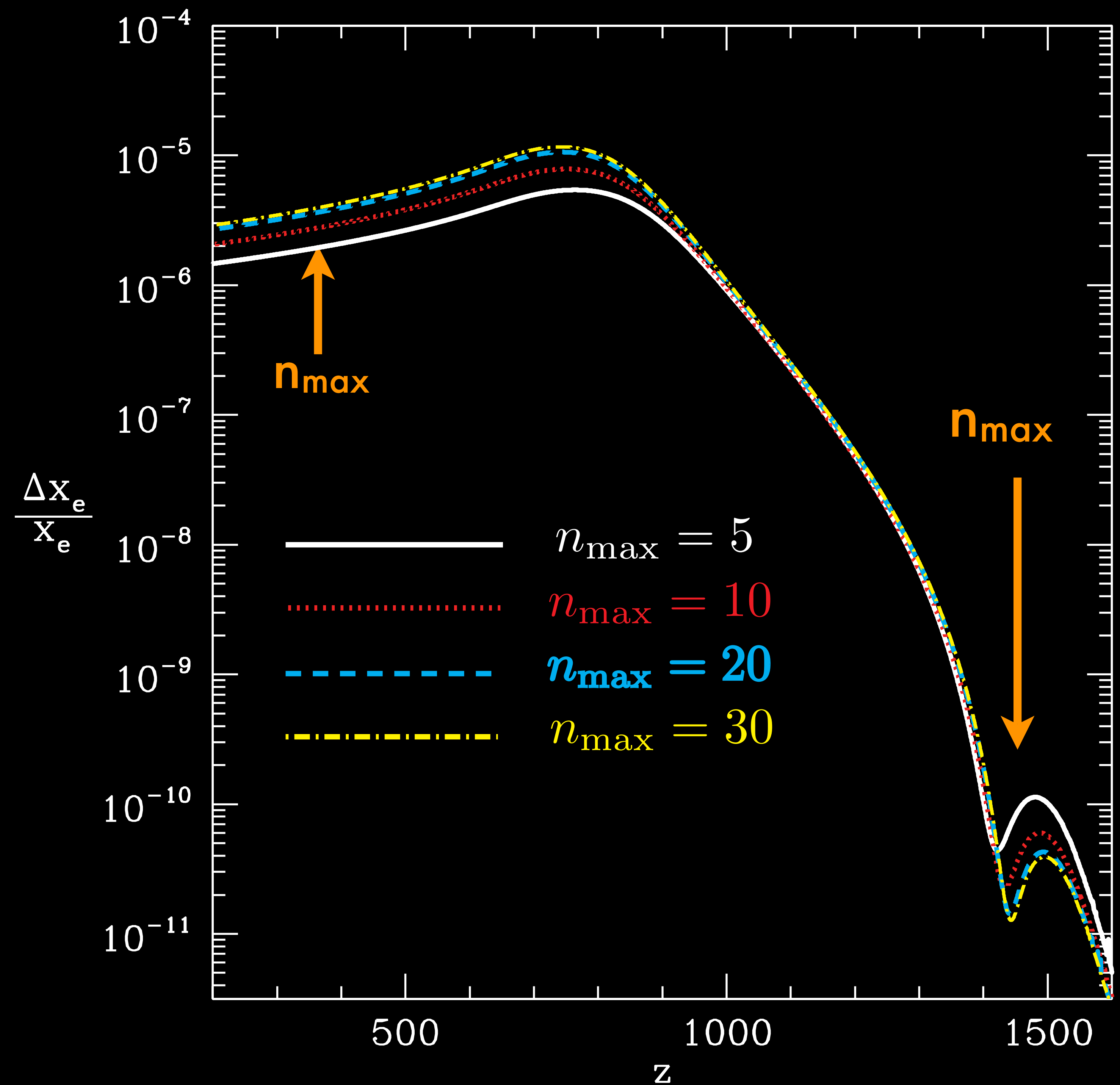
Masing could make spectral distortions detectable!

RESULTS: RECOMBINATION HISTORIES INCLUDING HIGH-n



- $x_e(z)$ falls with increasing $n_{\rm max} = 10 \rightarrow 250$, as expected.
- Rec Rate>downward BB Rate> Ionization, upward BB rate
- For $n_{max} = 100$, code computes in only 2 hours

RESULTS: RECOMBINATION WITH HYDROGEN

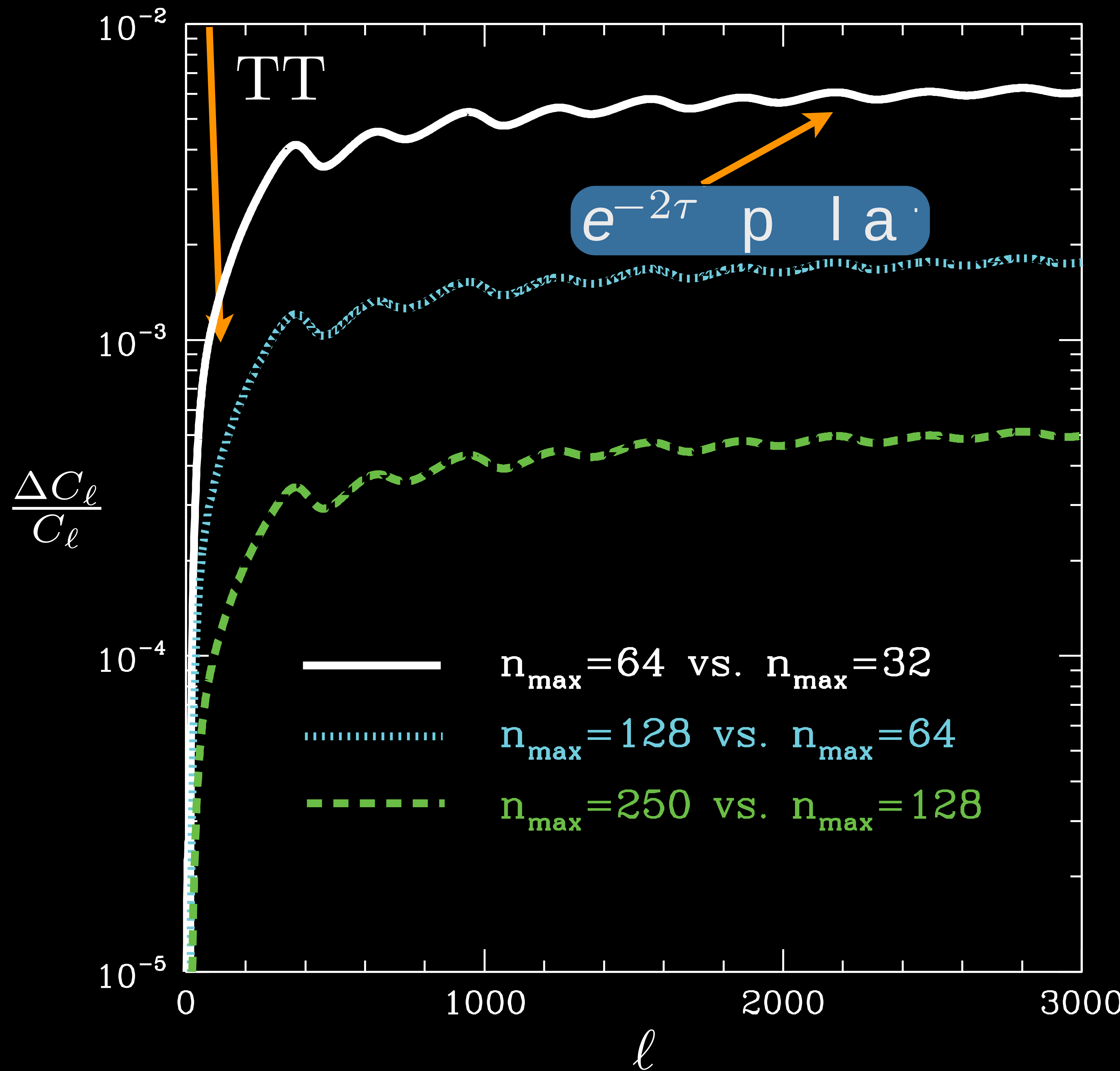


$$\Delta x_e \equiv x_e\big|_{\text{no } E2 \text{ transitions}} - x_e\big|_{\text{with } E2 \text{ transitions}}$$

Negligible for Planck!

RESULTS: TT $C_l s$ WITH HIGH-N STATES

Super-horizon scales don't care about recombination



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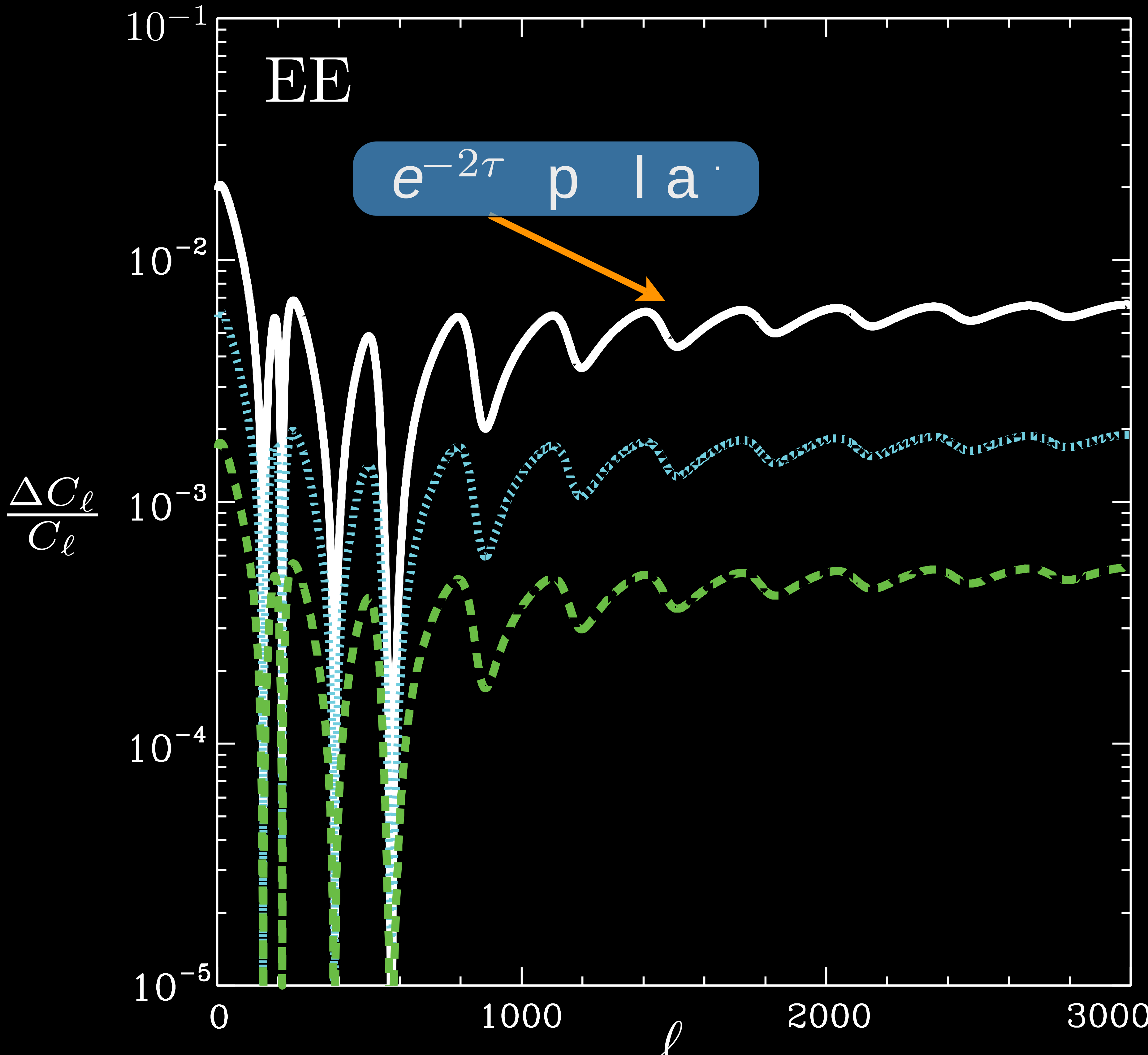
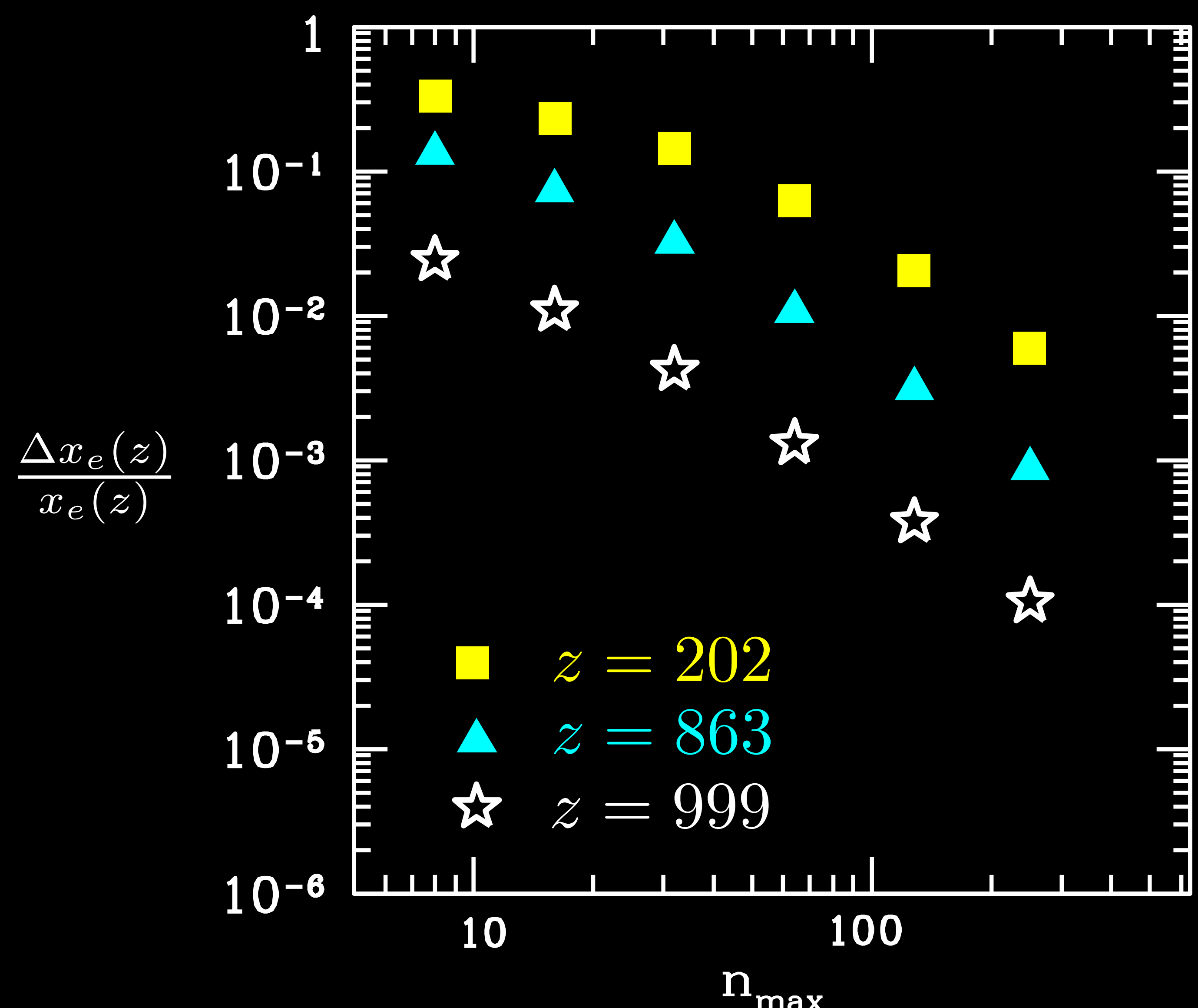
RESULTS: EE $C_l s$ WITH HIGH-N STATES



CONVERGENCE



- Relative error well described by power law at high $n_{\max}$

$$\Delta x_e / x_e \propto n_{\max}^{-1.9}$$

- Can extrapolate to absolute error

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THE UPSHOT FOR COSMOLOGY

- Can explore effect on overall Planck likelihood analysis

$$Z^2 = \sum_{ll',X,Y} F_{ll'} \Delta C_l^{\mathrm{X}} \Delta C_l^{\mathrm{Y}}$$

$$Z = 1.8 \text{ if } n_{\max} = 64,$$
$$Z = 0.50 \text{ if } n_{\max} = 128,$$
$$Z = 0.14 \text{ if } n_{\max} = 250.$$

WRAPPING UP

- RecSparse: a new tool for MLA recombination calculations
  - Highly excited levels (n~64 and higher) are relevant for CMB data analysis
  - E2 transitions in H are not relevant for CMB data analysis
- Future work:
  - Include line-overlap
  - Collisions/high-n cutoff
  - Fisher/Monte-Carlo analyses
  - Compute rec. line. spectra (masers?)